Thermal Instability of Radiative Thermally Conducting Viscous Plasma with FLR Corrections for Star formation

Sachin Kaothekar¹*
¹Department of Physics, Mahakal Institute of Technology, Ujjain (M.P.) - 456664, India.

*Corresponding authors e-mail: sac_kaothekar@rediff.com, sackaothekar@gmail.com,

The effect of radiative heat-loss function and finite ion Larmor radius (FLR) corrections on thermal instability of infinite homogeneous viscous plasma has been investigated incorporating the effects of thermal conductivity, finite electrical resistivity for star formation in interstellar medium (ISM). A general dispersion relation is derived using the normal mode analysis method with the help of relevant linearized perturbation equations of the problem. The wave propagation along and perpendicular to the direction of magnetic field has been discussed. Stability of the medium is discussed by applying Routh Hurwitz's criterion. We find that the presence of FLR corrections, radiative heat-loss function and thermal conductivity modifies the fundamental criterion of thermal instability. Numerical calculations have been performed to show the effect of various parameters on the growth rate of the thermal instability. From the curves we find that heat-loss function and FLR corrections have stabilizing effect on the growth rate of thermal instability. Our results are applicable in understanding the star formation in galaxies.

KEY WORDS: Star formation Thermal instability, radiative heat-loss function, thermal conductivity, FLR corrections.

1. INTRODUCTION

Thermal instability is one of the most interesting phenomenon in the interstellar medium (ISM) for star formation. When a positive temperature perturbation is made in a thermal unstable medium, the perturbation grows and the emission rate decreases. This process is thought to be possible in a number of astrophysical situations such as the gas in clusters of the galaxies, in the solar corona and in the interstellar medium. What is less clear is the relative importance of this process in various circumstances. Thermal instability has many applications in astrophysical situations (e.g. a clumpy interstellar medium, stellar atmosphere, star formation, globular clusters and galaxy formation and many more situations Meerson 1966). The instability may be driven by radiative cooling of optically thin gas system or by exothermic nuclear reactions (Schwarzschild & Harm 1965).

Linear stability theory for a diluted gas medium with volumetric sources and sink of energy in thermal equilibrium was developed by Field (1965); he identified three unstable modes, the isobaric mode (the pressure driven formation of condensations not involving gravitation) and the two isentropic modes (the over stability of acoustic wave propagation in opposite directions). Hunter (1970, 1971) extended these results to an arbitrary non-stationary background flows, showing that cooling dominates media are potentially more unstable then that in equilibrium, while heating provides stabilization. The most common applications of thermal instability to interstellar medium and star formation deal with the isobaric mode that was employed to explain the observed multi phase structure of the interstellar medium (Field 1965, Pikel’ner 1968,
Goldsmith & Habing (1969), Wolfirc et al. (1995). In this direction Aggarwal and Talwar (1969) have discussed magneto-thermal instability in a rotating gravitating fluid. Sharm & Prakash (1975) have investigated radiative transfer and collisional effects on thermal convective instability of a composit medium. McCray & Stien (1975) have carried out the investigation of thermal instability in supernova shell. Nusel (1986) has discussed the thermal instability in cooling flows. Panavano (1988) has studied self regulating star formation in isolated galaxies: thermal instability in the interstellar medium. Labnez & Sanchez (1992) have studied the propagation of sound and thermal waves in plasma with solar abundance. Bora and Talwar (1993) have investigated the magneto-thermal instability with finite electrical resistivity and Hall current, both for self-gravitating and non-gravitation configurations. Prajapati et al. (2010) have discussed the effect of radiative heat-loss function and thermal conductivity on gravitational instability of fully ionized plasma with electron inertia, Hall current, rotation and viscosity. Szunzkiewicz & Millar (1997) have investigated the thermal stability of transonic accretion discs. Najad-Asghar & Ghanbari (2003) have carried out linear thermal instability and formation of clumpy gas clouds including the ambipolar diffusion. Vasiliev (2012) has investigated the thermal instability in a collisionally cooled gas. Najad-Asghar (2007) has investigated the formation of fluctuations in a molecular slab via isobaric thermal instability. Stiele et al. (2006) have carried out the problem of thermal instability in a weakly ionized plasma. Nipotic (2010) has investigated thermal instability in rotating galactic coronae. Hobbs et al. (2012) have discussed thermal instability in cooling galactic fuelling star formation in galactic discs. Nipoti & Posti (2013) have investigated thermal instability of a weakly magnetized rotating plasma. Choudhary & Sharma (2016) have discussed cold gas in cluster core: global stability analysis and non linear simulations of thermal instability.

Along with this in above discussed problems the effect of finite ion Larmor radius is not considered. In many astrophysical situations such as in interstellar and interplanetary plasmas the approximation of zero Larmor radiuses is not valid. Several authors Rosenbluth et al. (1962), Roberts and Taylor (1962), Jeffery and Taniuti (1966), Vandakurov (1964) have pointed out the importance of finite ion Larmor radius (FLR) effects in the form of magnetic viscosity, on the plasma instability. Recently Ferraro (2007) has shown the stabilizing effect of FLR on magneto-rotational instability. Marcu and Ballai (2007) have shown the stabilizing effect of FLR on thermosolutal stability of two-component rotating plasma. Sharma (1974) has shown the stabilizing effect of FLR on gravitationalal instability of rotating plasma. Bhatia and Chhonkar (1985) have investigated the stabilizing effect of FLR on the instability of a rotating layer of self-gravitating plasma. Herrnegger (1972) has studied the effects of collision and gyroviscosity on gravitational instability in a two-component plasma and concluded that the critical wave number becomes smaller with increasing gyroviscosity for finite Alfvén numbers and showed that Jeans criterion is changed by FLR for wave propagating perpendicular to magnetic field. Vaghela and Chhajlani (1989) have investigated the stabilizing effect of FLR on magneto-thermal stability of resistive plasma through porous medium with thermal conduction. Thus FLR effect is an important factor in discussion of thermal instability and other hydrodynamic instability.
In the light of above work, we find that Bora and Talwar (1993) have considered the effect of finite electrical resistivity, electron inertia, Hall current, thermal conductivity and radiative heat-loss function, but they neglect the effect of FLR corrections, viscosity, and permeability on thermal instability. Vaghela and Chhajlani (1989) have considered the effect of finite electrical resistivity, viscosity, permeability and thermal conductivity, but they neglect the effect of radiative heat-loss function on thermal instability. Aggarwal and Talwar (1969) have considered the effect of viscosity, rotation, finite electrical resistivity, thermal conductivity and radiative heat-loss function, but they neglect the effect of FLR corrections, on thermal instability. Thus we find that in these studies, Aggarwal and Talwar (1969) and Bora and Talwar (1993), the joint influence of permeability, FLR corrections, radiative heat-loss function, viscosity, electrical resistivity, thermal conductivity and magnetic field on the thermal instability is not investigated. Therefore in the present work thermal instability of magnetized plasma with FLR corrections, radiative heat-loss function, viscosity, thermal conductivity and finite electrical resistivity for the configuration is studied. The stability of the system is discussed by applying Routh-Hurwitz criterion. The above work is applicable to dense molecular clouds and star formation in interstellar medium.

2. BASIC EQUATIONS OF THE PROBLEM

We assume an infinite homogeneous, magnetized, thermally conducting, radiating, viscous plasma having (FLR) corrections in the presence of magnetic field $\mathbf{B} (0, 0, B)$. The MHD equations of the problem with these effects are written as

$$\frac{dv}{dt} = -\frac{\nabla p}{\rho} - \frac{\nabla \cdot \mathbf{P}}{\rho} + \frac{1}{4\pi \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{v},$$  \hspace{1cm} (1)

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0,$$  \hspace{1cm} (2)

$$\frac{1}{\gamma - 1} \frac{dp}{dt} - \frac{\gamma - 1}{\gamma - 1} \frac{d\rho}{dt} + \rho L \nabla \cdot (\lambda \nabla T) = 0,$$  \hspace{1cm} (3)

$$p = \rho RT,$$  \hspace{1cm} (4)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),$$  \hspace{1cm} (5)

$$\nabla \cdot \mathbf{B} = 0,$$  \hspace{1cm} (6)
where $P$ is the pressure tensor stands for finite ion gyration radius as given by Robert and Taylor (1962) is

$$P = -\rho \nu \left( \frac{\partial \nu}{\partial x} + \frac{\partial \nu}{\partial y} \right),$$

$$P = \rho \nu \left( \frac{\partial \nu}{\partial x} - \frac{\partial \nu}{\partial y} \right),$$

$$P = 0,$$

$$P = P = \rho \nu \left( \frac{\partial \nu}{\partial x} + \frac{\partial \nu}{\partial y} \right),$$

$$P = P = 2 \rho \nu \left( \frac{\partial \nu}{\partial x} + \frac{\partial \nu}{\partial z} \right).$$

The parameter $\nu$, has the dimensions of the kinematics viscosity and called as magnetic viscosity defined as $\nu = \Omega L R^2 / 4$, where $R_i$ is the ion-Larmor radius and $\Omega_i$ is the ion gyration frequency. Also $p$, $\rho$, $\nu$, $T$, $\lambda$, $R$, and $\gamma$ denote the fluid pressure, density, kinematic viscosity, temperature, thermal conductivity, gas constant and ratio of two specific heats respectively. $L(p, T)$ is the radiative heat-loss function and depends on local values of density and temperature of the fluid. The convective derivative operator is given as

$$\frac{d}{dt} = \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right),$$

where $\partial_t$ stands for $\partial / \partial t$.

### 3. LINEARIZED PERTURBATION EQUATIONS

The perturbation in fluid velocity, magnetic field, density, pressure, temperature and heat-loss function is given as $u(x, y, z)$, $h(x, y, z)$, $\delta \rho$, $\delta p$, $\delta T$ and $L$ respectively. The linearized perturbation equations for such medium are

$$\frac{1}{\gamma - 1} \partial_t \delta \rho - \frac{\gamma p}{\gamma - 1 \rho} \partial_t \delta \rho + \rho \left[ L_p \delta \rho + L_T \delta T \right] - \lambda \nabla^2 \delta T = 0,$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta \rho}{\rho},$$

$$\partial_t \rho + \rho \nabla \cdot \mathbf{v} = 0,$$
\[ \partial \delta B = \nabla \times \left( \mathbf{v} \times \mathbf{B} \right), \quad (13) \]

\[ \nabla \cdot \delta \mathbf{B} = 0, \quad (14) \]

where \( L_T, L_{\rho} \) are the partial derivatives of temperature dependent heat-loss function \( (\partial L/\partial T)_T \) and density dependent heat-loss function \( (\partial L/\partial \rho)_T \) respectively.

We assume that all the perturbed quantities vary as

\[ \exp \left( i(\sigma t + k_x x + k_z z) \right), \quad (15) \]

where \( \sigma \) is the frequency of harmonic disturbance, \( k_x \) and \( k_z \) are the wave numbers of the perturbations along \( x \) and \( z \) axes.

The components of equation (13) may be given as

\[ \delta B_x = \frac{iB}{\omega} k \mathbf{v}, \quad \delta B_y = \frac{iB}{\omega} k \mathbf{v}, \quad \delta B_z = -\frac{iB}{\omega} k \mathbf{v}. \quad (16) \]

Combining equations (10) and (11), we obtain

\[ \delta p = \left( A + \omega c^2 \right) \left( \alpha + \omega \right) \delta \rho, \quad (17) \]

where \( \omega = i\sigma \) and \( c = (\gamma \rho / \rho)^{1/2} \) is the adiabatic velocity of sound in the medium. The parameter \( A \) and \( B \) are given as

\[ A = (\gamma - 1) \left( TL_T - \rho L_{\rho} + \frac{\lambda k^2 T}{\rho} \right), \]

\[ \alpha = (\gamma - 1) \left( T \rho L_T + \frac{\lambda k^2 T}{\rho} \right). \quad (18) \]

Using equations (13)-(19) in equation (10) with equation (8), we may write the following algebraic equations for the components of equation (10)
Taking divergence of equation (10) and using equations (13) to (19), we obtain as

\[
\left[ \omega + \nu k^2 + \frac{V^2 k^2}{\omega} \right] v_x + \left[ \nu \left( k^2 + 2 k^2 \right) \right] v_y + \frac{i k}{k^2} T \Omega^2 s = 0 ,
\] (21)

\[- \left[ \left( k^2 + 2 k^2 \right) \right] v_x + \left[ \omega + \nu k^2 + \frac{V^2 k^2}{\omega} \right] v_y - 2 \nu k x z v = 0 ,
\] (22)

\[2 \nu k x z y + \left[ \omega + \nu k^2 \right] u + \frac{i k}{k^2} T \Omega^2 s = 0 .
\] (23)

where \( s = \delta \rho / \rho \) is the condensation of the medium.

To obtain the dispersion relation, we have made following substitutions in above equations

\[\Omega^2_T = \frac{\omega \Omega^2_j + \Omega^2_i}{\omega + B} , \quad \Omega^2_j = c^2 k^2 , \quad \Omega^2_i = k^2 (\gamma - 1) \left\{ TL_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right\} ,
\]

\[\begin{align*}
N &= \omega + \nu k^2 , & Q &= N + \frac{V^2 k^2}{\omega} , & M &= N + \frac{V^2 k^2}{\omega} , \\
P_1 &= \omega N + \Omega^2_T , & D &= \nu_0 (k_x^2 + 2 k^2) , & E &= 2 \nu_0 k_x k_z , & E_1 &= i k_x \frac{V^2 k^2}{\omega} , \\
N_1 &= i k_x \nu_0 \left( k_x^2 + 4 k^2 \right) , & F &= \frac{i k_x}{k^2} \Omega^2_T , & F_1 &= \frac{i k_x}{k^2} \Omega^2_T , & V^2 &= B^2 / 4 \pi \rho .\end{align*}
\]

(25)

4. \textbf{DISPERSION RELATION}

The nontrivial solution of the determinant of the matrix obtained from equations (21) - (24) with \( v_x , v_y , v_z , s \) having various coefficients should vanish to give the following dispersion relation

\[
P Q N M + 4 N M \nu^2 k^2 k^2 0 x z - \frac{2 \nu^2 k^2 k^2 0 x z}{k^2} \Omega^2 T \Omega (k_x^2 + 2 k_z^2) + N P \nu^2 0 x z (k_x^2 + 2 k_z^2) + 2 \nu^2 k^2 k^2 (k_x^2 + 2 k_z^2) \]

\[
= 0 .
\]
\[
\frac{V^2}{\omega} \Omega^2 - \frac{V^2 k^2}{k^2} \frac{0}{\omega} \frac{x}{k^2} \Omega^2 (k_x^2 + 2k_z^2) N (k_x^2 + 4k_z^2) - \frac{V^2 k^2}{\omega} \frac{x}{k^2} \Omega^2 N M - 4V^2 k^4 k^2 \frac{\omega^2}{\omega} \Omega^2 = 0.
\]

(26)

The dispersion relation (26) represents the simultaneous inclusion of radiative heat-loss function, FLR corrections, thermal conductivity, finite electrical resistivity, viscosity, and magnetic field on thermal instability of plasma. In absence of radiative heat-loss function the general dispersion relation (26) is identical to that of Vaghela and Chhajlani (1989). On neglecting the effect of thermal conductivity and radiative heat-loss function dispersion relation (26) is identical to Sanghvi and Chhajlani (1986). In absence of radiative heat-loss function, thermal conductivity, finite electrical resistivity and viscosity the general dispersion relation (26) is identical to Sharma (1974) for non-rotational case. In absence of FLR corrections, viscosity and dispersion relation (26) is identical to Bora and Talwar (1993) neglecting Hall current and electron inertia in that case. Also in absence of FLR corrections, viscosity, finite conductivity and dispersion relation (26) reduces to that obtained by Field (1965) for non-gravitating medium. Now we discuss the general dispersion relation (26) for longitudinal and transverse wave propagation.

5. ANALYSIS OF THE DISPERSION RELATION

5.1 LONGITUDINAL MODE OF PROPAGATION (k \parallel B)

In this case the perturbations are taken parallel to the direction of the magnetic field (i.e. \( k_x = 0, k_z = k \)). The dispersion relation (26) reduces to

\[
\left[ \omega + \nu k^2 \right] \left\{ \left[ \omega + \nu k^2 + \frac{V^2 k^2}{\omega} \right] + 4 \nu^2 k^4 \right\} \left[ \omega^2 + \omega \nu k^2 + \frac{\omega \Omega^2 + \Omega^2}{\omega + \alpha} \right] = 0.
\]

(27)

This dispersion relation represents the combined effect of, viscosity, magnetic field strength, thermal conductivity, radiative heat-loss function and FLR corrections on thermal instability of plasma. On comparing this dispersion relation (27) with dispersion relation (20) of Vaghela and Chhajlani (1989) we find that two factors are the same but the third factor is different and gets modified because of radiative heat-loss function. Also on multiplying all the components of equation (27) we get the dispersion relation, which is an equation of degree eight in \( \omega \) and it is cumbersome to write such a lengthy equation. If we remove the effect of FLR corrections and viscosity in the above relation then we recover the relation given by Bora and Talwar (1993) excluding Hall current and electron inertia in their case. Hence the above dispersion relation is the modified form of equation (21) of Bora and Talwar (1993) due to the inclusion of, FLR corrections and viscosity, in our case and by neglecting Hall current and electron inertia in their case for longitudinal propagation in dimensional form. In present case we
have considered the effects of, FLR corrections and viscosity, but Bora and Talwar (1993) have not considered these effects. Thus the dispersion relation in the present analysis is modified due to the presence of, FLR corrections and viscosity, but condition of instability is unaffected by the presence of FLR corrections, viscosity, and . Thus we conclude that the, FLR corrections and viscosity of the medium have no effect on the condition of instability. Also it is clear that the growth rate of dispersion relation given by Bora and Talwar (1993) gets modified due to the presence of FLR corrections and viscosity in the present case. Thus we conclude that medium, FLR corrections and viscosity, modify the growth rate of instability in the present case. Hence these are the new findings in our case than that of Bora and Talwar (1993).

The dispersion relation (27) has three different components and we discuss each component separately. The first component of the dispersion relation (27) gives

\[ \omega + \nu k^2 = 0. \]  

(28)

This represents a stable damped mode modified by the presence of viscosity, and of the medium. Thus viscous is capable to stabilize the growth rate of the considered system. The above mode is unaffected by the presence of FLR corrections, magnetic field strength, thermal conductivity and radiative heat-loss function. This dispersion relation is identical to Vaghela and Chhajlani (1989).

The second factor of equation (27) on simplification gives

\[ \omega^4 + 2\nu k^2 \omega^3 + \left( \nu k^2 + 2V^2 k^2 + 4\nu_0^2 k^4 \right) \omega^2 + \left( 2\nu k^2 V^2 k^2 + 8\eta k^2 V_0^2 k^4 \right) \omega + V^2 k^2 = 0. \]  

(29)

The above dispersion relation shows the viscous magnetized medium having finite electrical resistivity, and FLR corrections. This dispersion relation is identical to Vaghela and Chhajlani (1989). The above relation is independent of thermal conductivity and radiative heat-loss functions. Equation (29) is a four degree equation in power of \( \omega \) having its all coefficients positive which is a necessary condition for the stability of the system. To achieve the sufficient condition the principal diagonal minors of Hurwitz matrix must be positive. On calculating we get all the principal diagonal minors positive. Hence equation (29) always represents stability.

For inviscid, infinitely conducting medium in absence of FLR corrections \( (\nu = \nu_0 = 0) \) equation (29) becomes

\[ \omega^2 + V^2 k^2 = 0. \]  

(30)
This represents the pure Alfven mode.

For inviscid medium \((\nu = 0)\) equation (29) becomes

\[
\omega^4 + 2(V^2 k^2 + 2\nu^2 k^4)\omega^2 + V^4 k^4 = 0.
\] (31)

The roots of equation (31) are

\[
\omega = \left[ -\left(V^2 k^2 + 2\nu^2 k^4\right) \pm 2\nu k^2 \left(V^2 k^2 + \nu^2 k^4\right) \right].
\] (32)

Hence FLR corrections modify the Alfven mode by changing the growth rate of the system. Equations (31) and (32) are the modified form of Vaghela and Chhajlani (1989) by medium.

The third component of the dispersion relation (27) on simplifying gives

\[
\omega^3 + \left[\nu k^2 + (\gamma - 1) \left(\frac{T\rho L_T}{p} + \frac{\lambda k^2 T}{p}\right)\right] \omega^2 + \left[(\gamma - 1) \left(\frac{T\rho L_T}{p} + \frac{\lambda k^2 T}{p}\right) \nu k^2 + (\nu^2 k^4)\right] \omega + \left\{k^2 (\gamma - 1) \left(TL_T - \rho L_T + \frac{\lambda k^2 T}{\rho}\right)\right\} = 0.
\] (33)

This dispersion relation (33) represents the combined influence of, \(\nu\), radiative heat-loss function, thermal conductivity and viscosity on the thermal instability of plasma. But there is no effect of FLR corrections, finite electrical resistivity and magnetic field on the thermal instability of the considered system. In absence of radiative heat-loss function the above relation (33) is identical to Vaghela and Chhajlani (1989). If the constant term of cubic equation (33) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (33) is given as

\[
\left\{k^2 (\gamma - 1) \left(TL_T - \rho L_T + \frac{\lambda k^2 T}{\rho}\right)\right\} < 0.
\] (34)

The medium is unstable for wave number \(k < k_{j1}\). Here it may be noted that the modified critical wave number involves the derivatives of temperature dependent, density dependent heat-loss function and thermal conductivity of the medium. \(c' = (p/\rho)^{1/2}\) is the isothermal velocity of sound in the medium. In absence of \(\nu\) and viscosity, equation (33) is identical to Field (1965), as the viscosity and of the medium have no effect on the condition of instability. It is clear that the
growth rate of the dispersion relation given by Field (1965) is getting modified due to the presence of viscosity and in our present case. Hence these are the new findings in our case than that of Field (1965).

Figure 1 shows the effect of $k^*_\lambda$ on the growth rate of thermal instability for fixed values of other parameters. From curves it is clear that as the value of $k^*_\lambda$ increases both the peak value and the growth rate of thermal instability decreases. Thus the parameter $k^*_\lambda$ moves the present system towards the stabilization. In Fig. 2 we have plotted the growth rate of thermal instability against wave number for different values of the parameter $k^*_\tau$. From figure we conclude that as the value of $k^*_\tau$ increase, the peak value of curves decreases and the area of growth rate also decrease. Hence, the presence of $k^*_\tau$ also stabilizes the system. In Fig. 3 we have shown the effect of viscosity on the growth rate of thermal instability. Figure displays that on increasing the value of viscosity the growth rate of thermal instability decreases. Therefore, the parameters $k^*_\lambda$, $k^*_\tau$ and $\nu^*$ viscosity stabilize the system.

To discuss the effect of each parameter on the growth rate of thermal instability we solve equation (33) numerically by introducing the following dimensionless quantities

To study the effects of viscosity, and radiative heat-loss functions on the growth rate of thermal instability, we solve Eq. (33) numerically. Therefore Eq. (33) can be written in non-dimensional form with the help of following dimensionless quantities

$$
\omega^* = \frac{\omega}{k_{\rho}c_s}, \quad \nu^* = \frac{\nu k_{\rho}}{c_s}, \quad k^* = \frac{k}{k_{\rho}}, \quad k^*_\lambda = \frac{k^*_\lambda}{k_{\rho}}, \quad k^*_\tau = \frac{k^*_\tau}{k_{\rho}}.
$$

Using Eq. (35), we write Eq. (33) in non-dimensional form as

$$
\omega^* + \left[ \nu^* k^{*2} + k^*_\tau + k^*_\lambda k^{*2} \right] \omega^* + c_s^2 \left[ \nu^* k^{*2} \left( k^*_\tau + k^*_\lambda k^{*2} \right) + k^{*2} \right] \omega^* + \frac{k^{*2}}{\gamma} \left( k^*_\tau - 1 + k^*_\lambda k^{*2} \right) = 0.
$$
Fig. 1: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $\nu^*$ with $k_r^* = 0.5$ and $K_1^* = 1$, $k_j^* = 0.1$. 
Fig. 2: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $k^*_T$ with $k^*_\lambda = 0.01$ and $K^*_1 = \nu^* = 1.0$.

Fig. 3: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $k^*_\lambda$ with $k^*_T = 0.5$ and $K^*_1 = \nu^* = 1.0$. 

$T^*_k = 0.15$

$T^*_k = 0.25$

$T^*_k = 0.45$
To discuss the stability of the system given by equation (33), if constant term of cubic equation (33) is greater than zero, then all the coefficients of the equation (33) must be positive. Equation (33) is a third degree equation in the power of $\omega$ having its coefficients positive, which is a necessary condition for the stability of the system. To achieve the sufficient condition the principal diagonal minors of Hurwitz matrix must be positive. The principal diagonal minors are

$$
\Delta = \left[ vk^2 + (\gamma - 1) \left( \frac{T \rho L_T}{p} + \frac{\lambda k^2 T}{p} \right) \right] > 0,
$$

$$
\Delta_2 = vk^2 \left[ \gamma (\gamma - 1) \right] \left( \frac{T \rho L_T}{p} + \frac{\lambda k^2 T}{p} \right) + c^2 k^2 \right] + (\gamma - 1) k^2 \rho L_T > 0,
$$

$$
\Delta_3 = \Delta_2 \left( k^2 (\gamma - 1) \left( T L_T - \rho L_T + \frac{\lambda k^2 T}{p} \right) \right) > 0.
$$

If $\Omega^2_j > 0, \Omega^2_i > 0$ and $\gamma > 1$, then it is clear that all the $\Delta$s are positive hence system represented by equation (33) is stable system.

For viscous, radiating and thermally non-conducting medium $(\omega = L_T, p = 0, \lambda = 0)$ equation (33) becomes

$$
\omega^3 + \left[ vk^2 + \frac{\gamma L_T}{c_p} \right] \omega^2 + \left[ vk^2 \frac{\gamma L_T}{c_p} + c^2 k^2 \right] \omega + \frac{\gamma L_T}{c_p} \left( c^2 \left( \frac{-p L_T}{T L_T} \right) \right) = 0.
$$

The condition of instability from constant term of equation (38) is

$$
\left\{ k^2 \left( c^2 \frac{-p L_T}{T L_T} \right) \right\} < 0,
$$

Thus we conclude that for longitudinal wave propagation as given by equation (27) the system is unstable only for Jeans condition, else it is stable. Also for longitudinal wave propagation the Jeans criterion remains unaffected by FLR corrections, viscosity, magnetic field, finite electrical resistivity and, but thermal conductivity and radiative heat-loss function modify the Jeans expression and the fundamental Jeans instability criterion becomes radiative instability criterion.
5.2 TRANSVERSE MODE OF PROPAGATION (k ⊥ B)

In this case the perturbations are taken perpendicular to the direction of the magnetic field (i.e. \( k_x = k, k_z = 0 \)). The dispersion relation (26) reduces to

\[
\left\{ \omega + \nu k^2 \right\} \left[ \omega + \nu k^2 \right] \left\{ \omega^2 + \omega v k^2 + \frac{\omega v^2 k^2}{\nu} + \frac{\omega \Omega_j^2 + \Omega_i^2}{\omega + B} \right\} + \omega^2 v_0^2 k^4 = 0. \tag{40}
\]

This dispersion relation (40) is modified due to the presence of radiative heat-loss function, FLR corrections, thermal conductivity, viscosity, finite electrical resistivity and magnetic field. The dispersion relation (40) has two different components. The first component of the dispersion relation (40) represents a stable viscous mode modified by the presence of the medium as discussed in equation (28).

The second component of the dispersion relation (40) on simplifying gives

\[
\begin{align*}
\omega^4 &+ \left\{ 2\nu k^2 + (\gamma - 1) \left( \frac{T \rho L}{p} + \frac{\lambda k^2 T}{p} \right) \right\} \omega^3 + \left\{ 2\nu k^2 \left[ (\gamma - 1) \frac{T \rho L}{p} + \frac{\lambda k^2 T}{p} \right] \right\} \omega^2 \quad \text{+} \quad \left\{ v^2 k^2 \left[ \nu k^2 + v_0^2 k^4 \right] \right\} + \omega^2 v_0^2 k^4 \quad \text{+} \quad \left\{ k^2 \left[ (\gamma - 1) \left( \frac{T \rho L}{p} + \frac{\lambda k^2 T}{p} \right) \right] \right\} \omega \\
+ V^2 k^2 \left[ \nu k^2 + (\gamma - 1) \left( \frac{T \rho L}{p} + \frac{\lambda k^2 T}{p} \right) \right] + c^2 k^2 \left[ \nu k^2 \right] + \left\{ k^2 \left[ (\gamma - 1) \left( TL_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right) \right] \right\} \omega \\
+ \left\{ V^2 k^2 (\gamma - 1) \left( \frac{T \rho L}{p} + \frac{\lambda k^2 T}{p} \right) + k^2 (\gamma - 1) \left( TL_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right) \right\} = 0. \tag{41}
\end{align*}
\]

The above dispersion relation represents the combined influence of thermal conductivity, radiative heat-loss function, FLR corrections, finite electrical conductivity, viscosity, and magnetic field on thermal instability of plasma through porous medium. In absence of radiative heat-loss function equation (41) is identical to Vaghela and Chhajlani (1989). When constant term of equation (41) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (41) is given as
\[
\left\{ V^2 k^2 (\gamma - 1) \left( \frac{T \rho L_T}{p} + \frac{\lambda k^2 T}{\rho} \right) + k^2 (\gamma - 1) \left( T L_T - \rho L_T + \frac{\lambda k^2 T}{\rho} \right) \right\} < 0.
\]

Thus to discuss the effect of each parameter (viz. heat-loss function, viscosity, and FLR corrections) on the growth rate of unstable modes, we solve equation (41) numerically by introducing the following dimensionless quantities

\[
\omega^* = \frac{\omega}{k^*_\rho c_s}, \quad \nu^* = \frac{\nu_0 k^*_\rho}{c_s}, \quad k^* = \frac{k}{k^*_\rho}, \quad k^*_\lambda = \frac{k^*}{k^*_\lambda}, \quad k^*_T = \frac{k^*}{k^*_T}, \quad \nu^*_0 = \frac{\nu_0 k^*_\rho}{c_s}.
\]

Using Eq. (43), we write Eq. (41) in non-dimensional form as

\[
\omega^{*4} + \left\{ 2\nu^* k^{*2} + k^*_\lambda k^{*2} \right\} \omega^{*3} + \left\{ 2\nu^* k^{*2} \left[ k^{*2} + k^*_T k^{*2} \right] + \nu^* k^{*4} \left[ k^{*2} + k^*_T k^{*2} \right] + \nu^* k^{*2} + \nu^*_0 k^{*2} + c^* k^{*2} \right\} \omega^{*2}
\]

\[
+ \left\{ \nu^* k^{*2} + \nu^*_0 k^{*2} + c^* k^{*2} \right\} \omega
\]

\[
+ \left\{ \nu^* k^{*2} \left[ k^{*2} + k^*_T k^{*2} \right] + \nu^*_0 k^{*2} \left[ k^{*2} + k^*_T k^{*2} \right] \right\} = 0.
\]

In Figs. 4-8 the dimensionless growth rate (\(\omega^*\)) has been plotted against the dimensionless wave number (\(k^*\)) to see the effect of various physical parameters such as viscosity, radiative heat-loss function and FLR corrections. It is clear from Fig. 4 that growth rate decreases with increasing the value of viscosity. Thus the effect of viscosity is stabilizing. From Fig. 5 we see that as the value of \(k^*_\lambda\) increases the growth rate decreases. Thus the effect of parameter \(k^*_\lambda\) is also stabilizing.
Fig. 4: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $\nu^*$ with $k_T^* = 0.3$ and $k_1^* = 0.2$, $K_1^* = \nu_0^* = 1.0$.

Fig. 5: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $k_\lambda^*$ with $k_T^* = 0.5$ and $K_1^* = \nu_0^* = \nu^* = 1.0$. 
Fig. 6: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $k_T^*$ with $k_\lambda^* = 0.2$ and $K_1^* = \nu_0^* = \nu^* = 1.0$.

Fig. 7: The normalized growth rate ($\omega^*$) as a function of normalized wave number ($k^*$) for different values of $\nu_0^*$ with $k_\lambda^* = 0.2$, $k_T^* = 0.3$ and $K_1^* = \nu^* = 1.0$. 
From Fig. 6 we conclude that growth rate decreases with increasing parameter $k_T$. Thus the presence of $k_T$ stabilizes the growth rate of the system. Figure 7 displays the influence of FLR corrections on the growth rate of thermal instability. From figure it is clear that the FLR correction has a stabilizing effect on the growth rate of thermal instability. Therefore, the parameters viscosity, radiative heat-loss functions and FLR corrections have stabilizing influence on the system.

For non-viscous, radiating, thermally conducting, magnetized, finitely conducting, medium with FLR corrections ($\nu = 0$, $T_{T, \rho} = V = \lambda = \nu = 0$) equation (41) becomes

$$\omega^3 + \left\{ (\gamma - 1) \left[ \frac{T \rho L_T}{p} + \frac{\lambda k_T^2 T}{p} \right] \right\} \omega^2 + \left\{ V^2 k^2 + \nu_0^2 k^4 + c^2 k^2 \right\} \omega + \left\{ \nu k \left[ (\gamma - 1) \left[ \frac{T \rho L_T}{p} + \frac{\lambda k_T^2 T}{p} \right] \right] + V^2 k^2 \left[ \frac{\gamma L_T}{c_p} \right] \right\} = 0. \quad (45)$$

The above equation is modified form of Vaghela and Chhajlani (1989) by inclusion of radiative heat-loss function. When constant term of equation (45) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (45) is given as

$$\left\{ k^2 \left[ T_{L_T} - \rho L_{p} + \frac{\lambda k_T^2 T}{\rho} \right] \right\} < 0. \quad (46)$$

From the above condition of instability given by equation (46) we conclude that FLR corrections try to stabilize the system. Also on comparing equations (41) and (46) we see that inclusion of viscosity removes the effects of FLR corrections and medium from condition of instability. So in both the cases either the system is viscous or non-viscous, FLR corrections and stabilizes the growth rate of thermal instability.

For inviscid, thermally non-conducting, radiating, magnetized, finitely conducting, medium with FLR corrections ($\nu = \lambda = 0$, $T = V = \nu = 0$) equation (41) becomes

$$\omega^3 + \left( \frac{\gamma L_T}{c_p} \right) \omega^2 + \left( V^2 k^2 + \nu_0^2 k^4 + c^2 k^2 \right) \omega + \left( \nu_0^2 k^4 \left( \frac{\gamma L_T}{c_p} \right) + V^2 k^2 \frac{\gamma L_T}{c_p} \right)$$
\[ + \frac{\gamma \omega}{c_p} \left( k^2 \left( c^2 L_T - \frac{pL_p}{T} \right) \right) = 0. \quad (47) \]

When constant term of equation (47) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (47) is given as

\[ k^2 \left( c^2 L_T - \frac{pL_p}{T} \right) < 0, \quad (48) \]

or critical Jeans wave number is given as

For inviscid, infinitely conducting, radiating, thermally conducting, magnetized, medium with FLR corrections \( (\nu = 0, L_{T,\rho}, V = \lambda = \nu_0 \neq 0) \) equation (41) becomes

\[ \omega^2 + \left( \gamma - 1 \right) \left( \frac{T \rho L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) \omega + \left( V^2 k^2 + \nu_0^2 k^2 + c^2 k^2 \right) + \left( \nu_0^2 k^4 + V^2 k^2 \right) (\gamma - 1) \]

\[ + \left( \frac{T \rho L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) + k^2 \left( TL_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right) = 0. \quad (49) \]

When constant term of equation (49) is less than zero this allows at least one positive real root which corresponds to the instability of the system. The condition of instability obtained from constant term of equation (49) is given as

\[ \left( \nu_0^2 k^4 + V^2 k^2 \right) \left( \frac{T \rho L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) + k^2 \left( TL_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right) < 0. \quad (50) \]

The above condition of instability (50) is the modified form of equation (41) of Prajapati et al. (2010) by and FLR corrections, excluding electron inertia in their case. From the condition of instability given by equation (50) we conclude that, FLR corrections and magnetic field try to stabilize the system. Also on comparing equations (41) and (49) we see that inclusion of viscosity remove the effect of FLR corrections, and magnetic field from condition of instability. So in both the cases whether the system is viscous or non-viscous FLR corrections stabilize the growth rate of thermal instability.
Thus we conclude that FLR corrections, heat-loss function, thermal conductivity, magnetic field strength and viscosity have stabilizing influence on the growth rate of thermal instability.

6. CONCLUSION

In the present problem we have studied the effects of FLR corrections on the thermal instability of infinite homogeneous viscous plasma with thermal conductivity, radiative heat-loss function. The general dispersion relation is obtained which is modified due to the presence of considered physical parameters and is discussed for longitudinal and transverse mode of propagation to the direction of magnetic field. We find that the fundamental criterion of thermal instability regarding the size of initial break up is considerably modified due to radiative heat-loss function, and FLR corrections. The effect of heat-loss function parameters is found to stabilize the system in both the longitudinal mode and transverse mode of propagation.

In the case of longitudinal mode of propagation, we find Alfvén mode modified by the presence of, FLR corrections and viscosity. The thermal mode is obtained separately which is modified by the presence of, radiative heat-loss function, thermal conductivity and viscosity. The condition of thermal instability is unaffected by the presence FLR corrections, and viscosity. From the curves we find that the heat-loss function has a stabilizing role on the growth rate of the system in longitudinal mode of propagation.

In the case of transverse mode of propagation, we obtain a thermal mode modified by the presence of, FLR corrections, radiative heat-loss function, thermal conductivity and viscosity. We find that the condition of instability is independent of FLR corrections and viscosity, and depends only on thermal conductivity and radiative heat-loss function. But the growth rate is affected by the presence of all the considered parameters. For the case of inviscid and thermally non-conducting medium it is found that the condition of instability modified due to the presence of FLR corrections and radiative heat-loss function. It is observed that for an inviscid medium the condition of instability is modified due to the presence of FLR corrections, magnetic field, thermal conductivity and radiative heat-loss function, and it is independent of and viscosity. From the curves we find that the heat-loss function has stabilizing effect on the growth rate of thermal instability. Also it is interesting to see that in both the cases the peak value of the curves decreases on increasing heat-loss function this means that the system becomes more stable on increasing the value of heat-loss function. The effect of FLR corrections is to stabilize the system.

When the cloud density reaches critical value, the cloud fragments into cool dense condensations via thermal instability. When the critical density increase as metallicity decrease, and also as radiation increase. Condensations collide with each other and self-gravitating clumps will be produced when the mean cloud density becomes sufficiently high; then stars will form. Expansion of the H II region around the massive star and supernova explosions will blow off surrounding gas and end star formation process. When the mean density at the time of star formation is high, high virial velocity prevents expansion of the H II region. Also, in such high-density environments, the star formation timescale is shorter than the lifetime of a massive star.
Then the gas in cluster-forming region will be converted into stars efficiently, before the gas is detached by expanding H II region or supernova explosions. High density is realized in the contracting low-metallicity gas, and if the formation of a contracting gas cloud is possible, a strong-radiation environment is another candidate. Thus, it is suggested that high star formation efficiency and bound cluster formation are expected achieved in low-metallicity and/or strong-radiation environments. Such environments exist in dwarf galaxies, the early stage of our Galaxy and starburst galaxies.

ACKNOWLEDGEMENTS
One of the author’s (SK) is grateful to Er. Praveen Vashishtha, Chairman Mahakal Institute of Technology for continuous support.

REFERENCES


Sharm, R.C., & Prakash, K., Radiative transfer and collisional effects on thermal convective instability of a composit medium, Progress of Theoretical Physics, vol. 54, pp. 409-414, 1975.


