Smart structure application for Euler – Bernoulli beam vibration damping

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Abstract — Active control methods can be used to eliminate undesired vibration in engineering structures like beams and blades. In this paper, the dynamic behavior of a smart beam which consists of an aluminum cantilever beam with piezoelectric patches works as actuators and sensors is studied. The Lead Zirconate Titanate (PZT) is used. The natural frequency of the beam was obtained using analytical method and also by finite element method (FEM) (ANSYS software). The transient analysis of the smart beam is conducted then the active vibration control using proportional-integral-derivative controller (PID controller) is applied to suppress the beam vibration.

Index Terms — piezoelectric, smart material, cantilever beam, vibration damping, FEM, PID controller, smart beam

1 INTRODUCTION

Smart structures (Fig. 1) are systems that incorporate particular functions such as sensing, processing and actuation. They have the ability to sense certain stimuli and respond in a controlled manner in order to carry out these activities, a smart structure must have sensor, actuator and Control System. Smart structures are important because of their relevance to structural health monitoring and structural vibration control.

The major problem in the dynamic operation of structures is undesirable vibrations. Therefore vibration control and active damping are among the most studied areas using smart materials and structures.

![Smart Structure Diagram](Image)

Fig. 1 Smart structure

Smart materials used as actuator and sensor in smart structures and there are many types such as Shape Memory Alloys (SMA), Magnetostriective Materials, Magneto-rheological Materials (MR) and Piezoelectric Materials. Piezoelectric Materials are used as actuators as well as sensors. Piezoelectric behavior can be revealed in two distinct ways. There are two broad classes of piezoelectric materials used in vibration control: ceramic and polymers. The types of piezoelectric materials are Single Crystals, Ceramics, Polymers and Composites [1].

The piezoelectric patches are either bonded to or embedded within the structures, whereas the piezoelectric laminas are stacked together with a substrate lamina to form a piezoelectric composite laminate [2]. However, there are several factors that limit the use of piezoceramic materials, such as their brittle nature and low tensile strength, therefore limiting their ability to conform to curved shapes, and the large add-on mass associated with using typical lead-based piezoceramic[3]. The use of arrays of piezoelectric sensors and actuators embedded within the structure would remedy the above mentioned restrictions. Due to their small size, these sensors/actuators have the flexibility to conform to curved shapes, and they add little mass to the structure [4]. In addition, these piezoelectric sensors and actuators can be tailored to achieve a particular smart structure design. Owing to the small size of the piezoelectric sensors and actuators relative to the size of the host structure, these sensors/actuators can be analyzed as inclusions in a non-piezoelectric matrix (host structure) by using a micromechanics approach [5].

Piezoelectric sensors operate using the direct effect, i.e., electric charge is generated when a piezoelectric material is stressed causing deformation. The deformation of the sensor can be measured by measuring the voltage across its electrodes. Piezoelectric actuators operate using the converse effect of piezoelectric materials. This effect states that when a piezoelectric material is placed into an electric field i.e., a voltage is applied across its electrodes; a strain is induced in the material. PZT, a piezoceramic, is commonly used for actuation because it has a maximum actuation strain [6].

Provenza and Morrison used a wireless system to control the vibrations of a rotating plate. The system was actuated using rotating bearing excitation; the frequency of the control was identical to the excitation frequency, while the amplitude and phase were tuned to optimize the response. The investigation showed that the three bending plate vibration modes were suppressed [7].
Kauffman and Lesieutre used analytical simulations and experimental data applied to turbomachinery. They used semi active approach by switching electrical boundary conditions of surface mounted piezoelectric patches applied to a titanium flat plate. They concluded that resonance frequency detuning can reduce vibration by altering the structural stiffness to detune it [8].

Duffy et al. used flexible macro-fiber-composite piezoelectric patches applied to polymer matrix fiber composite fan blades; the piezoelectric material was surface-mounted on the blades. Three thin piezoelectric patches were applied to each blade–two actuator patches and one small sensor patch. These were placed in a location of high resonant strain for the first bending mode that taken from finite element simulation at non-rotating condition. The results showed that the vibrations induced to the fan blades at different speeds were damped by using active and shunted piezoelectric patches [9].

Zhou et al. used an adaptive control strategy based on passive piezoelectric shunt techniques for mistuned bladed disks. Resonant shunted piezoelectric patches were attached onto the disk between adjacent blades to reduce the blade vibration through blade–disk coupling. Numerical simulation shows that a good performance was achieved with respect to reducing the vibration of timevariant mistuned bladed disks [10].

Zhou et al. used an essentially nonlinear piezoelectric shunt circuit for the practical realization of nonlinear energy sink (NES), and then applied to a mistuned bladed disk for blade vibration reduction. A numerical method was developed to calculate quasi-periodic responses arising in the electromechanical system under harmonic forcing. Shunted piezoelectric were attached onto the disk surface damping strategy in order to reduce blade vibrations. The numerical simulation shows that the (NES) was able to compensate for a drift of the system characteristics in each blade–disk sector. Consequently, a series of NES in the full structure allow the blade vibration level to be reduced significantly in the presence of mistuning caused by blade cracks, fretting or even foreign object damage [11].

Extensive research has been done on the vibration control and suppression of structures using piezoelectric materials, as evident from numerous review articles (for example [12],[13],[14],[15],[16],[17],[18],[19]).

In this paper, the dynamic behavior of a smart beam which consists of an aluminum cantilever beam with piezoelectric patches works as actuators and sensors is studied. The Lead ZirconateTitanate (PZT) is used. The natural frequency of the beam is obtained using analytical method and also by finite element method (ANSYS software). The transient analysis of smart beam is conducted then the active vibration control using proportional-integral-derivative controller (PID controller) is applied to suppress the beam vibration where the results are compared.

2 THEORETICAL ANALYSIS OF BEAMS AND MODEL VALIDATION:

First, the analytical results for Euler-Bernoulli beam natural frequencies are given by:

$$\omega_n = \left(\frac{\beta_i^2}{\lambda}\right) \sqrt{\frac{EI}{\rho A t^4}} \quad (1)$$

Where:
- A: the cross section area
- L: the beam length
- \( \rho \): the density
- E: Young’s Modulus
- I: the first four natural frequencies the constant \( \beta_i \) is given by 1.875104, 4.69409, 7.85475, and 10.99554.
- Error %

Table 1: validation of FEM Beam model

<table>
<thead>
<tr>
<th>Analytical</th>
<th>ANSYS (present results)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 9.569</td>
<td>9.6164</td>
<td>0.5</td>
</tr>
<tr>
<td>2 60.89</td>
<td>60.247</td>
<td>1.1</td>
</tr>
<tr>
<td>3 167.72</td>
<td>168.76</td>
<td>0.6</td>
</tr>
<tr>
<td>4 328.92</td>
<td>331</td>
<td>0.6</td>
</tr>
</tbody>
</table>

3 VIBRATION DAMPING USING PIEZOELECTRIC TRANSDUCERS

For vibration suppression of the beam model validated in the above case study, four PZT-5H are used as sensors and actuators attached to the beam and the PID controller is used. First, the piezoelectric constitutive equations are illustrated.

4 PIEZOELECTRIC CONSTITUTIVE EQUATIONS

For a piezoelectric material, the electrical and mechanical constitutive equations are coupled.

$$S = s^e T + d^f E \quad (2)$$

$$D = d T + \varepsilon^e E \quad (3)$$

Where:
- D: the electric displacement (charge per unit area, expressed in C/m2)
- E: the electric field (V/m)
- S: the strain
- T: the stress (N/m2)
- s^e : the compliance when the electric field is constant.
- d: the compliance of the material (inverse of the Young modulus)
- s^f : The compliance when the electric field is constant.
- In Equation (2), the piezoelectric constant d relates the strain to the electric field in the absence of mechanical stresses and s^e is the compliance when the electric field is constant.
- In Equation (3), d relates the electric displacement to the stress under a zero electric field; d is expressed in (m/V or Cou-
5 PROPORTIONAL–INTEGRAL–DERIVATIVE CONTROLLER PID CONTROLLER

PID controller is a control loop feedback mechanism (controller) commonly used in industrial control systems. A PID controller continuously calculates an error value $e(t)$ as the difference between a desired set point and a measured process variable and applies a correction based on proportional, integral, and derivative terms, respectively (sometimes denoted P, I, and D) which give their name to the controller type.

6 APPLICATION OF PIEZOELECTRIC PATCHES FOR VIBRATION DAMPING

The four PZT-5H patches are used on the center of the beam three of them on the top surface as actuators and the fourth on the bottom surface of the beam as sensor, Table (2) indicates the dimension and position of the four PZT patches.

TABLE (2) THE DIMENSION AND POSITION OF THE FOUR PZT PATCHES

<table>
<thead>
<tr>
<th>Position from fixed end mm</th>
<th>Dimension mm</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st actuator</td>
<td>14mm</td>
<td>(44×20×0.25) PZT-5H</td>
</tr>
<tr>
<td>2nd actuator</td>
<td>90mm</td>
<td>(44×20×0.25) PZT-5H</td>
</tr>
<tr>
<td>3rd actuator</td>
<td>322mm</td>
<td>(44×20×0.25) PZT-5H</td>
</tr>
<tr>
<td>1st sensor</td>
<td>14mm</td>
<td>(44×20×0.25) PZT-5H</td>
</tr>
</tbody>
</table>

The PZT-5H properties are entered to the ANSYS package as illustrated below:

```plaintext
mp,dens,2,7500 ! Density for piez. material
mp,perx,2,15.03E-9 ! Permittivity in x direction
mp,pery,2,15.03E-9 ! Permittivity in y direction
mp,perz,2,13E-9   ! Permittivity in z direction
tb,piez,2         ! Define piez. Table
tbdata,16,17       ! E16 piezoelectric constant
tbdata,14,17       ! E25
tbdata,3,-6.5      ! E31
tbdata,6,-6.5      ! E32
tbdata,9,23.3      ! E33
tb,anel,2          ! Define structural table
tbdata,1,126E9,79.5E9,84.1E9 ! C11, C12, C13
tbdata,7,126E9,84.1E9 ! C22, C23
tbdata,12,117E9    ! C33
tbdata,16,23.3E9   ! C44
tbdata,19,23E9     ! C55
tbdata,21,23E9     ! C66
```

After adding the PZT patches to the ANSYS model, the analysis is conducted by the following steps:

- Modal analysis of the beam to get the mode shapes and natural frequencies
- Application of PZT for damping and the results is obtained by transient analysis of the structure with and without the PZT effect, the control of PZT is applied using PID controller.

7 RESULTS AND CONCLUSIONS

For the above case study the modal analysis results show the first four mode shapes and natural frequencies Fig. (2 to 5)
By applying the PZT the damping of natural frequencies is shown in Table (3).

<table>
<thead>
<tr>
<th>Natural Frequencies</th>
<th>Without PZT</th>
<th>With PZT</th>
<th>Damping %</th>
</tr>
</thead>
<tbody>
<tr>
<td>N 1</td>
<td>9.6164</td>
<td>10.167</td>
<td>5.7</td>
</tr>
<tr>
<td>N 2</td>
<td>60.247</td>
<td>62.75</td>
<td>4.15</td>
</tr>
<tr>
<td>N 3</td>
<td>168.76</td>
<td>173.74</td>
<td>3</td>
</tr>
<tr>
<td>N 4</td>
<td>331</td>
<td>338.82</td>
<td>2.4</td>
</tr>
</tbody>
</table>

8 TRANSIENT ANALYSIS

Transient analysis for the beam model without and with PZT patches is used to suppress the vibration of the beam. A reduction in the settling time obtained, first beam without PZT settling time is about 10 s, then beam with the 4 PZT patches which controlled by PID controller the settling time is about 3 s Fig. 6.

![Fig. 6 The effect of PID control on beam tip displacement](image)

9 CONCLUSION

The FEM model is validated by comparison with analytical method and shows a good agreement. The PZT patches added to the model and the first natural frequency increases up to 5.7% which is the important mode for vibration. Then the transient analysis is conducted for the model without and with PZT by using PID controller, the results show reduction of settling time from 10 s to 3 s which is about 70% of the settling time.

REFERENCES