Spectrum Sensing Techniques for Cooperative Cognitive Radio Networks to Guarantee Primary User Performance in AWGN Channel

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Abstract—Cognitive radio technology has been proposed to improve spectrum efficiency by having the cognitive radios act as secondary users to opportunistically access under-utilized frequency bands. Frequency spectrum sharing between licensed primary users (PUs) and unlicensed secondary users (SUs) requires the SUs reliably detect the spectrum occupancy. Spectrum sensing, as a key enabling functionality in cognitive radio networks, needs to reliably detect signals from PUs to avoid harmful interference. However, due to the effects of channel fading/shadowing, individual SUs may not be able to reliably detect the existence of a PU so that multiple SUs can cooperate to conduct spectrum sensing. In this paper, we derive an optimal voting rule for any detector applied to cooperative spectrum sensing such that minimizing the Bayes risk function. Furthermore, we derive an algorithm to optimize the energy detection threshold for the cognitive users for any fusion rule. Furthermore, we propose a new algorithm that determines the optimal fusion rule and optimum threshold that minimizes the false alarm probability such that the missing probability under constrains (bounded error).

Index Terms—Cognitive radio, cooperative spectrum sensing, Bayes risk function, optimization, energy detection, error rate probability

1 INTRODUCTION

Recent years, with the rapid development of wireless communication technology, more and more spectrum resources are needed to support the high data rate. Spectrum scarcity becomes a problem. Recent studies by the Federal Communications Commission (FCC) Spectrum Policy Task Force (SPTF) have demonstrated that the actual licensed spectrum is largely unoccupied most of the time [1]. Another recent work on spectrum occupancy measurements showed that the average spectrum occupancy from 30 MHz to 3 GHz over six cities is 5.2% [2]. The cognitive radio (CR) has been proposed [3], [4] to mitigate the conflict between spectrum scarcity and low spectrum efficiency.

One of the most challenging tasks in CR networks is spectrum sensing, which is required to opportunistically access the idle radio spectrum. Generally, the spectrum sensing techniques can be classified as energy detection, matched filter detection, and cyclostationary feature detection [5]. Among these techniques, energy detection has low complexity, low implementation cost, and demands none a priori about the PU signal [6]. So in this paper, we use energy detection as the local spectrum sensing scheme.

One of the great challenges of implementing spectrum sensing is the hidden terminal problem, which occurs when the cognitive radio is shadowed, in severe multipath fading or inside buildings with high penetration loss, while a primary user (PU) is operating in the vicinity [7]. Due to the hidden terminal problem, a cognitive radio may fail to notice the presence of the PU and then will access the licensed channel and cause interference to the licensed system.

In order to deal with the hidden terminal problem in cognitive radio networks, cooperate spectrum sensing is proposed. It has been shown that spectrum sensing performance can be greatly improved with an increase of the number of cooperative partners [8]–[12].

It should be mentioned that optimal spectrum sensing under data fusion was investigated in [13], where the optimal linear function of weighted data fusion has been obtained. In other recent works [14], [15], optimal sensing throughput tradeoff was studied. Optimal distributed signal detection with likelihood ratio test using reporting channels from the CRs to the fusion center has been dealt with in [16]. It should be mentioned that optimal voting rule and the optimal detection threshold are discussed in [17].

The rest of the paper is organized as follows. Section II the system model of the paper and define the symbols that we will use in the paper. Section III discussing how to achieve our objectives and show the simulation results of the proposed technique. Section IV proposes another technique that guarantees certain performance. Finally, section V concludes the paper.

2 SYSTEM MODEL

We consider a cognitive radio network where there are K secondary users and central node. We assume that each secondary user perform spectrum sensing independently and takes
its local decision then sends the binary decision 0 or 1 which represent the channel is occupied or the channel is unoccupied respectively to the central node which fuse the K binary decisions from the K secondary users to decide whether the primary user is present or absent.

Then the spectrum sensing can be modeled as a binary hypothesis testing problem with hypothesis $H_0$ and $H_1$ denoting the absence and presence of a primary user, respectively. In the proposed model, the low-pass equivalent of the $i^{th}$ sample of the received signal at the $k^{th}$ radio is written as:

$$r_k (i) = \begin{cases} w_k (i), & \text{if } i \leq \lambda_k \\ h_k (i) s (i) + w_k (i), & \text{if } i > \lambda_k \end{cases}$$

For $i = 1, 2, ..., I$ and $k = 1, 2, ..., K$ and; that is, the observation window of each radio has I samples, and sensing is performed with K radios. We assume sampling at the symbol rate.

Where $r_k (i)$ is the received signal at the $k^{th}$ CR at the $i^{th}$ sample, $w_k (i)$ is the additive white Gaussian noise (AWGN) of the $k^{th}$ CR at the $i^{th}$ sample, $h_k (i)$ is the complex gain of the sensing channel between the PU and the $k^{th}$ CR at the $i^{th}$ sample, and $s (i)$ is the PU signal at the $i^{th}$ sample. Assume the spectrum sensing performed by energy detector as mentioned in [18] If the secondary user has limited information on the primary signals (e.g., only the local noise power is known) then the energy detector is optimal. The sensing channel can be viewed as time-invariant during the sensing process. Assume the reporting channel is an error free.

For the $k^{th}$ CR with the energy detector, the average probability of false alarm, the average probability of detection, and the average probability of missed detection over AWGN channels are given, respectively, by [19].

$$P_{f,k} = \frac{\Gamma \left(u, \frac{\lambda_k}{2} \right)}{\Gamma (u)}$$

$$P_{d,k} = Q_u \left( \sqrt{2\gamma_k}, \sqrt{\lambda_k} \right)$$

And

$$P_{m,k} = 1 - P_{d,k}$$

Where $\gamma_k$ and $\lambda_k$ denote the instantaneous signal-to-noise ratio (SNR) and the energy detection threshold of the $k^{th}$ CR respectively, $u$ is time-bandwidth product of the energy detector, $\Gamma (a, x)$ is the incomplete gamma function, $\Gamma (a)$ is the gamma function, and $Q_u (a, b)$ is the generalized Marcum Q-function.

In cooperative spectrum sensing each cognitive radio device make a binary decision depending on its local observation and then forwards one bit of the decision $D_k$ (1 standing for the presence of the PU, 0 for the absence of the PU) to the central node through an error-free channel. At the central node, all 1-bit decisions are fused together according to logic rule

$$Z = \sum_{i=1}^{K} D_k \begin{cases} \geq n, & \text{if } \text{true decision} \\ < n, & \text{if } \text{false decision} \end{cases}$$

Where the threshold $n$ is an integer, representing the “n-out-of-K” voting rule.

3.1 Optimum Fusion Rule with Minimum Risk function

In [17] the author defined, the total error rate as:

Total error rate $= Q_f + Q_m$ (i.e. $Q_f$ and $Q_m$ are equal weights).

Then he got an expression for the optimal value of $n$ for the “n-out-of-K” rule.

As known in cognitive radio the missing probability $Q_m$ is more important than the false alarm probability $Q_f$ because the missing probability $Q_m$ means that the channel is occupied by the primary user and the cognitive user can’t detect it, then this error will make the cognitive user to operate and cause harmful interference to the primary user, but the probability of false alarm $Q_f$ means that while the channel is idle, the cognitive user decide the channel is occupied, then this error will make to decrease the utilization of the spectrum, for this reason $Q_m$ is still more dangerous than $Q_f$.

Then we want to derive an expression for the optimal voting rule that minimizes the Bayes risk function in which $Q_f$ and $Q_m$ are unequal weights(weight of $Q_f$ is greater than weight of $Q_m$), and this expression for the optimal value of $n$ will be a very interesting issue in cognitive radio technology.
In this paper we will define the Bayes risk function $\mathcal{R}$ as:

$$
\mathcal{R} = C_{00}P_0P_0 + C_{11}P_1P_1 + C_{10}P_0P_1 + C_{01}P_0P_1
$$

(7)

Where,

$c_{ij}$ = the cost of deciding the channel is $H_i$, while the channel is $H_j$.

$p_{ij}$ = the probability of deciding the channel is $H_i$, while the channel is $H_j$.

$p_i$ = probability of the channel is $H_i$.

Now assume that $C_{00} = C_{11} = 0$ (i.e., the correct decision costs are equal zero), and let $P_{10} = Q_f$ and $P_{01} = Q_m$, then

$$
\mathcal{R} = C_{10}P_0Q_f + C_{01}P_1Q_m
$$

Let $C_{10}P_0 = K_1$ and $C_{01}P_1 = K_2$

Let $\beta = \frac{K_1}{K_1 + K_2}$

Then

$$
\mathcal{R} = (K_1 + K_2) \left[ \beta Q_f + (1 - \beta)Q_m \right]
$$

(8)

We want to minimize the risk function $\mathcal{R}$ with respect to $n$

Let

$$
G(n) = \sum_{l=n}^{K} \left[ \beta p_f^l (1 - p_f)^{K - l} - (1 - \beta) \left(1 - p_m\right)^{l} P_m^{K - l} \right]
$$

$$
\mathcal{R} = (K_1 + K_2) \left[ 1 + G(n) \right]
$$

$$
\frac{\partial \mathcal{R}}{\partial n} = (K_1 + K_2) \frac{\partial G}{\partial n} = (K_1 + K_2) \left[ G(n + 1) - G(n) \right] = 0
$$

$$
\beta p_f^n (1 - p_f)^{K - n} = (1 - \beta) \left(1 - p_m\right)^n P_m^{K - n}
$$

$$
n \left( \ln \left( \frac{p_f}{1 - p_m} \right) + \ln \left( \frac{p_m}{1 - p_f} \right) \right) = K \left( \ln \left( \frac{p_m}{1 - p_f} \right) - \ln \left( \frac{1}{1 - \beta} \right) \right)
$$

Then

$$
n_{\text{opt}} = \frac{K \ln \left( \frac{p_m}{1 - p_f} \right) - \ln \left( \frac{1}{1 - \beta} \right)}{\ln \left( \frac{p_f}{1 - p_m} \right) + \ln \left( \frac{p_m}{1 - p_f} \right)}
$$

Let

$$
\alpha = \frac{\ln \left( \frac{p_f}{1 - p_m} \right)}{\ln \left( \frac{p_m}{1 - p_f} \right)}
$$

Then we get an expression for the optimal value of $n$ for the “$n$-out-of-$K$” rule in case of $Q_f$ and $Q_m$ are unequal weights. Then $n_{\text{opt}}$ depends on the total number of users $K$ and energy detection threshold $\lambda$, fig. (1) illustrates $n_{\text{opt}}$ versus $K$ with constant energy detection threshold $\lambda = 5$ and $\text{SNR}=2 \text{dB}$ at $\beta = \{0.01, 0.5\}$.

Fig. (2) illustrates that the optimal voting rule “$n_{\text{opt}}$” depends on the energy detection threshold, for each value of $\lambda$ has $n$ where $K=20$ for $\beta=0.01$ and 0.5, from this figure also, we can note that as the detection threshold increases the optimal $n$ decreases this is a reasonable result.

From the figure also we note, as $\beta$ decreases, $n$ decreases at the same energy detection threshold because of $\beta$ is the weight of $Q_f$ while $(1 - \beta)$ is the weight of $Q_m$ then, when $\beta$ decreases the weight of $Q_m$ increases (i.e. $Q_m$ should be decreased) then decrease $n$ to decrease $Q_m$. This can be illustrated in
fig.(3) and fig.(4) where the energy detection threshold \( \lambda = 8 \) from these figures we note \( Q_m \) is almost equal to \( Q_f \) at \( \beta = 0.5 \), and \( Q_m \) is decreased and \( Q_f \) is increased as \( \beta \) decreases as shown in fig.(3) and fig.(4). And from these figures, we can note that as \( K \) increases the probability of false alarm and the missing probability decrease, this note illustrates the improvement from no cooperative (\( K = 1 \)) and cooperative system (\( K > 1 \)).

From (8) the Bayes risk function.

\[
\mathcal{R} = (K_1 + K_2) \text{ERP} 
\]  

(10)

Where error rate probability (ERP) is

\[
\text{ERP} = \beta Q_f + (1 - \beta) Q_m
\]  

(11)

Fig. (2) Optimal voting rule versus detection threshold of cooperative spectrum sensing in AWGN channel with SNR = 2 dB, \( K = 20 \), \( \beta = \{0.01, 0.5\} \)

In fig.(5) illustrates the ERP at \( \beta = \{0.01, 0.1, 0.5\} \) and \( \lambda = 8 \). The figure illustrates the ERP at different values of \( \beta \). The improvement with decreasing \( \beta \) is placebo and non-real because of the energy threshold is fixed.

3.2 Optimal Energy Detection Threshold

If \( K \) and SNR \( \gamma \) are known and \( n_{opt} \) is expressed as in (9) then we need to know \( \lambda \) that minimizes the ERP.

Now we determine the optimum energy detection threshold \( \lambda_{opt} \) where \( K \), \( n \), and SNR \( \gamma \) are known such that

\[
\lambda_{opt} = \arg \min (\beta Q_f + (1 - \beta) Q_m).
\]

In [17] we note that the error rate probability ERP in terms of \( \lambda \) has a global minimum in \( \lambda \) for each \( n \). This means that there exist only one value of \( \lambda \) that minimizes the risk
function(\(\lambda_{opt}\)).

\[
\lambda_{opt} = \text{argmin} \left( \beta Q_f + (1 - \beta)Q_m \right)
\]

(12)

Then, to get an expression for \(\lambda_{opt}\), differentiate the risk function and equal it to zero.

\[
\beta \frac{\partial Q_f}{\partial \lambda} + (1 - \beta) \frac{\partial Q_m}{\partial \lambda} = 0
\]

(13)

The expressions for \(\frac{\partial Q_m}{\partial \lambda}\) and \(\frac{\partial Q_f}{\partial \lambda}\) were derived in [17, Eq.(9) and Eq.(11)]. However, it is quite difficult to get the closed-form solution of \(\lambda\). Therefore, a search for the optimal \(\lambda\) is required.

Because of there exist only one value of \(\lambda\) that minimizes the risk function(\(\lambda_{opt}\)), we can employ one dimensional optimization algorithms (e.g., Bisection search, Fibonacci search, Golden section search, Newton search) to find \(\lambda_{opt}\).

An example to find \(\lambda_{opt}\) is given in algorithm 1.

**Algorithm 1** Find \(\lambda_{opt}\) assume \(K\), \(n\) and SNR \(\gamma\) are known.

| Input: K, n, SNR \(\gamma\), and \(\Delta\) (\(\Delta\) is the tolerance of accuracy of \(\lambda_{opt}\) |
|---|---|
| \(\lambda^{(i-1)} = 0, \lambda^{(0)} = 1, l = 0\) | while \(\lambda^{(i)} - \lambda^{(i-1)} > \Delta\) |
| calculate \(\lambda^{(i+1)},\) \[
\lambda^{(i+1)} = \frac{\partial \text{ERP}}{\partial \lambda} \left( \lambda^{(i)} \right) - \frac{\partial \text{ERP}}{\partial \lambda} \left( \lambda^{(i-1)} \right) \]

\[
\frac{\partial \text{ERP}}{\partial \lambda} \left( \lambda^{(i)} \right) - \frac{\partial \text{ERP}}{\partial \lambda} \left( \lambda^{(i-1)} \right)
\]

<table>
<thead>
<tr>
<th>(l = l + 1)</th>
<th>end while</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output: (\lambda_{opt} = \lambda^{(i)})</td>
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Algorithm 1 in this paper is faster than algorithm 1 in [20] to find \(\lambda_{opt}\) with certain tolerance of accuracy.

### 3.3 Optimum Fusion Rule and Energy Threshold

The pair \((\lambda, n)\) that yields the lowest error rate probability will be chosen as optimal parameters for the scheme. This process is shown in the following algorithm.

**Algorithm 2** Find \(\lambda_{opt}\) and \(n_{opt}\) assume \(K\), \(\beta\), and \(\gamma\) are known.

| Input: K, \(\beta\), and SNR \(\gamma\)|
|---|---|
| \(n_{opt} = 1, \lambda_{opt} = 1, \text{ERP}_{min} = 1\) | for \(n = 1\) to \(K\) |
| Find \(\lambda^*\) using Algorithm 1 | Find \(\text{ERP}(n, \lambda^*)\) |
| if \(\text{ERP}(n, \lambda^*) < \text{ERP}_{min}\) | \(n_{opt} = n, \lambda_{opt} = \lambda^*, \text{ERP}_{min} = \text{ERP}(n, \lambda^*)\) |
| end if | end for |
| Output: \((n_{opt}, \lambda_{opt})\) |

In the following an example for \(K\) varies from 3 to 30 users at SNR=7 and \(\beta\) has two values (0.01 and 0.1), at each value of \(K\) there exist an optimum voting rule “\(n_{opt}\)” and optimum energy detection threshold \(\lambda_{opt}\) that minimizes the ERP. In fig.(6) illustrates the ERP versus \(K\) for \(\beta = 0.01, 0.1\), where ERP evaluated for an optimum voting rule “\(n_{opt}\)” and optimum energy detection threshold \(\lambda_{opt}\).

From the figure we can note the improvement on ERP comparing with the ERP which evaluated for an optimum voting rule “\(n_{opt}\)” and fixed \(\lambda\) as in fig.(5).

In fig.(7) illustrates \(Q_m\) and \(Q_f\) versus \(K\) for \(\beta = 0.01\) and 0.1, the figure depicts how weights of \(Q_m\) and \(Q_f\) in ERP effect on the values of \(Q_m\) and \(Q_f\) and depicts also the improvement on \(Q_m\) and \(Q_f\) comparing to \(Q_m\) and \(Q_f\) in fig(4) with fixed energy detection threshold \(\lambda\).
4 Objective-Constraint Optimization

Define As we discuss previously the missing probability $Q_m$ is more important than the probability of false alarm $Q_f$. In this section, we will introduce a technique that optimize the fusion rule and optimum energy detection threshold $\lambda_{opt}$. Supposing that $K$ is fixed, i.e. what is the optimum fusion rule and optimum threshold which we denote as $n_{opt}$ and $\lambda_{opt}$ respectively that minimizes the false alarm probability $Q_f$ with constraint on $Q_m$ (subject to $Q_m \leq \xi$) ? i.e.

Minimizing $Q_f = \sum_{l=1}^{K} \left( \frac{K}{l} \right) P_f^l \left( 1 - P_f \right)^{K-l}$ (objective function)

Subject to $Q_m \leq \xi$ (constraint function)

We can note the $Q_m$ and $Q_f$ are depending on two parameters $n$ and the energy detection threshold $\lambda$. Then this problem is an optimization problem with constraint.

$$F = Q_f + \Psi (Q_m - \xi)$$  (14)

Where $\Psi = $Lagrangean multiplier

$$\left( \frac{\partial Q_f}{\partial n} \right) = \left( \frac{\partial Q_f}{\partial \lambda} \right) = \Psi$$  (15)

To get $n_{opt}$ and $\lambda_{opt}$ solve (15), it is quite difficult to get the closed-form solution of $\lambda$ and $n$. Therefore, we propose an algorithm to find $\lambda$ and $n$ which get minimum $Q_f$ such that $Q_m \leq \xi$.

Algorithm 3 find $\lambda$ and $n$ which get minimum $Q_f$ such that $Q_m \leq \xi$ assume $K$ and SNR $\gamma$ are known.

Input: $K$, SNR $\gamma$, $\xi$, and $\Delta$ ($\Delta$ is the tolerance of accuracy of $\lambda$)

$Q_{f_{min}} = 1$, $n_{opt}=1$, $\lambda_{opt}=1$

for $n = 1$ to $K$

step = 2, $\lambda = 1$

while step > $\Delta$

find $Q_m (\lambda, n)$

if $Q_m (\lambda, n) \leq \xi$

$\lambda = \lambda + \text{step}$

else if step=2

at this value of $n$ there is no $\lambda$ grantees the constraint

$\text{step} = \frac{\text{step}}{2}$, $\lambda = \lambda - \text{step}$

end if

end while

find $Q_f (\lambda, n)$

if $Q_f (\lambda, n) \leq Q_{f_{min}}$

$n_{opt} = n$, $\lambda_{opt} = \lambda$

end if

end for

Output: ($n_{opt}$, $\lambda_{opt}$)

By simulations, In fig.(8) illustrates $n_{opt}$ versus $\lambda_{opt}$ at $K=20$, $Q_m \leq 10^{-4}$ in AWGN channel with SNR=7 dB.

In fig.(9) illustrates $Q_m$ and $Q_f$ versus $n_{opt}$ at
K=20, SNR=7 dB , and $Q_m \leq 10^{-4}$, from the figure we note $Q_m \leq 10^{-4}$ and $n_{opt} = 6$ that minimizes $Q_f$ then the optimum energy detection threshold $\lambda_{opt} = 10$ from fig.(8).

Fig.(9) $Q_m$, $Q_f$, and ERP versus $n_{opt}$ at K=20 , $Q_m \leq 10^{-4}$ in AWGN channel with SNR=7dB

5 CONCLUSION

In this paper we proposed new techniques that guarantee a certain performance of the primary system. The problem turned into optimization problem to determine the two parameters optimal fusion rule and optimum threshold to guarantee a certain performance of the primary system. We defined Bayes risk function, optimal parameters to minimize error rate probability were investigated. We derive an expression to optimum fusion rule and proposed an algorithm to evaluate optimum threshold. In addition, an optimal spectrum sensing was formulated by optimizing both fusion rule and threshold. We proposed another technique that guarantee $Q_m$ to be less than certain threshold and minimizes $Q_f$ as soon as possible. We proposed an algorithm to evaluate optimum fusion rule and optimum threshold that minimize the $Q_f$ subject to $Q_m$ to be less than certain threshold.

REFERENCES