Solving Linear Programming Problems and Transportation Problems using Excel Solver

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Abstract- This paper outlines the steps required for installing Excel Solver in Microsoft Word 2010 for use in solving linear programming problems. It provides a step-by-step procedure with snapshots for improved performance. Several questions are solved including transportation problems using Excel Solver.

Index Terms- Excel Solver, linear programming, maximization, minimization, optimization, profit, transportation problem.

1 INTRODUCTION

THE use of Excel Solver for analysis of operations research problems is important and useful in present day technological world. It is difficult to solve linear programming problems using the manual method in organizations that solve problems with over fifty variables. A work that can take days or weeks to solve could be done in a matter of seconds using Excel Solver. Excel Solver has proven to be relevant in other disciplines such as finance, production management, etc. In this paper, I shall present a step-by-step procedure to follow in the installation and use of Excel Solver for solving linear programming problems and transportation problems.

2. Literature Review

Linear Programming

I will skip the definition of terms in linear programming and the assumptions and go straight to problem solving with Excel Solver. It is believed that the reader has prior knowledge of the subject matter. If you haven't installed Excel Solver in your Microsoft Excel, then follow the steps below:

a. Launch Microsoft Excel.

b. Go to “File” click on it and select “Options” (figure 1).

c. A dialog box will be displayed. Select “Add-Ins” (figure 2).

d. Choose “excel solver” and click “Go” and “OK” (figure 3).

e. Close and re-launch Microsoft Excel. Select the “Data” column. You can see “Solver” being displayed (figure 4).

Figure 1

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Let's begin with a simple illustration:

Example 1: Max. $z = 20x_1 + 15x_2$

s.t.

\[ 50x_1 + 35x_2 \leq 6000 \]
\[ 20x_1 + 15x_2 \geq 2000 \]
\[ x_1 \leq 100 \]
\[ x_2 \leq 100 \]
\[ x_1, x_2 \leq 0 \]

Input your data into Microsoft Excel worksheet as you can see in the figure 5 below. Then add the other items as displayed.

In the total column for maximization (i.e. in D3) input the following command: B3*$B$10+C3*$C$10. You can either use upper case or lower case to insert the command. When you are done, click on D3, place the pointer at the lower right hand tip of the cell and drag it down to D7. The formulae for the
constraints will be automatically produced. By now, your excel page should look like this:

![Formula in Excel](image)

You can see the formula on D3 cell being displayed in the formula bar. The formulae for D4 to D7 are:

$$D4 = b4 \times b10 + c4 \times c10$$
$$D5 = b5 \times b10 + c5 \times c10$$
$$D6 = b6 \times b10 + c6 \times c10$$
$$D7 = b7 \times b10 + c7 \times c10$$

You can as well insert them one after the other if it’s more convenient.

In cell D10, type “=D3”. Now that your data is ready, you solve the linear programming problem using Excel Solver. Click on Data on the menu bar and select Solver.

In the objective column, type $D3$. By default, max is selected. In minimization problems, you change to min. In the next column, (i.e. “by changing variable cells”) type $b10:c10$. To insert the constraints, select “Add” (figure 8) and input the following command, the right hand side command on the “Cell reference” box and the left hand side command on the “Constraint” box. Then select “OK.”

$$B10:C10 \geq 0$$
$$D4:D7 \leq F4:F7$$

This is how the Solver Parameter should look like after inputing the instructions above:

![Solver Parameter](image)

Then click on “Solve”. The values of $x_1$, $x_2$ and the objective function are: 64, 48 and 2000 respectively. The model and the solution are shown below:
Exercise 1: Min z = 0.3x1 + 0.9x2

s.t.

x1 + x2 \geq 800
0.21x1 - 0.3x2 \geq 0
0.03x1 - 0.1x2 \geq 0
x1, x2 \geq 0

The question and solution should look like this:

The non-negativity added is insignificant since it is already included as one of the variables. Did you get the result right? It is very interesting. More exercises will help you master how to solve linear programming problems using Excel Solver with ease.

Now, try this question:

Exercise 2: Max z = 5x1 + 4x2

s.t.

6x1 + 4x2 \leq 24
x1 + 2x2 \leq 6
-x1 + x2 \leq 1
x2 \leq 2
x1, x2 \leq 0

The question and solution to the problem in the excel worksheet is given below:

Now, let’s solve a real life problem by first formulating the model.

Example 2: Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. The following table provides the basic data of the problem:

<table>
<thead>
<tr>
<th>Raw material</th>
<th>Maximum daily available (tons)</th>
<th>Tons of raw material per tons</th>
<th>Profit per ton ($1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>M2</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton.
Also, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit [Taha (2011), p.47].

Let \( x_1 \) represent the number of tons of exterior paints produced and \( x_2 \) the number of interior paints produced.

Maximize \( z = 5x_1 + 4x_2 \) (in $1000)

s.t.

\[
\begin{align*}
6x_1 + 4x_2 &\leq 24 \quad (M1) \\
-x_1 + x_2 &\leq 1 \quad \text{(Market limit)} \\
x_2 &\leq 2 \quad \text{(Maximum daily demand)} \\
x_1, x_2 &\geq 0
\end{align*}
\]

The solution to the problem is given in figure 13 below.

Exercise 3: An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, then 40 per day could be painted. If the body shop were only producing cars, then it could process 50 per day. Each truck contributes $300 to profit, and each car contributes $200 to profit. Use linear programming to determine a daily production schedule that will maximize the company’s profit (Winston, 2004).

Solution: Let \( x_1 \) and \( x_2 \) represent the number of trucks and cars produced respectively.

Fraction of day paint shop works on trucks = \( \frac{1}{40}x_1 \)

Fraction of day body shop works on trucks = \( \frac{1}{50}x_1 \)

Fraction of day body shop works on cars = \( \frac{1}{50}x_2 \)

Hence, the constraints are:

\[
\begin{align*}
\frac{1}{40}x_1 + \frac{1}{50}x_2 &\leq 1 \quad \text{(Paint shop constraint)} \\
\frac{1}{50}x_1 + \frac{1}{50}x_2 &\leq 1 \quad \text{(Body shop constraint)}
\end{align*}
\]

The model for the problem is:

Max \( z = 3x_1 + 2x_2 \)

s.t.

\[
\begin{align*}
\frac{1}{40}x_1 + \frac{1}{50}x_2 &\leq 1 \\
\frac{1}{50}x_1 + \frac{1}{50}x_2 &\leq 1 \\
x_1, x_2 &\geq 0
\end{align*}
\]

Insert the model into a Microsoft Excel worksheet and solve.

TRANSPORTATION PROBLEM

Transportation problems can be solved using Excel Solver. What is required is to change the problem into a linear programming problem and solve it as a minimization problem following the same procedure as explained above. Before you proceed, you may need to study transportation problem first for better understanding.

Excel Solver and TORA can be used for solving different forms of transportation problem. Excel Solver can only compute the least transportation cost without giving credence to its computation using three methods: Least Cost Method, North West Corner Method and Vogel Approximation; which are exemplified by TORA. This is made possible because the problem is first changed to a LP problem and solved using the simplex method. According to Taha (2011),” TORA handles all necessary computations in the background using the simplex method and uses the transportation model format only as a screen ‘veneer’”. The two methods, however, do not solve transportation problems using the MODI method.

Example 1: MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The quarterly capacities of the three plants are 1000, 1500, and 1200 cars, and the demands at the two distribution centers for the same period are 2300 and 1400 cars [Taha (2011), p.209].
This problem can be changed to a linear programming problem as follows:

\[
\text{Minimize } Z = 80x_{11} + 215x_{12} + 100x_{21} + 108x_{22} + 102x_{31} + 68x_{32}
\]

Subject to:
\[
\begin{align*}
&x_{11} + x_{12} \geq 1000 \quad \text{(Los Angeles)} \\
&x_{21} + x_{22} \geq 1500 \quad \text{(Detroit)} \\
&x_{31} + x_{32} \geq 1200 \quad \text{(New Orleans)} \\
&x_{11} + x_{21} + x_{31} \geq 2300 \quad \text{(Denver)} \\
&x_{12} + x_{22} + x_{32} \geq 1400 \quad \text{(Miami)}
\end{align*}
\]

Insert the model into an excel worksheet. This is what you should have:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Denver</th>
<th>Miami</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles</td>
<td>80</td>
<td>215</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detroit</td>
<td>100</td>
<td>108</td>
<td>1500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Orleans</td>
<td>102</td>
<td>68</td>
<td>1200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>2300</td>
<td>1400</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This problem can be changed to a linear programming problem as follows:

Click on “Solve” when you have supplied the values. Your result will show in the worksheet as you can see in figure 16 below.

The result shows that 1000 units of the product should be shipped to Denver from Los Angeles, 1300 units from Detroit to Denver, 200 units from Detroit to Miami and 1200 units from New Orleans to Miami to minimize cost.

Excel Solver only features the least cost method thus it cannot solve the question using North-West Corner Method or Vogel Approximation method. However, TORA software features all three methods.

Below is a transportation question you can solve and the solution there in.
Exercise 1: Powerco has three electric power plants that supply the needs of four cities. Each power plant can supply the following numbers of kilowatt-hours (kwh) of electricity: plant 1-35 million; plant 2-50 million; plant 3-40 million. The peak power demands in these cities, which occur at the same time (2p.m.), are as follows (in kwh): city 1-45 million; city 2-20 million; city 3-30 million; city 4-30 million. The costs of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel. The cost of shipping is shown in the table 1 below. Formulate an LP to minimize the cost of meeting each city’s peak power demand (Winston, 2004, p.360).

The shipping cost, supply and demand for power is shown in table 2 below. The table shows the cost of shipping in dollars per million kwh from each plant to each city. The supply constraints ensure that the total amount produced at each plant does not exceed the plant's maximum capacity. The demand constraints ensure that the total amount sent to each city meets the city’s demand. The objective function is to minimize the total cost of shipping, which is given by the formula:

Max z = $8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} + 9x_{21} + 12x_{22} + 13x_{23} + 7x_{24} + 14x_{31} + 9x_{32} + 16x_{33} + 5x_{34}$

Subject to:

Supply constraints:
- $x_{11} + x_{12} + x_{13} + x_{14} \leq 35$
- $x_{21} + x_{22} + x_{23} + x_{24} \leq 50$
- $x_{31} + x_{32} + x_{33} + x_{34} \leq 40$

Demand constraints:
- $x_{11} + x_{21} + x_{31} \leq 45$
- $x_{12} + x_{22} + x_{32} \leq 20$
- $x_{13} + x_{23} + x_{33} \leq 30$
- $x_{14} + x_{24} + x_{34} \leq 30$

Insert the model into an excel worksheet. Then go to the “solver parameter” and input the required command as shown in figure 4 below.
the problem are shown in table 5 below.

<table>
<thead>
<tr>
<th>No</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>11</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>7</td>
<td>9</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>14</td>
<td>16</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>5</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3

Figure 5

The formula for cell N9 is shown on the formula box. Always check your input to confirm it is right before solving the model. Optimal solution to this LP is \( z = 1020, x_{12}=10, x_{13}=25, x_{21}=45, x_{23}=5, x_{32}=10, x_{34}=30 \).

This method of computation involves the generation of variables that make the calculations cumbersome as the demand and supply centers increase.

Let’s try a different method for solving transportation problem below:

Example 2: SunRay Transport Company ships truckloads of grain from three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in table . the unit transportation costs (shown in the northeast corner of each box) are in hundreds of dollars. The model seeks the minimum-cost shipping schedule between the silos and the mills.

<table>
<thead>
<tr>
<th>Mill</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>11</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Silo 2</td>
<td>12</td>
<td>7</td>
<td>9</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>Demand</td>
<td>5</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

Table 3

Fill in the information as shown in figure below. To fill in the range names to cells, select the appropriate cell and right click.

Figure 18

Double-click on any cell. Select “define name” and a dialog box like the one in figure 19 appears. Fill in the necessary details as shown on the range items in table below.

Figure 19

Open the Excel Solver and insert the information in the appropriate order as shown in figure 20 below.
Click on Solve. The solution to the problem is displayed in the worksheet as you can see below in figure 21.

<table>
<thead>
<tr>
<th>Input data:</th>
<th>SunRay Transportation Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit cost</td>
<td>D1</td>
</tr>
<tr>
<td>S1</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>12</td>
</tr>
<tr>
<td>S3</td>
<td>4</td>
</tr>
<tr>
<td>Demand</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D1</td>
</tr>
<tr>
<td>S1</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>5</td>
</tr>
<tr>
<td>Column Sum</td>
<td>5</td>
</tr>
</tbody>
</table>

References
