The Application of the most suitable Impact Model(s) for simulating the Seismic Response of a Straight Bridge under Impact due to Pounding

Avishek Chanda[1], Arnab Banerjee[2] and Raj Das[3]
1 Department of Mechanical Engineering, The University of Auckland, New Zealand, acha553@aucklanduni.ac.nz/a.chanda91@gmail.com
2 Department of Mechanical Engineering, The University of Auckland, New Zealand, aban991@aucklanduni.ac.nz
3 Department of Mechanical Engineering, The University of Auckland, New Zealand, r.das@auckland.ac.nz
Corresponding Author: Avishek Chanda - acha553@aucklanduni.ac.nz/a.chanda91@gmail.com

ABSTRACT

The impact is a phenomenon which is most significant in the field of multi-body dynamics, that is, the dynamics prevalent between two or more bodies, in close proximity, experiencing contact. This results in an impulsive force between the interacting bodies, depending on the geometry and the interaction properties, for an infinitesimal time duration. Simulating the impact phenomenon has formed an intrinsic part of the modern day technology in structural pounding, robotics and bio-mechatronic applications and all other engineering aspects involving contacts between two or more bodies. Literature, containing models for simulating the impact phenomenon, can be found from the time of Newton and, therefore, it is of utmost importance to understand which models give the ideal response for a system experiencing impact. This work only concentrates with the simulation of the post-impact response of two rigid bodies. Soft-body impact, stress wave propagation and impact damage in composites are excluded from the scope of this paper.

One of the most important aspect of structural pounding is the pounding of bridge segments, during a seismic event, causing catastrophic failures due to deck un-seating, torsional, shear failure of the columns and other local failures. Therefore, the post-impact behaviour of a bridge segment is extremely significant for the purpose of simulation, in order to estimate the failure possibility of the bridge. This simulation can be performed using both compliance based-models, having the compliance forces as the function of the penetration experienced, and also by non-smooth models which simulate the post-impact phenomenon using linear complementarity problem (LCP). However, the most conventional method, for analysing the post-impact response during structural pounding, has always been the compliance based methods.

This work concentrates in the estimation of the ideal impact analysing model(s), among all the available general models dependent mainly on the coefficient of restitution, only in the normal direction. A critical comparison is carried out on a unified non-dimensional frame, so that the models can be compared on the same platform. The ideal model(s), critically achieved, is further deployed to simulate the response of a straight bridge under a seismic event. The straight bridge is idealised as a single degree of freedom system, having a lump mass or inverted pendulum configuration. This work will help in understanding the efficiency of each model, for the purpose of impact analysis, and also in forming a guideline for the most efficient method(s) that can be used to simulate the response of the system, experiencing structural pounding.

Key Words: Multibody dynamics; Structural pounding; Impact analysis; Unilateral Contact.

1 INTRODUCTION

Impact occurs when two or more bodies comes in contact, with each other, for an infinitesimal time duration, resulting in different responses depending on the various material properties, the geometry of the bodies and also the individual pre-impact conditions. The study of the dynamic systems is known as the multibody dynamics, which can be further defined as the system of multiple bodies where the system's relative motion, due to the external force, is constrained by the constituent kinematic pairs [1-3]. The forces, acting on the multi-body system, may include inertia or gravitational forces, state-dependent forces, concentrated forces or others and often result in impacting events [4-7]. The impact phenomenon may eventually lead to failure due to vibration, load propagation, fatigue, cracks, wear and other detrimental events leading to the inefficient functioning of the system. The pioneer works in this field can be found more than a century ago and still has a significant importance in the modern research and other engineering activities [8-13].

In the present days, the most challenging aspect, for engineers, lies in the selection of the most suitable constitutive model for the impacting system in hand, due to the influential geometries and kinematics of the bodies under impact [14-16]. The other challenges include estimation of the influential contact parameters [4, 17, 18] and the quantification of the energy transfer [19-21]. The first ever method was proposed by Hertz [22, 23] based on the linear spring model and was completely elastic in nature, with no energy dissipation. Over the century, many other models have been developed, to achieve the desired coefficient of restitution, impulse and energy loss according to Newton's laws, leading to the different compliance and non-smooth models. The main problem lies with the selection of the appropriate contact parameters for complex cases [24, 25] and also in the introduction of iterative high-frequency dynamics, which needs a significant reduction of step time, and thus, greatly increases the total simulation time [26].

The main disparity in the models lies in the identification of the contact points and the penetration of the bodies under impact. The geometric constraint-based models, determined through Lagrangian multipliers [27], experience co-incident contacting points [7]; whereas, a certain amount of penetration of the impacting bodies is allowed with the penalty method [28] and absolutely no penetration in the case of the non-smooth system. One of the most important aspects of all the non-smooth and compliance models is that they belong to a system of rigid body impact. When the net deformation is negligible in comparison to the total dynamics of the body, the body can be classified as a rigid body [29]. The first ever experimental test was conducted by Mier et al. [30] who analysed the response of an inverse pendulum, under impact, for different shapes of the impactor. The applications of the multibody dynamics and the impact phenomenon range from the modelling of the civil and infrastructural applications, such as, bridge segments [31-36] or closely spaced buildings [37-41] to the modelling of granular structures, like, sand, clay and others, that is, in the simulation of the contact problem experienced in the Discrete Element Method [42-44] and others. In the modern days, the application field has even extended to the field of robotics [45-47], bio-mechatronics [48-52], simulating systems with smooth particle hydrodynamics [53-55] and many others.

One of the major fields of application, of the problem of contact mechanics, include bridges, where, the challenge lies in the formulation of the responses in a robust way that is valid for most of the cases of impact. They have always been the main aspect of the highway infrastructure. Several severe earthquakes have resulted in the structural damage of many bridges due to pounding between the bridge segments and the abutments. The pounding or impact is caused when the out-of-phase vibration of the deck, caused by the earthquake, is higher than the at-rest separation or gap, which is generally around 4 cm [56, 57].

Severe structural damages in bridges and other structures, in close proximity, were primarily recorded in the San Fernando earthquake in 1971 [58, 59], which was followed by several such disasters during the Loma Prieta earthquake in 1989 , the Northridge earthquake in 1994 [59, 60], the Chile earthquake in 2010 [61], the Japan and Christchurch earthquakes in 2011 [39, 41, 62, 63] and the Nepal earthquake in 2015 [64]. These failures have motivated many researchers to analyze the response of the structures, experiencing pounding, during earthquakes.

This has led to the application of impact models in analysing the pounding responses of bridges. The impact phenomenon is in general modelled with the help of compliance methods and thus leads to a subsequent time lag. This has led to the development of an alternate method, known as the non-smooth method, which uses unilateral contact theory to calculate the response of structures under impact. Moreau [65] and Panagiotopoulos [66] were the first to implement this method which involved the impact laws, in the inequality form, that can easily be transformed into linear complementarity.

This work deals with the critical study of the different available impact models in the normal direction. The main aim is to conclude the most efficient and accurate model for simulating the response of a multi-body system experiencing impact, a process analogous to previous study [67]. This finding is further used to simulate the response of a straight bridge-abutment system, idealised into a system having single degree of freedom; process which is again adopted from previous studies [33, 68]. This work will help in understanding the most accurate model, as far as simulating the rigid multibody system is concerned, and in understanding the response of a straight bridge, idealised as an inverted pendulum, experiencing impact, when simulated with the most ideal model.

2 COMPARATIVE STUDY BETWEEN THE EXISTING MODELS

The pioneer in the study of contact kinematics was Hertz [22, 23, 69], who studied the effects due to contact of two perfectly elastic bodies, back in 1881[69]. This theory, famous as the Hertzian contact theory, was invented while studying the fringes of the optical interface in between the two lenses and the possible deformation of the surfaces of the lenses due to contact[70]. The law proposed by Hertz can be expressed as:

$$F = K\delta^n \tag{1}$$

where, $n = 3/2$, n being the non-linear power exponent, F is the contact force, δ stands for the indentation experienced and K represents the parameter for contact stiffness. The main feature of the model is the dependency of K on the material properties and geometry of the surfaces in contact [71]. Thus, the stiffness parameter varies with the variation in the geometry of the impacting surfaces. Many more advancements were made in the elastic non-linear and linear models with studies having variation in stiffness parameters with respect to the geometry [72], dependency of the model on the plastic deformation or indentation [72], application in spur gears [73] and many other fields.

Further modification led to the development of models dependent on the area of impact. The first model proposed was [74]:

$$F = 2E^* \frac{A}{\pi r_a}\delta \tag{2}$$

where, A is the impacting area and r_a is the average radius from the centre of the polygon. This was followed by further developments where different kinds of surfaces were accommodated in the models.

Studies showed that energy dissipation, during the compression and expansion stages of the contact, is always experienced in the practical field. Thus, the Hertzian contact model and the ones evolved from it failed to accommodate this dissipation criterion, which characterises the events of a mechanical structure under impact [18]. The type of such models is known as Kelvin model and the first ever energy dissipative model in contact mechanics was the Kelvin-Voigt's approach [70], which can be stated as:

$$F = K\delta + C\dot{\delta} \tag{3}$$

where, the initial term of the right-hand side (RHS) of the equation represents the linear force component in the elastic state and the second half of the RHS corresponds to the energy dissipative part of the contact. The term C represents the coefficient of damping, δ is the indentation experienced and $\dot{\delta}$ is the relative velocity, during impact, in the normal direction. Further development was carried out by Anagnostopoulos [37] who modified the first model into a model that can be used for studying the pounding of adjacent buildings, during severe earthquakes. The model proposed was:

$$F = K\delta + C\dot{\delta} \tag{4}$$

where, $C = 2\xi\sqrt{KM} = 2\xi\omega$ and $C = 2(\ln\varepsilon_N)\sqrt{KM}$, with ξ being the damping ratio and ω is the natural frequency. Advancements were made on the same type by Goyal et al. [75, 76] and Brogliato [71].

Although many models were proposed based on the linear kelvin element, the models lacked realistic and physical approach. This requirement led to the inclusion of the non-linear element into the linear equation of Kelvin. Kuwabara *et al.* [77] proposed the first Hertz-damp model, which is independent of the pre-impact velocity of the colliding bodies. The model was used to predict the coefficient of restitution for two colliding spheres and can be represented as:

$$F = K\delta^n + C\delta^m\dot{\delta} \tag{5}$$

where, the indices $n = 3/2$, $m = 1/2$ and C can be calculated from:

$$C = \frac{4D}{5}\left(R_{eff}\right)^{1/2}$$
$$D = \frac{3\left(1-\sigma_1^2\right)\left(1-\sigma_2^2\right)}{4E_1E_2} \;\&\; R_{eff} = \frac{R_1R_2}{R_1+R_2} \tag{6}$$

The pre-impact velocity, which is the velocity with which the bodies approach each other, has considerable potential to change the response of the entire multi-body system, under impact. They proposed a model combining the

IJSER

non-linear viscoelastic component and the linear Hertz law, which can be stated as:

$$F = K\delta^n + C\delta^m\dot{\delta} \quad (7)$$

where, $n = m = 3/2$ and $C = 3(1-\varepsilon_N)K/2\dot{\delta}^-$, $\dot{\delta}^-$ is the pre-impact relative velocity and K is the parameter of the contact stiffness having the same value as the one proposed by Hertz. This type of contact model is still the most used and have thus experienced maximum evolution till the model proposed by Khatiwada et al. [78] as:

$$f = K\delta^n + C\delta^m\dot{\delta} \quad (8)$$

where, $n = m = 3/2$ and $(1+\varepsilon_N) = (K/C\dot{\delta}^-)\ln\left|\left((K/C\dot{\delta}^-)+1\right)/\left((K/C\dot{\delta}^-)-\varepsilon_N\right)\right|$. The continuous models have proved their inability in calculating the exact same coefficient of restitution and the same impulse after completion of the contact phase. This led to the development of an alternative impact model considering for Hertz-damp element, which was known as the piecewise method or the multi-linear gap element. Valles and Reinhorn [79] model experiences energy dissipation in two different contact phases and thus, consists of two separate contact stiffness parameters, without any damping constant. The model can be represented as:

$$F = \begin{cases} K_1\delta & (\dot{\delta} > 0) \\ K_2(\delta - \delta_P) & (\dot{\delta} \le 0) \end{cases} \quad (9)$$

where, δ_P is the remaining displacement experienced by the model. Jankowski [80] further modified the piecewise model and incorporated the non-linear damping element, for the purpose of simulating the results of structural pounding, induced by earthquakes. The proposed model can be given as:

$$F = \begin{cases} K\delta^{3/2} + C\delta^{1/4}\dot{\delta} & (\dot{\delta} > 0) \\ K\delta^{3/2} & (\dot{\delta} \le 0) \end{cases} \quad (10)$$

where, $C = 2\xi\sqrt{KM_{eff}}$ and $\xi = \sqrt{5}\left(1-\varepsilon_N^2\right)/2\pi\varepsilon_N$ when the relationship between the rebounding and approaching velocities and $\xi = 9\sqrt{5}\left(1-\varepsilon_N^2\right)/2\varepsilon_N\left(\varepsilon_N(9\pi-16)+16\right)$ when the restitution period's relative velocity and also the relationship between the post and pre-impact velocities are considered.

The main drawbacks of the compliance method based models are the time lag experienced, making it non-suitable for instantaneous contacts, and the compulsion of the velocities and positions to be continuous with time. This problem can be addressed by the non-smooth technique where neither the model experiences any time lag nor there is any compulsion of smooth time evaluation of the velocity and position. According to Newton, the solution of linear complementarities is $\upsilon_N = 0$ because $\dot{\delta}_N^- < 0$, which yields:

$$\begin{aligned} \Lambda_N &= -(1+\varepsilon_N)M\dot{\delta}_N^- \\ \dot{\delta}_N^+ &= -\varepsilon_N\dot{\delta}_N^- \end{aligned} \quad (11)$$

Hence, Eq. (11) gives the relationship of the impulse (Λ_N) and the post-impact velocity $(\dot{\delta}_N^+)$ with the pre-impact velocity $(\dot{\delta}_N^-)$. Another law was proposed by Poisson and the solution of linear complementarity is $\Lambda_{NP} = 0$ because $\gamma_N^- < 0$, which yields:

$$\begin{aligned} \dot{\delta}_N^+ &= -\varepsilon_N\dot{\delta}_N^- \\ \Lambda_{NE} &= \Lambda_{NP} + \varepsilon_N\Lambda_{NC} \\ &= -\varepsilon_N M\dot{\delta}_N^- \end{aligned} \quad (12)$$

So, the total impulse throughout the Poisson's impact process is:

$$\begin{aligned} \Lambda_N &= \Lambda_{NC} + \Lambda_{NE} = -M\dot{\delta}_N^- - \varepsilon_N M\dot{\delta}_N^- \\ &= -(1+\varepsilon_N)M\dot{\delta}_N^- \end{aligned} \quad (13)$$

A detailed evolution of each type of contact model with the proper description of each is provided in the study done by Banerjee et al. [67]. A detailed critical analysis of the model, on a single non-dimensional frame, shows the accuracy and efficiency of each model and eventually provides the most significant and efficient one.

Figure 1 represents the impulse obtained as an output when the non-dimensional force and time are compared for all the available models. Impact models, which represent the models capable of calculating the response of a multi-body system under impact, are generally considered to be ideal when, after experiencing the energy dissipation, the final coefficient of restitution is exactly similar to the initial value previously assumed for solving the governing differential equation of motion. Another important aspect of an ideal model is that the value of the post-impact impulse being analogous to Newton's law [81]. This also implies that the area of the

ISSN 2229-5518

hysteresis loop, derived from plotting the force and displacement of the impacting body, which equals the energy lost, should be equal to the value obtained according to Newton's formulation.

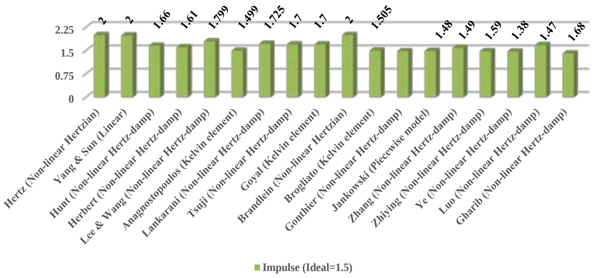

Figure 1: The post-impact impulse, obtained from the study between the dimensionless force and time, observed in each model

Most of the models are observed to fail in meeting the criterion calculated from the Newton's law and thus lack practicality and usability. Although, no models were exactly found to replicate the exact response, the models proposed by a few authors can be classified as the most plausible ones. Figure 2 represents the response of the system in the form of the post impact energy loss when the dimensionless force and displacement are studied for the different models in order to calculate the final area of the hysteresis loop.

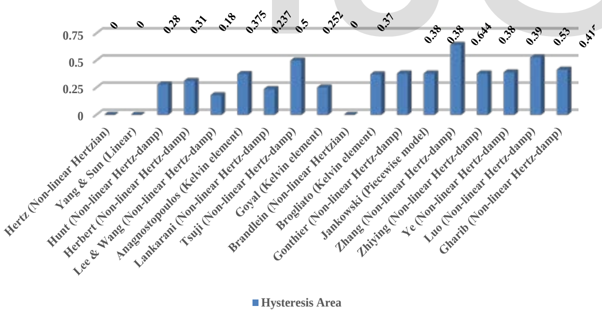

Figure 2: The post-impact energy loss, obtained from the study between the dimensionless force and displacement, resulted from each model

The final or post-impact coefficient of restitution, given by the models, can be calculated from the study conducted between the non-dimensional force and time. Figure 3 represents the comparison of the various coefficient of restitutions obtained by the models.

Figure 3: The post-impact coefficient of restitution, obtained from the study between the dimensionless force and time, given by each model

Figure 4 illustrates the percentage of error experienced by each available system when the coefficient of restitution is 0.25. The error calculation is performed over four different values of coefficient of restitution (0.25, 0.5, 0.75 and 1) and the plotted values correspond to the most unstable value of coefficient of restitution.

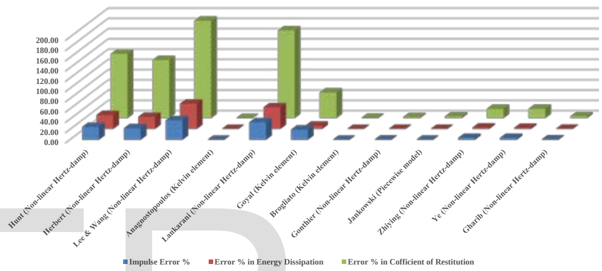

Figure 4: The percentage of error observed in each model, for all the three responses, when compared with Newton's law

It can be observed that the model proposed by Anagnostopoulos has the best response among all the available compliance models; although, the only great disadvantage of the model is the impractical aspect of negative force attained at the end of the expansion phase. Thus the most stable models can be inferred to be those proposed by Brogliato and Jankowski.

In spite of all these advantages, the models fail to perform when the coefficient of restitution tends to zero, i.e., for the cases which can be classified as the completely plastic collisions. Moreover, the small variation in the most suitable models can also be inferred as inappropriate for precise cases. These issues can be directly eliminated by considering unilateral contacts.

3 IDEALISED SYSTEM FOR STUDYING THE RESPONSE WITH THE IDEAL MODEL

A longitudinal earthquake is assumed to affect a straight bridge and abutment system, as illustrated in Figure 5. The structure is simplified into a SDOF system, demonstrated in Figure 6, experiencing a base excitation due to the earthquake. The response spectrum for analysis is adapted according to Eurocode 8 (1994) [31]. The spectrum compatible artificial accelerogram is represented in Figure 7.

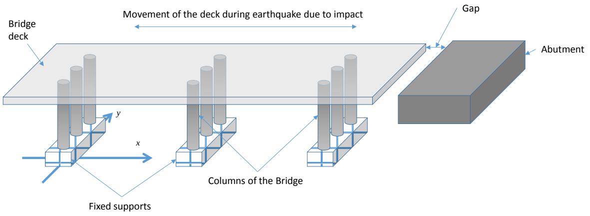

Figure 5: Illustration of a bridge-abutment system, having straight orientation

A standard straight bridge with a length of 50 m and a width of 20 m is considered for the analysis. The thickness of the bridge is considered to be 0.5 m with equal columns of 15 m height and 0.4 m radius. The deck and the columns of the bridge are idealized into a single fixed to fixed column supporting the lump mass of the bridge, for representing it as a single degree of freedom system. A structural damping of 5% for concrete is also considered for the analysis, as commonly adopted [57].

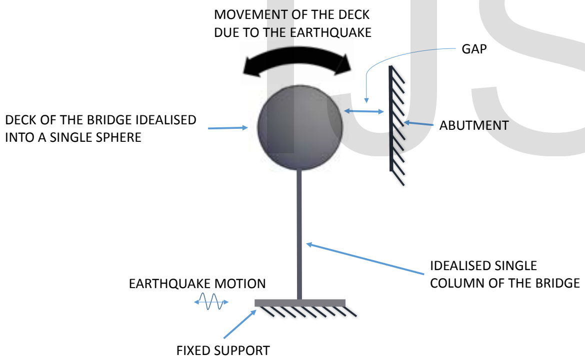

Figure 6: Idealisation of the straight bridge-abutment structure into a single degree of freedom system

The equation of motion of the system under study is:

$$M\ddot{u} + 2\xi\omega M\dot{u} + Ku = -M\ddot{u}g \qquad (14)$$

where, M is the mass of the system, u is the displacement in the longitudinal direction, ξ is the damping coefficient, K is the lateral stiffness of the column and ω is the frequency. The response of the bridge is analysed for different structures with various gaps and columns. A comparative study is carried out for the straight bridge with different coefficients of restitution, namely 0.2, 0.4, 0.6, 0.8 and 1. For each value, the gaps of 0.01, 0.03 and 0.05 along with a non-impacting case were considered. All the cases were considered for a bridge having 6, 9 and 12 columns.

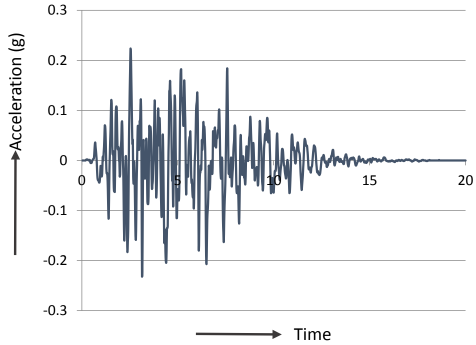

Figure 7: Earthquake motion, used for the analysis, which is spectrum compatible [82]

4 RESULTS AND DISCUSSIONS

The non-smooth method, incorporating unilateral contact, observed to be the most accurate for simulating the phenomenon of impact of rigid segments, is used to analyze the response of the system in consideration. The response of the equivalent single degree of freedom, bridge deck-abutment, system is analysed by comparing the displacement, velocity and acceleration time histories of the impacting system. The responses are compared for systems with different coefficients of restitution, gaps between the deck and the abutment and the number of columns. The gap between the deck and the abutment is varied, in order to understand how the responses vary for the different kinds of impact cases and also for the case of no impact. Similarly, the number of columns are varied to understand the effect of stiffness on the responses. The simulation is an extension of the work carried out by Chanda et al. [68] with the main difference being in the number of columns. The number of columns was assumed to a single entity without any change; whereas, in the present work, the number of columns are varied as well.

Figure 8 illustrates the variation observed when the displacement time history is plotted. The increase in stiffness can be observed to reduce the amount of displacement. The maximum amount of displacement can be observed when the gap is extremely small and with less number of columns. The lesser the gap the more is the possibility of displacement, therefore signifying the fact that the bridge becomes vulnerable when the gap is very small (e.g. 0.01 m). For more realistic cases (e.g. 0.03 m and 0.05 m) the displacement increases gradually with the increase in gap.

IJSER

ISSN 2229-5518

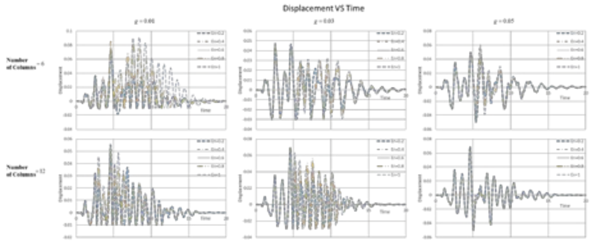

Figure 8: Displacement-time history of the bridge-abutment system with different coefficients of restitution for different gaps and columns

Figure 9 illustrates the time history analysis of the post-impact velocity experienced by the system under consideration. A similar response is observed with the maximum velocity being experienced when the gap between the deck and the abutment is 0.01 m. Also, increase in stiffness, results in lesser post-impact velocity of the structure.

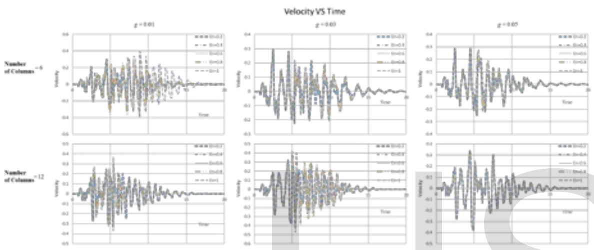

Figure 9: Velocity-time history of the bridge-abutment system with different coefficients of restitution for different gaps and columns

Figure 10 shows the variation in the acceleration of the straight bridge segment due to impact with different coefficients of restitution and gaps. It can be observed that the duration of the acceleration fluctuation increases with an increase in the coefficient of restitution for a given gap. It can also be observed that the fluctuation is the highest when the gap is minimum (0.01 m), although, in the other cases, it increases with the gap; a result which is obtained because of the repetitive pounding experienced by the deck.

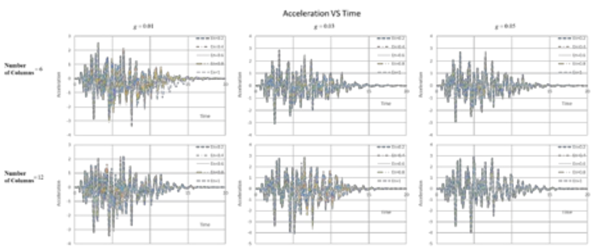

Figure 10: Acceleration-time history of the bridge-abutment system with different coefficients of restitution for different gaps and columns

With the number of columns being 9, the structure fails to give response for the entire 20 seconds of the earthquake motion. This is a phenomenon which requires further investigation. For other gaps (0.03 m and 0.05 m), it behaves analogous to the cases experienced by structures having 6 and 12 columns. In the case of no-impact, the response of the bridge is also quite high, when compared to those with gaps of 0.03 m and 0.05 m, and is independent of the coefficient of restitution.

5 CONCLUSION

A proper critical evaluation of the efficiency of the available impact response analysing models, in the normal direction, have been presented to find out the most efficient one. Generally, the errors in compliances model increase with the decrement of the coefficient of restitution and solution of most of the Hertzian model becomes unstable for the purely plastic collision; however, the non-smooth models and Kelvin elements are stable in all ranges of coefficients of restitution.

The most practically efficient model was observed to be non-smooth and was used to simulate the seismic response of a straight bridge abutment structure, idealised into a single degree of freedom system. It can be observed that the vulnerability of the bridge increases with the increase in the coefficient of restitution because of energy loss. Also, an increase in the velocity was observed with the increase in gap; although, it is very high for very small gaps. The increase in the number of columns results in the bridge becoming safer due to the increase in stiffness. The no-impact case has a very smooth response similar to the earthquake excitation, though amplified. Thus, a low coefficient of restitution leads to more energy dissipation and is preferable for bridges to be safe, the bridge-abutment gap should be standardized to around 0.03 m and the number of columns should be maximized for a particular design.

REFERENCES

1. Flores, P., et al., *Kinematics and dynamics of multibody systems with imperfect joints: models and case studies*. Vol. 34. 2008: Springer Science & Business Media.
2. Nikravesh, P.E., *Newtonian-based methodologies in multi-body*

IJSER

dynamics. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2008. **222**(4): p. 277-288.

3. Nikravesh, P.E., *Computer-aided analysis of mechanical systems*. 1988: Prentice-Hall, Inc.
4. Ambrósio, J. and P. Verissimo, *Improved bushing models for general multibody systems and vehicle dynamics*. Multibody System Dynamics, 2009. **22**(4): p. 341-365.
5. Flores, P., *Modeling and simulation of wear in revolute clearance joints in multibody systems*. Mechanism and Machine Theory, 2009. **44**(6): p. 1211-1222.
6. Machado, M., et al., *Development of a planar multibody model of the human knee joint*. Nonlinear Dynamics, 2010. **60**(3): p. 459-478.
7. Alves, J., et al., *A comparative study of the viscoelastic constitutive models for frictionless contact interfaces in solids*. Mechanism and Machine Theory, 2015. **85**: p. 172-188.
8. Bhalerao, K. and K. Anderson, *Modeling intermittent contact for flexible multibody systems*. Nonlinear Dynamics, 2010. **60**(1-2): p. 63-79.
9. Choi, J., et al., *An efficient and robust contact algorithm for a compliant contact force model between bodies of complex geometry*. Multibody System Dynamics, 2010. **23**(1): p. 99-120.
10. Ebrahimi, S., G. Hippmann, and P. Eberhard, *Extension of the polygonal contact model for flexible multibody systems*. International Journal of Applied Mathematics and Mechanics, 2005. **1**(1): p. 33-50.
11. Flores, P., R. Leine, and C. Glocker, *Application of the nonsmooth dynamics approach to model and analysis of the contact-impact events in cam-follower systems*. Nonlinear Dynamics, 2012. **69**(4): p. 2117-2133.
12. Hirschkorn, M., J. McPhee, and S. Birkett, *Dynamic Modeling and Experimental Testing of a Piano Action Mechanism*. Journal of Computational and Nonlinear Dynamics, 2005. **1**(1): p. 47-55.
13. Minamoto, H. and S. Kawamura, *Moderately high speed impact of two identical spheres*. International Journal of Impact Engineering, 2011. **38**(2–3): p. 123-129.
14. Dopico, D., et al., *Dealing with multiple contacts in a human-in-the-loop application*. Multibody System Dynamics, 2011. **25**(2): p. 167-183.
15. Gonzalez-Perez, I., J.L. Iserte, and A. Fuentes, *Implementation of Hertz theory and validation of a finite element model for stress analysis of gear drives with localized bearing contact*. Mechanism and Machine Theory, 2011. **46**(6): p. 765-783.
16. Pham, H.-T. and D.-A. Wang, *A constant-force bistable mechanism for force regulation and overload protection*. Mechanism and Machine Theory, 2011. **46**(7): p. 899-909.
17. Zhu, S., S. Zwiebel, and G. Bernhardt, *A theoretical formula for calculating damping in the impact of two bodies in a multibody system*. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 1999. **213**(3): p. 211-216.
18. Machado, M., et al., *Compliant contact force models in multibody dynamics: Evolution of the Hertz contact theory*. Mechanism and Machine Theory, 2012. **53**: p. 99-121.
19. Dietl, P., J. Wensing, and G. Van Nijen, *Rolling bearing damping for dynamic analysis of multi-body systems—experimental and theoretical results*. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2000. **214**(1): p. 33-43.
20. Hunt, K.H. and F.R.E. Crossley, *Coefficient of Restitution Interpreted as Damping in Vibroimpact*. Journal of Applied Mechanics, 1975. **42**(2): p. 440-445.
21. Moreira, P., P. Flores, and M. Silva. *A biomechanical multibody foot model*

IJSER

for forward dynamic analysis. in *Bioengineering (ENBENG), 2012 IEEE 2nd Portuguese Meeting in*. 2012. IEEE.

22. Hertz, H., *Über die Berührung fester elastischer Körper*. Journal f¨ur die Reine und Angewandte Mathematik, 1882. **29**: p. 156–171.
23. Hertz, H., *On the contact of elastic solids*. J. reine angew. Math, 1881. **92**(156-171): p. 110.
24. Mahmoud, S. and R. Jankowski, *Modified linear viscoelastic model of earthquake-induced structural pounding*. Iran. J. Sci. Technol, 2011. **35**: p. 51-62.
25. Shivaswamy, S. and H.M. Lankarani, *Impact Analysis of Plates Using Quasi-Static Approach*. Journal of Mechanical Design, 1997. **119**(3): p. 376-381.
26. Flores, P. and J. Ambrósio, *On the contact detection for contact-impact analysis in multibody systems*. Multibody System Dynamics, 2010. **24**(1): p. 103-122.
27. Greenwood, D.T., *Principles of dynamics*. 1988: Prentice-Hall Englewood Cliffs, NJ.
28. Lankarani, H. and P. Nikravesh, *Application of the canonical equations of motion in problems of constrained multibody systems with intermittent motion*. Advances in Design Automation, 1988. **1**: p. 417-423.
29. Shabana, A.A., *Dynamics of multibody systems*. 2013: Cambridge university press.
30. van Mier, J.G., et al., *Load-time response of colliding concrete bodies*. Journal of Structural Engineering, 1991. **117**(2): p. 354-374.
31. Dimitrakopoulos, E., *Analysis of a frictional oblique impact observed in skew bridges*. Nonlinear Dynamics, 2010. **60**(4): p. 575-595.
32. Dimitrakopoulos, E., A.J. Kappos, and N. Makris, *Dimensional analysis of yielding and pounding structures for records without distinct pulses*. Soil Dynamics and Earthquake Engineering, 2009. **29**(7): p. 1170-1180.
33. Dimitrakopoulos, E., N. Makris, and A. Kappos, *Dimensional Analysis of the Earthquake Response of a Pounding Oscillator*. Journal of Engineering Mechanics, 2010. **136**(3): p. 299-310.
34. Dimitrakopoulos, E., N. Makris, and A.J. Kappos, *Dimensional analysis of the earthquake-induced pounding between adjacent structures*. Earthquake Engineering & Structural Dynamics, 2009. **38**(7): p. 867-886.
35. Dimitrakopoulos, E.G., *Nonsmooth analysis of the impact between successive skew bridge-segments*. Nonlinear Dynamics, 2013. **74**(4): p. 911-928.
36. Julian, F.D.R., T. Hayashikawa, and T. Obata, *Seismic performance of isolated curved steel viaducts equipped with deck unseating prevention cable restrainers*. Journal of Constructional Steel Research, 2007. **63**(2): p. 237-253.
37. Anagnostopoulos, S.A., *Pounding of buildings in series during earthquakes*. Earthquake Engineering & Structural Dynamics, 1988. **16**(3): p. 443-456.
38. Anagnostopoulos, S.A. and K.V. Spiliopoulos, *An investigation of earthquake induced pounding between adjacent buildings*. Earthquake engineering & structural dynamics, 1992. **21**(4): p. 289-302.
39. Cole, G., et al., *Interbuilding pounding damage observed in the 2010 Darfield earthquake*. Bulletin of the New Zealand Society for Earthquake Engineering, 2010. **43**(4): p. 382.
40. Cole, G.L., *The effects of detailed analysis on the prediction of seismic building pounding performance*, in *Civil Engineering*. 2012, University of Canterbury: Canterbury, New Zealand.
41. Cole, G.L., R.P. Dhakal, and F.M. Turner, *Building pounding damage*
IJSER

observed in the 2011 Christchurch earthquake. Earthquake Engineering & Structural Dynamics, 2012. **41**(5): p. 893-913.

42. Fleischmann, J., *DEM-PM Contact Model with Multi-Step Tangential Contact Displacement History*. 2015, Simulation-Based Engineering Laboratory: University of Wisconsin-Madison.
43. Mishra, B.K., *A review of computer simulation of tumbling mills by the discrete element method: Part II—Practical applications.* International Journal of Mineral Processing, 2003. **71**(1–4): p. 95-112.
44. Williams, J.R. and R. O'Connor, *Discrete element simulation and the contact problem.* Archives of Computational Methods in Engineering, 1999. **6**(4): p. 279-304.
45. Bi, S.S., X.D. Zhou, and D.B. Marghitu, *Impact modelling and analysis of the compliant legged robots.* Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2012. **226**(2): p. 85-94.
46. Marhefka, D.W. and D.E. Orin, *A compliant contact model with nonlinear damping for simulation of robotic systems.* Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 1999. **29**(6): p. 566-572.
47. Verscheure, D., et al., *Identification of Contact Parameters from Stiff Multi-point Contact Robotic Operations.* The International Journal of Robotics Research, 2010. **29**(4): p. 367-385.
48. Argatov, I., *Development of an asymptotic modeling methodology for tibio-femoral contact in multibody dynamic simulations of the human knee joint.* Multibody System Dynamics, 2012. **28**(1-2): p. 3-20.
49. Argatov, I. and G. Mishuris, *Articular Contact Mechanics*, in *Contact Mechanics of Articular Cartilage Layers*. 2015, Springer International Publishing. p. 229-259.
50. Askari, E., et al., *Study of the friction-induced vibration and contact mechanics of artificial hip joints.* Tribology International, 2014. **70**: p. 1-10.
51. Morales-Orcajo, E., J. Bayod, and E.B. de Las Casas, *Computational Foot Modeling: Scope and Applications.* Archives of Computational Methods in Engineering, 2015: p. 1-28.
52. Silva, P., M. Silva, and J. Martins, *Evaluation of the contact forces developed in the lower limb/orthosis interface for comfort design.* Multibody System Dynamics, 2010. **24**(3): p. 367-388.
53. Das, R. and P.W. Cleary, *Effect of rock shapes on brittle fracture using Smoothed Particle Hydrodynamics.* Theoretical and Applied Fracture Mechanics, 2010. **53**(1): p. 47-60.
54. Das, R. and P.W. Cleary, *Modelling 3D fracture and fragmentation in a thin plate under high velocity projectile impact using SPH*, in *3rd SPHERIC workshop*. 2008: Lausanne, Switzerland.
55. Liu, M. and G. Liu, *Smoothed particle hydrodynamics (SPH): an overview and recent developments.* Archives of computational methods in engineering, 2010. **17**(1): p. 25-76.
56. (AASHTO), A.A.o.S.H.a.T.O., *Section 6: Foundation and Abutment Design*, in *(AASHTO) Guide Specifications of Bridges*. 2011: Washington DC.
57. (AASHTO), A.A.o.S.H.a.T.O., *Chapter 4: Analysis and Design Requirements*, in *(AASHTO) Guide Specifications of Bridges*. 2011: Washington DC.
58. Jennings, P.C., *Engineering features of the San Fernando earthquake of February 9, 1971*. 1971.
59. Yashinsky, M. and M. Karshenas, *Fundamentals of seismic protection for bridges*. 2003: National Information Centre of Earthquake Engineering.
IJSER

60. Priestley, M.N., F. Seible, and G.M. Calvi, *Seismic design and retrofit of bridges*. 1996: John Wiley & Sons.
61. Elnashai, A.S., et al., *The Maule (Chile) earthquake of February 27, 2010: Development of hazard, site specific ground motions and back-analysis of structures.* Soil Dynamics and Earthquake Engineering, 2012. **42**: p. 229-245.
62. CENTER, J.B.E., *Damage to Highway Bridges Caused by the 2011 Tohoku-Oki Earthquake*. 2011: Tokyo.
63. Chouw, N. and H. Hao, *Pounding damage to buildings and bridges in the 22 February 2011 Christchurch earthquake.* International Journal of Protective Structures, 2012. **3**(2): p. 123-140.
64. Thapa, M.B., N. Giri, and M. Basnyat, *Experience of Nepal: Implication to Risk Reduction*, in *Disaster Management and Private Sectors*. 2015, Springer. p. 301-313.
65. Moreau, J.J., *Unilateral Contact and Dry Friction in Finite Freedom Dynamics*, in *Nonsmooth Mechanics and Applications*, J.J. Moreau and P.D. Panagiotopoulos, Editors. 1988, Springer Vienna. p. 1-82.
66. Panagiotopoulos, P.D., *Nonconvex energy functions. Hemivariational inequalities and substationarity principles.* Acta Mechanica, 1983. **48**(3-4): p. 111-130.
67. Banerjee, A., A. Chanda, and R. Das, *Historical Origin and Recent Development on Normal Directional Impact Models for Rigid Body Contact Simulation: A Critical Review.* Archives of Computational Methods in Engineering, 2016: p. 1-26.
68. Chanda, A., A. Banerjee, and R. Das, *Sensitivity Analysis of the Impact Parameters on the Seismic Response of Straight Bridges*, in *2nd Australasian Conference on Computational Mechanics*. 2015: Brisbane, Australia.
69. Johnson, K., *One hundred years of Hertz contact.* proceedings of the Institution of Mechanical Engineers, 1982. **196**(1): p. 363-378.
70. Goldsmith, W., *Impact*. 2001: Courier Corporation.
71. Brogliato, B., *Nonsmooth mechanics: models, dynamics and control*. 1999: Springer.
72. Goldsmith, W., *Impact: The theory and physical behavior of colliding solids*. 1960: E. Arnold.
73. Yang, D. and Z. Sun, *A rotary model for spur gear dynamics.* Journal of Mechanical Design, 1985. **107**(4): p. 529-535.
74. Luo, L. and M. Nahon, *A Compliant Contact Model Including Interference Geometry for Polyhedral Objects.* Journal of Computational and Nonlinear Dynamics, 2005. **1**(2): p. 150-159.
75. Goyal, S., E. Pinson, and F. Sinden, *Simulation of dynamics of interacting rigid bodies including friction I: General problem and contact model.* Engineering with Computers, 1994. **10**(3): p. 162-174.
76. Goyal, S., E.N. Pinson, and F.W. Sinden, *Simulation of dynamics of interacting rigid bodies including friction II: Software system design and implementation.* Engineering with computers, 1994. **10**(3): p. 175-195.
77. Kuwabara, G. and K. Kono, *Restitution coefficient in a collision between two spheres.* Japanese journal of applied physics, 1987. **26**(8R): p. 1230.
78. Khatiwada, S., N. Chouw, and J.W. Butterworth, *A generic structural pounding model using numerically exact displacement proportional damping.* Engineering Structures, 2014. **62–63**: p. 33-41.
79. Valles, R.E. and A.M. Reinhorn, *Evaluation, prevention and mitigation of pounding effects in building structures*. 1997, National Center for Earthquake Engrg. Res., University of Buffalo: Buffalo, USA.
IJSER

80. Jankowski, R., *Non-linear viscoelastic modelling of earthquake-induced structural pounding*. Earthquake engineering & structural dynamics, 2005. **34**(6): p. 595-611.
81. Newton, I., *The principia: mathematical principles of natural philosophy*. 1999: Univ of California Press.
82. Banerjee, A., D.K. Paul, and A. Acharyya, *Optimization and safety evaluation of concrete gravity dam section*. KSCE Journal of Civil Engineering, 2015. **19**(6): p. 1612-1619.

IJSER