

Thermal Buckling Analysis Of Functionally Graded Plates Using Power Law Function

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Abstract— Functionally Graded Materials (FGM) are those materials which have continuous variation of material properties from metal phase to ceramic phase. Due to the continuous change in material properties of FGMs, the stress singularity at the interface between the two different materials is eliminated and thus the bonding strength is enhanced. They are widely used in high temperature environment such as nuclear reactors and rocket heat shields. The material property of FGM plate varies along the thickness direction and the variation is idealised by different mathematical idealisation techniques. This paper deals with the buckling behaviour of clamped PFGM plate under linear temperature field. Thermal buckling behaviour of FGM plate has been obtained numerically through ANSYS software. The convergence study of the results is optimized by changing the mesh size. The critical buckling temperature rise obtained for functionally graded plates using ANSYS software are compared with the available literature. The effect of different parameters such as power-law index, thickness ratio, and aspect ratio on critical buckling temperature rise for temperature independent and dependent material properties of each constituent is also discussed.

Index Terms— Functionally Graded Material, Power-law function, Thermal Buckling

1 INTRODUCTION

FUNCTIONALLY Graded Materials (FGMs) are a comparatively a new class of materials with continuously changing material properties from metal to ceramic surface, in the thickness direction. Because of its low thermal conductivity, those ceramic part from the material offers high-temperature resistance and protects those metal starting with oxidation. The ductile constituent of the metal keeps crack caused by stresses due to high-temperature gradient. This new sort of material gives an edge over traditional composite materials by eliminating inter-laminar thermal stress concentration and in addition delamination. Because of these novel characteristics, FGM structures are appropriate for thermal barrier applications as in aerospace, energy, defence industries etc. The introduction of FGM structures to high temperature field results loss of their structural integrity due to the thermal load and which causes the geometrical variability. Therefore, many researchers are interested to find the buckling behaviour of FGM structures under thermal environment.

A.M. Zenkour and D. S. Mashat [1] determined the thermal buckling response of functionally graded plates using sinusoidal shear deformation plate theory (SPT). Ghannadpour et al. [2] examined the buckling responses of FG flat panels by applying a finite strip method under different thermal loadings such as uniform, linear and nonlinear temperature distribution across the thickness. G. Srinivas and U. Shiva Prasad [3] focused on analysis of FGM flat plates under thermal loading in order to understand the effect variation of material properties has on structural response. Results are compared to published results in order to show the accuracy of modelling

FGMs using ANSYS software. Zhao et al. [5] used the first order shear deformation theory (FSDT) and the element-free kp-Ritz method for mechanical and thermal buckling analyses of FG plates.

Based on the available literature, it is observed that very few works were done on thermal buckling behaviour of FGM plate with temperature dependent material properties. This study aims to predict the buckling strength of FGM flat plate under linear temperature field with/without temperature dependent material properties using the finite element software ANSYS. The convergence study of current plate model has been executed and following that the validation of ANSYS results with published results is implemented. The effect of different parameters such as power-law index, aspect ratio and thickness ratio on critical buckling temperature rise of functionally graded flat plates are examined and discussed in detail.

2 MATERIAL PROPERTIES OF FGM PLATES

According to the power-law function, the material properties of the plate are assumed to vary through the thickness of the plate. The top surface ($z = h/2$) of the plate is assumed to be rich in ceramic whereas the bottom surface ($z = -h/2$) to be rich in metal.

The material properties of PFGM can be determined by the rule of mixture:

$$P(z, T) = [P_c(T) - P_m(T)] V_c(z) + P_m(T) \quad (1)$$

where, subscript 'c' and 'm' denote ceramic and metal, respectively and $V_c(z)$ is the ceramic volume fraction and expressed as

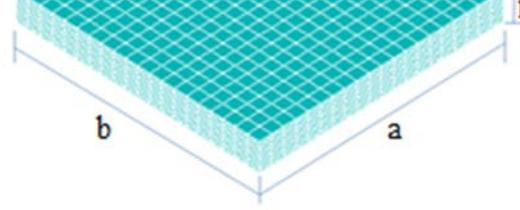
$$V_c(z) = \left[\frac{z}{h} + \frac{1}{2} \right]^n \quad (0 \leq n < \infty) \quad (2)$$

where, n is the power law index which shows the material profile across the thickness.

For temperature-dependent materials, the corresponding properties are given by [9]

$$P_{c,m}(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (3)$$

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Where, P_0, P_{-1}, P_1, P_2 and P_3 are the temperature coefficients. Unless otherwise specified, the material properties are computed at standard room temperature ($T=300K$). The variation of volume fraction of the ceramic phase through the dimensionless thickness ($Z=z/h$) is plotted in Fig.1 for different values of power law indices. Temperature dependent (TD) and Temperature independent (TID) material properties of ceramic and metal are shown in Table 1

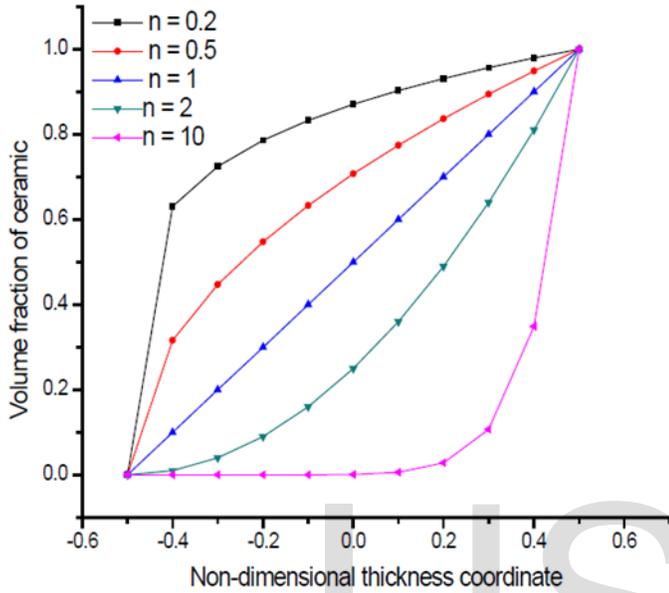


Fig.1 Material distribution of PFGM plate along the non-dimensional thickness coordinate

TABLE 1
TEMPERATURE DEPENDENT MATERIAL PROPERTIES OF CERAMIC AND METAL^[9]

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3	TID(at 300K)
alumina(Al_2O_3)	E(Pa)	3.50E+11	0	-3.85E-04	4.03E-07	-1.67E-10	3.80E+11
	$\alpha(1/K)$	6.83E-06	0	1.84E-04	0	0	7.40E-06
aluminum(Al)	E(Pa)	6.71E+10	0	9.20E-04	-2.70E-06	1.45E-09	7.00E+10
	$\alpha(1/K)$	1.72E-05	0	1.16E-03	0	0	2.30E-05

3 FINITE ELEMENT MODELLING

In this study, a functionally graded flat plate of sides a and b with uniform thickness, h is considered as shown in Fig. 2. The plate is fully clamped on all its edges and is modelled and analysed in ANSYS through ANSYS Parametric Design Language (APDL) code. An eight node serendipity shell element (SHELL281), characterized in the ANSYS environment is utilised to discretise the FG flat plate. This element is suitable for analysing thin to moderately-thick shell structures. The element has translations and rotations in the x,

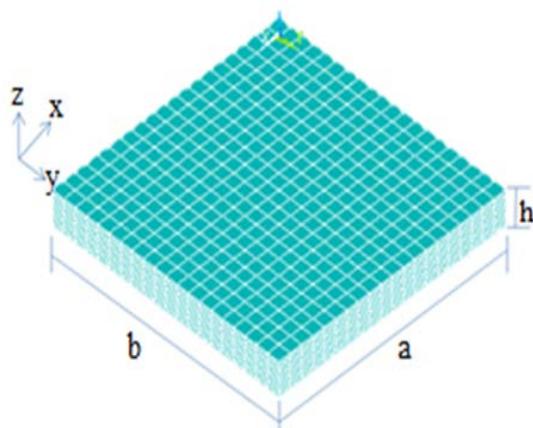


Fig.2 Geometry and dimension of FG Flat Plate

The thermal buckling response of FG flat plate is analysed under linear temperature field for both TD and TID material properties. A clamped boundary condition is used, to constrain the all four edges of FG plate. The FG flat plate is discretised and solved using finite element steps in ANSYS APDL platform. The reference temperature of FG plate is considered here is T_0 (300 K) and thermal buckling analysis of the FG plate is examined in temperature range of 300K to 800K. The temperature field is assumed to vary linearly from bottom (metal) surface to the top (ceramic) surface of the plate and is given by:

$$T=T_0 + (T_1-T_0) g(z) \tag{4}$$

Where, $g(z) = \left(\frac{z+h}{h}\right)^n$ and $n=1$ (5)

T_0 and T_1 represent the temperatures at bottom (metal) and top (ceramic) surface of an FGM plate.

4 CONVERGENCE AND VALIDATION STUDY

Fig.3 shows the convergence study by varying the mesh size and Fig.4 shows the validation of clamped FG flat plate ($a=b=1m$ and $h = 0.01m$) for different power-law indices under uniform temperature field. Aluminum (Al) and Alumina (Al_2O_3) are taken as metal and ceramic materials, respectively. The material properties are taken as same as in Zhao, Lee and Liew(2009). It is found from the mesh refinement that the values are well converging at a (20x20) mesh. From the validation study, it is observed that the present results are showing well agreement with the published results.

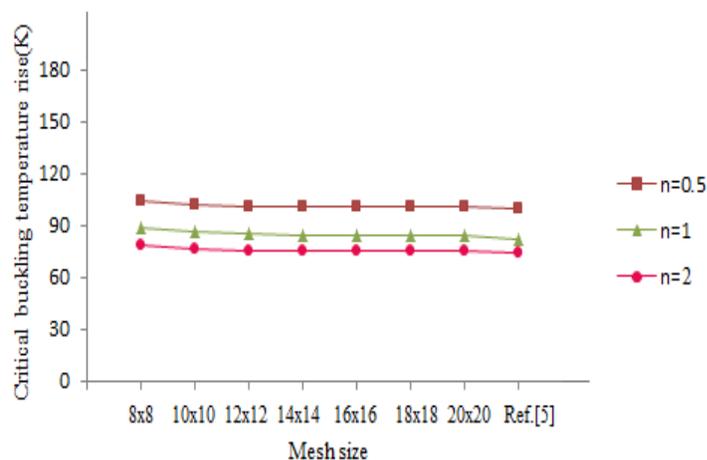


Fig.3 Mesh Convergences for PFGM with various mesh counts

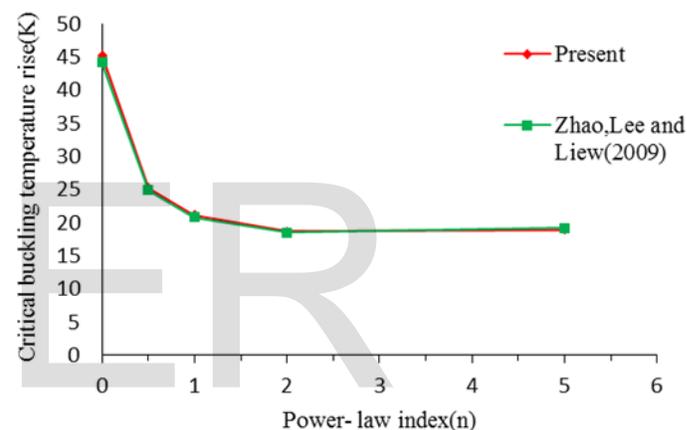


Fig.4 Critical buckling temperature rise of clamped (Al/Al_2O_3) FG flat square plate($a/h=100$)

5 PARAMETRIC STUDIES

In this section, few problems have been carried out of FG flat panels under linear thermal loading. Aluminum(Al) and alumina(Al_2O_3) are considered as FGM constituents, throughout in the section. Temperature dependent and Temperature independent material properties are mentioned in Table 1. Poisson's ratio is taken constant as 0.3. Fig. 5 shows the variation of critical buckling temperature rise of clamped FG flat plate ($a/b=1$, $n=2$) along with the thickness ratio for different temperature values. It is seen that the critical buckling temperature rise increases with the decrease in thickness ratios, because as the thickness ratio increases the plate geometry changes from thick to thin and the thin structures have lower structural stiffness. Fig. 6 shows the variation of critical buckling temperature rise of clamped FG flat plate ($a/b=1$, $a/h=100$) along with the power-law index for different temperature values. It is observed that the critical buckling temperature rise decreases with the increase in power-law indices because as n value increases the FG flat plate turns to metal. Fig. 7 shows the variation of critical buckling temperature rise

of clamped FG flat plate ($n=2$, $a/h=100$) along with the aspect ratios for different temperature values. It is clear that the critical buckling temperature rise increases with increase in aspect ratio. This is because, a flat plate with large aspect ratio exhibit higher stiffness value. In all the three cases, the critical buckling temperature rise decreases with the increase in applied linear temperature from reference temperature.

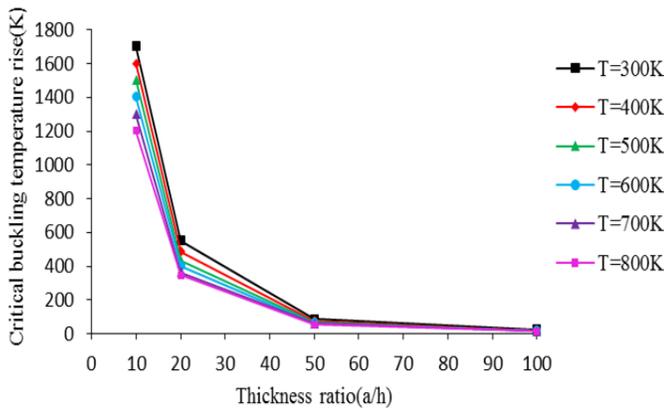


Fig.5 Variation of critical buckling temperature rise with the thickness ratios for Clamped FG flat plate ($a/b=1$ & $n=2$)

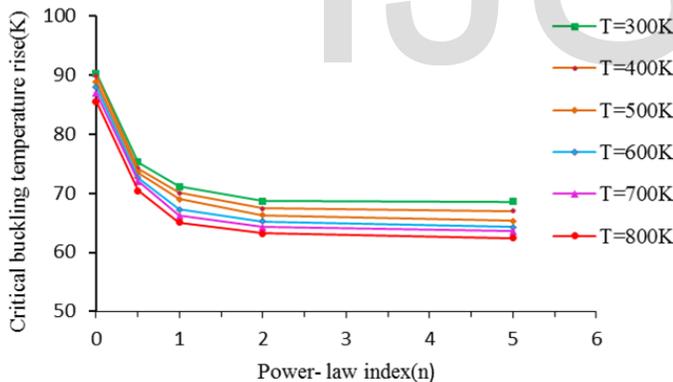


Fig.6 Variation of critical buckling temperature rise with the power-law indices for Clamped FG flat plate ($a/b=1$ & $a/h=100$)

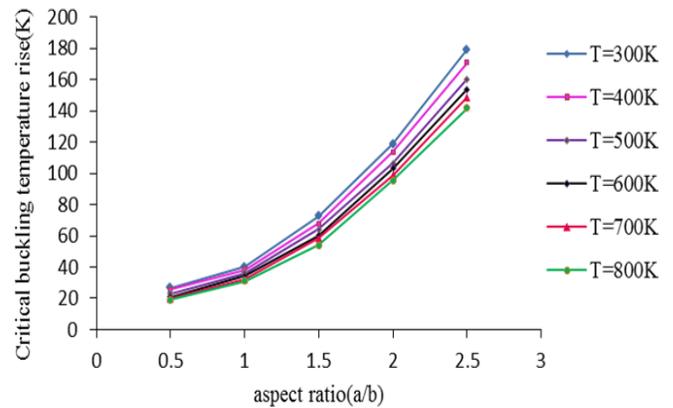


Fig.7 Variation of critical buckling temperature rise with the aspect ratios for Clamped FG flat plate ($n=2$ & $a/h=100$)

6 CONCLUSION

The thermal buckling behaviour of clamped FG flat plate under linear temperature loading is examined. The material properties of FGM constituents are temperature dependent and the effective material property is evaluated using the power-law distribution of the volume fractions. The present model is discretised and then solved in ANSYS based on APDL code.

The results were compared with the published results. The effects of geometrical and material parameters on the thermal buckling of FG flat plate are illustrated. It is found that the critical buckling temperature rise decreases with the increase in power-law indices and thickness ratio and increases with increase in aspect ratio. Lower value of applied linear temperature results the larger critical buckling temperature rise.

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