SUH and GIUH for Flood Estimation

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Abstract—The Synthetic Unit Hydrograph method is widely used for the flood estimation in ungauged catchments. One of the popular method that has wide spread application in the design of bridges in Railway track is the method suggested by the Central Water Commission (CWC). It is established that Geomorphic Instantaneous Unit hydrograph (GIUH) is a viable method for estimating the flood in ungauged watersheds. Hence it becomes imperative to compare the well-established method and recently developed method for assessing its estimating capability. Hence, the current study assessed the two procedures of flood estimation, viz., synthetic unit hydrograph and geomorphic instantaneous unit hydrograph.

Keywords—Flood Assessment, SUH, GIUH

I. INTRODUCTION

Flood estimation in ungauged catchment is one of the most frequent applications of surface hydrology in general and rainfall-runoff modelling in particular [7]. Unit Hydrograph (UH) is the most popular and widely used method for predicting flood hydrograph resulting from a known storm in a basin area. To develop a unit hydrograph for a catchment, detailed information about the rainfall and resulting flood hydrograph are needed. However, such information about the rainfall would be available only at few locations. In order to construct unit hydrographs for ungauged catchments, empirical equations of regional validity, which relate the salient hydrograph characteristics to the basin characteristics, are available. Unit hydrograph derived from such relationships are known as synthetic unit hydrographs [10]. CWC has developed a detailed procedure for generating synthetic unit hydrograph for Konkan region [4]. The geomorphological parameters are mostly time-invariant in nature and therefore, geomorphology based approach could be the most suitable technique for modelling the rainfall-runoff process for ungauged catchments [9]. The computation of Horton’s ratios and their application in generating the Geomorphic Instantaneous Unit Hydrograph (GIUH) can provide a solution for ungauged rivers. The drainage network analysis and application of GIUH can provide a significant contribution towards flood management program [5].

II. LITERATURE REVIEW

Geomorphologic instantaneous unit hydrograph (GIUH) is one of the possible approaches for predicting the hydrograph characteristics. GIS supported GIUH-based models have great potential in estimating floods from ungauged basins, or basins with scant data availability [5]. Rodriguez-Iturbe and Valdes [9] intimately linked the instantaneous unit hydrograph (IUH) with geomorphologic parameters of a basin. IUH is expressed in terms of Horton’s ratio and mean velocity of stream. Also, the paper derived general relationships which allow the estimation of peak and time of peak of the IUH of a watershed.

Amiry and Mohammad [1] used SCS, Snyder and Triangular model parameters in addition to GIUH method to estimate peak discharge estimation. Results obtained show that GIUH model is the best model for the estimation of instantaneous peak discharge among the four models. Hence in the current study, the evaluation of GIUH and SUH methods were carried out to assess its applicability. Indian railway customarily uses the synthetic unit hydrograph method as described in [4] for flood estimation. As it has been proved by many studies [3]; [6]; [1] that the GIUH can model floods with reasonable level of accuracy, it becomes essential to compare this method with the customarily used method, viz. the one suggested in [4].

III. METHODOLOGY

The current section describes the various methodologies and theoretical background that are required for the analysis within this study. They include GIUH and SUH methods.

A. GIUH Model

Nature of stream in a region is related to the rainfall characteristics and watershed geomorphology. GIUH is a hydrological model that relates geomorphological features of a basin to its response to rainfall.

1) Geomorphologic parameters

The geomorphological parameters- are Horton’s ratios; include area ratio ($R_A$), bifurcation ratio ($R_b$), length ratio ($R_L$), dynamic parameter (V) and also the drainage density. The Horton’s ratios and the drainage density of the catchment can be obtained from the drainage network, which can be extracted through processing and analysis of Digital Elevation Model (DEM) in a GIS environment.
2) Ordering of stream network

Strahler improved Horton’s ordering system to form the Horton-Strahler ordering system, which analyses the network as follows: Channels originating at a source are defined to be first order streams
- When two streams of order i join a stream of order (i+1) is created.
- When two streams of different order merge, the order of the downstream link is the highest of the two stream orders.
- The order of the basin is same as the highest order channel.

3) Laws of Geomorphology

The quantitative expressions of Horton-Strahler’s laws are summarized [4]
If W=Ω=order of basin network.
Ni (i=1, 2, 3…W/Ω) is the number of streams of order i, and Ai is the mean area contributing to streams of order w and its tributary (1≤i≤W)
- Law of stream number
  \[ \frac{N_{w-1}}{N_{w}} = R_B \]  (1)
  Where, \( N_w \)= Number of streams in order w and w+1
- Law of stream lengths
  \[ \frac{L_{w-1}}{L_{w}} = \frac{1}{R_B} \]  (2)
  Where, \( L_w \)= Average length of channels of order w
- Law of stream areas
  \[ \frac{A_{w-1}}{A_{w}} = \frac{1}{R_A} \]  (3)
  Where, \( A_w \)=Mean area of contributing watershed to streams of order w

The derivation of the probability density function of a drop reaching the outlet is tackled by defining set of terms and rules as follows [9].
State is the order of the stream in which the drop is located at time t, denoted by \( C_i \), (1≤i≤W).When the drop is still in overland phase, the state is the order of the stream to which the land drains directly, denoted by \( r_i \) (1≤i≤W)
- A drop may begin at any state, but all drops eventually terminate in the highest numbered state w+1
- Transition is the change of state
- If N=Number of states, N=W+1, where the extra state is the bucket or trapping state.

The above rules define a set of limited number of paths through which a drop may travel to reach the outlet. The number of possible paths is less than or equal to \( 2^{w-1} \).

The impulse response function denoted by \( u(t) \) which is represented as GIUH in this case is expressed [11] as
\[
GIUH = \sum_{i=1}^{2^w} (T_{s_i} \text{prob}(T_{s_i} \leq t) \ast \text{prob}(s_i))
\]  (4)

\( T_{si} \): the travel time in a particular path Si, must be equal to the sum of travel times in the element of that path,
\( \text{Prob}(Si) \): the probability of a drop which will travel possible paths Si to the outlet.
\( \text{Prob}(T_{si}) \): the probability density function of the total path travel time

The probability that the drop travel a path Si is given by
\[
\text{prob}(s_i) = \theta_i \cdot P_{i1} \cdot P_{ir} \cdot P_{i0}
\]

\( \theta_i \): The initial state probabilities and \( P_{i0} \) are the transition probabilities.

The initial probabilities as well as the transition probabilities can be defined as functions of only geomorphic and geometric parameters.

The transition probabilities are given by the following expression [11].
\[
R_{ij} = \begin{cases} \frac{2N_{i+1}}{N_i} & \text{if } j = i + 1 \\ \frac{N_{i+1}}{N_i} & \text{otherwise} \end{cases}
\]  (5)

In this expression, \( E(i, \Omega) \) represents the expected number of links of orders i in a network of order \( \Omega \).

Under the random topological model assumption, it is given by,
\[
E = (i, \Omega) \rightarrow N_i \left[ \frac{N_{i-1}}{2N_i} \right]
\]  (6)

For the fourth order stream network, the above expression were solved and the transition probabilities can be calculated using the following formula [11].
\[
P_{12} = \frac{2}{3} \frac{R_{32}}{R_{23}} + \frac{2}{3} \frac{R_{32}}{R_{23}} \left( \frac{2R_{23} - R_{22}}{R_{23}} \right)
\]  (7)

The initial state probability \( \theta_i \) is equal to the ratio of the area draining directly into streams of order I to the total basin area. It is given by the following expression [11].
\[
\theta_{w} = \frac{N_{w}}{A_{w}}
\]  (8)

For the fourth order stream network, solving the above expression, the initial state probabilities calculated using the following formula [11].
\begin{align*}
\theta_1 &= \frac{N_1 A_1}{\lambda_1} \\
\theta_2 &= \left[\frac{P_{12}}{\lambda_2^2} - \left(\frac{P_{12}}{\lambda_2}\right)^2\right] P_{12} \\
\theta_3 &= \left[\frac{P_{23}}{\lambda_3^2} - \left(\frac{P_{23}}{\lambda_3}\right)^2\right] P_{23} \\
\theta_4 &= \left[1 - \theta_1 - \theta_2 - \theta_3\right] \quad (9)
\end{align*}

Using the initial and transitional probabilities for each transition, probability of each path is calculated.

<table>
<thead>
<tr>
<th>Path Number</th>
<th>Path description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a_1 \rightarrow r_1 \rightarrow r_2 \rightarrow r_3 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>2</td>
<td>(a_1 \rightarrow r_1 \rightarrow r_2 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>3</td>
<td>(a_1 \rightarrow r_1 \rightarrow r_3 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>4</td>
<td>(a_2 \rightarrow r_2 \rightarrow r_3 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>5</td>
<td>(a_2 \rightarrow r_2 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>6</td>
<td>(a_3 \rightarrow r_3 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
<tr>
<td>7</td>
<td>(a_4 \rightarrow r_4 \rightarrow \text{outlet})</td>
</tr>
</tbody>
</table>

\(a_i\) - denotes when the rain drop is in hill slope state of order \(i\), and \(i = 1, 2, \ldots, \Omega\)

\(r_i\) - denotes when the rain drop is in channel state of order \(i\).

The probability density function of travel time of the eight possible paths, were calculated as a function of the hill slope velocities/stream flow velocities using the following equations.

\[
\alpha = \frac{V_s}{L_0} \quad \text{(10)}
\]

where \(L_0 = \frac{1}{\pi n}\)

\(L_0\) – average flow length

\(D\) – Drainage density

The value of travel time parameter \(\lambda\), which varies from stream to stream is given by

\[
\lambda = \frac{V_s}{L_i} \quad \text{(10)}
\]

Thus the geomorphological instantaneous unit hydrograph can be computed as [11]

\[
GIUH(E) = \sum_{i=1}^{\Omega} \left( f_{E_{1i}} \times f_{E_{1i+1}} \times \ldots \times prob(S) \right) \quad (11)
\]

The above equation can be solved as convolution of non identical exponential density function of a given path \(S_1\) [11]

\[
f_{S_1} = \sum_{j=1}^{\Omega} \lambda_i \ldots \lambda_j \exp(-\lambda_j t) \prod_{i=j+1}^{\Omega} \left( \lambda_i - \lambda_j \right) \left( \lambda_i - \lambda_{j-1} \right) \ldots \left( \lambda_i - \lambda_j \right)
\]

\(j \neq 1\) (12)

The probability density functions of each of the eight possible paths are computed. The density functions were then multiplied with the corresponding path probabilities in discretized time step using excel spread sheet and the GIUH is then determined. The IUH ordinates have the dimension of \(T^{-1}\); hence it is converted into cumecs by multiplying with the area of the watershed with appropriate conversions. Further the derived 1 hour unit hydrograph by lagging the ordinates by 1 hr and averaging the coordinates. Effective rainfall, computed as in [4] was used to simulate the runoff for hourly data.

B. Synthetic Unit Hydrograph [4]

The term ‘synthetic’ in synthetic unit hydrograph denotes the unit hydrograph derived from watershed characteristics rather than rainfall-runoff data. These methods utilize a set of empirical equations relating the physical characteristics of watershed to a few salient points of the hydrograph such as peak flow rate\(Q_p\), time to peak\(t_p\), time base\(t_B\), and IUH width at 0.5\(Q_p\) and 0.75\(Q_p\) i.e. \(W_{0.5}\) and \(W_{0.75}\) respectively [4]

1) Physiographic parameters

- Catchment Area\(A\)
  - A catchment is an area where water is collected by the natural landscape. The area enclosed upto the gauging site may be referred to as the catchment area \(A\).
- Length of the main stream \(L\)
  - This implies the longest length of the main river from the farthest watershed boundary of the catchment area to the gauging site.
- Equivalent Stream Slope \(S\)
  - One of the physiographic parameter is slope. The slope may be equivalent slope or statistical slope. Slope is calculated using analytical method.
  - L-section is broadly divided into 3-4 segments and the following formula is used to calculate the equivalent slope \(S\).

\[
S = \frac{L_i(D_{i-1} + D_i)}{L^2} \quad (13)
\]

Where, \(L_i\)=Length of the \(i^{th}\) segment in km.

\(D_{i-1}, D_i\)=Elevations of river bed at \(i^{th}\) intersection points of contours reckoned from the bed elevation at points of interest considered as datum, and \(D(i-1)\) and \(D\) are the heights of successive bed location at contour and intersections.

\(L\)=Length of the longest stream in km.
Fig. 1 Parameters of synthetic Unit hydrograph [4]

Synthetic unit graph parameters are computed using following equation[4].

(i) \( q_p = 0.9178(L/S)^{0.4313} \)

(ii) \( t_p = 1.5607(q_p)^{-1.0814} \)

(iii) \( W_{50} = 1.925(q_p)^{-1.0896} \)

(iv) \( W_{75} = 1.0189(q_p)^{-1.0443} \)

(v) \( W_{R50} = 0.5788(q_p)^{-1.0702} \)

(vi) \( W_{R75} = 0.3469(q_p)^{-1.0538} \)

(vii) \( T_B = 7.380(t_p)^{0.7343} \)

(viii) \( t_m = t_p + (t_p/2) \)

(ix) \( q_p^2 \cdot A \cdot L \cdot S \cdot T_m \cdot t_p \cdot t_r \cdot t_c \cdot t_f \cdot t_s \cdot t_b \cdot t_a \cdot t_d \cdot t_e \cdot t_h \cdot t_i \cdot t_j \cdot t_k \cdot t_l \cdot t_m \cdot t_n \cdot t_o \cdot t_p \cdot t_q \cdot t_r \cdot t_s \cdot t_t \cdot t_u \cdot t_v \cdot t_w \cdot t_x \cdot t_y \cdot t_z \cdot \)

Where, \( L \) = Length of longest main stream along the river course in km

\( S \) = Equivalent stream slope in m/km

\( q_p \) = Peak discharge of unit hydrograph per unit area in cumecs per sq. km

\( t_p \) = Time from the centre of unit rainfall duration to the peak of unit hydrograph in hours

\( W_{50} \) = Width of UG measured at 50% peak discharge ordinate \( Q_{50} \) in hours

\( W_{75} \) = Width of UG measured at 75% peak discharge ordinate \( Q_{75} \) in hours

\( W_{R50} \) = Width of the rising side of UG measured at 50% of peak discharge ordinate \( Q \) in hours.

\( T_B \) = Base width of unit hydrograph in hours

\( t_m \) = Time from the start of rise to peak of unit hydrograph in hours

\( t_p \) = Time from the centre of unit rainfall duration to the peak of unit hydrograph in hours

\( t_r \) = Unit rainfall duration adopted in a specific study in hours

3) Estimation of Design storm

- Design storm duration

The design storm duration \( (T_p) \) has been adopted as 1.1\( t_p \) as this value of storm duration gives the higher value of flood peak.

- Estimation of point rainfall and areal rainfall for storm duration

The catchment under study falls between three isohyets. Therefore the point rainfall may be computed for the catchment taking into account these three isohyets. The 50 year, 1 hour areal rainfall was split into 1 hour rainfall increments using the time distribution coefficients [4].

- Design loss rate

The loss rate is taken as 0.19 cm/hr as recommended by Central Water Commission [4].

- Design Base Flow

Base flow rate of 0.15 cumecs/sq. km may be adopted for estimating base flow for a catchment [4].

- Estimation of 50 year flood

For the estimation of peak discharge, the effective rainfall increments were re-arranged against the maximum unit hydrograph ordinate, the next lower value of effective rainfall against the next lower value of unit hydrograph ordinate and so on. Sum of the products of unit hydrograph ordinates and the effective rainfall increments gives the total direct runoff peak. Convolution is carried out to derive the direct runoff hydrograph. Total base flow was added and the ordinates were plotted against time to get the design flood hydrograph.

IV. STUDY AREA AND MODEL CUSTOMIZATION

Three sub-catchments in Thrissur district were taken as the study area.

A. Watershed Description

Thrissur city is located at 10.52°N 76.21°E and has an average altitude of 2.83 metres. The average annual rainfall is 3000 mm. The areas of the sub-catchments are more than 40 km².
In Fig.3, green colour shows Thrissur Corporation and other colour shows the sub-catchments. The sub-catchments are analysed using SUH and GIUH methods and the results were compared.

B. Model Inputs and Data Collection

Digital Elevation Model (DEM) data consist of a sampled array of elevations for a number of ground positions at regularly spaced intervals. DEM was downloaded from the site, http://www.jspacesystems.or.jp/ersdac/GDEM/E/4.html.

In order to make the DEM more accurate at the portion of the study area, the DEM of Thrissur Corporation area was created from contour map of the area. The contour map of the area was obtained from Thrissur Corporation. The conversion tool in ArcGIS was then used to create DEM from the available contour map. The DEM of Thrissur Corporation was mosaicked with the ASTER DEM. The mosaicked DEM was used to delineate the catchments. The accuracy of the delineated catchments was checked with the digitized streams from toposheet. In flat areas, it was difficult for the accumulation algorithm to determine the flow channel path. To improve the result a “burn-in” of the existing stream network has been used.

C. Model Set-Up

The first step in modelling for any of the methods involves delineating streams and watersheds and getting the basic watershed properties such as area, slope, flow length, stream network density etc. Delineation was done using ArcGIS hydrologic tools which involves the following process-filling the sinks, flow direction, flow accumulation, stream order derivation and stream link to feature conversion.

1) Synthetic Unit Hydrograph

The method was used for sub-catchments having area greater than 40 sq. km. Length of longest stream, area of the catchment and slope were determined from the delineated catchments. These parameters were used to determine the synthetic unit hydrograph parameters using the equations 14. Design storm duration \((T_D)\), point rainfall, areal rainfall, effective rainfall increments and base flow were calculated as described in [4].

2) GIUH Model

The same watershed modelled using SUH method was modelled with GIUH method and the discharge was compared. It was found that the watersheds come under third and fourth order. Geomorphologic analysis involves the computation of stream number, average stream length and average stream area of the watersheds. These parameters were used to determine the Horton’s ratio, required for the GIUH model. These were estimated from the drainage network, using the stream order and area of the watershed. The values of Transitional probability and initial state probability were obtained using the equations 1 and 12. The automated tool that is developed for the derivation of GIUH by [2] is used for the derivation of GIUH.

V. RESULT AND DISCUSSION

The aim of the study is to assess the runoff in the sub-catchments for which the SUH or GIUH are applicable.

A. Watershed Delineation

A number of sub watersheds having area more than 40 km\(^2\) contributing water to Thrissur Corporation were identified. The area of these sub watersheds are given in Table II. The name of the sub watersheds were identified by sub watershed number given in Fig.3.

<table>
<thead>
<tr>
<th>Name of the sub-catchment</th>
<th>Area (km(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.989</td>
</tr>
<tr>
<td>2</td>
<td>47.2664</td>
</tr>
<tr>
<td>3</td>
<td>203.702</td>
</tr>
</tbody>
</table>

B. Synthetic Unit Hydrograph

Synthetic Unit Hydrograph method was applied to sub-catchments outside Thrissur Corporation having area greater than 40 km\(^2\). The SUH method as indicated in earlier chapter were applied and the corresponding output were given Fig. 4 to 6.

C. GIUH Method

The same watersheds which were analyzed earlier using SUH method were analyzed and the flood hydrograph were computed from GIUH method. The unit hydrograph and flood hydrograph were for these basins were given in Fig 7 to 9.

D. Comparison between SUH and GIUH method

The unit hydrograph and flood hydrograph obtained from SUH method and GIUH method were compared for the sub-catchments 1, 2 and 3. The corresponding peak discharges from both the methods are given in Table III.
Fig. 4 Flood hydrograph for sub watershed 1 using SUH

Fig. 5 Flood hydrograph for sub watershed 2 using SUH

Fig. 6 Flood hydrograph for sub watershed 3 using SUH

Fig. 7 Flood hydrograph for sub watershed 1 using GIUH

Fig. 8 Flood hydrograph for sub watershed 2 using GIUH

Fig. 9 Flood hydrograph for sub watershed 3 using GIUH
The methods were applied to the sub-catchments having area greater than 40 km$^2$. The results from SUH and GIUH methods show large variations in their peak values. It may be noted that the variation is at the order of 5 to 8 times. Such variation cannot be considered as a valid variation. It may be noted that same rainfall values were used for the computation of flood peaks that are derived from [4]. However, the difference is attributed to the shape and peak of the derived unit hydrographs. Hence, one of the methods is seemed to be a valid proposition for these sub watersheds. It is demonstrated earlier in studies elsewhere [3][6][1][11][8][2] that the GIUH can simulate the flood hydrograph with good accuracy. Last two studies are applied to the region under the current study and hence the statement is applicable for these regions. Moreover, a value of flood of 1600 m$^3$ cannot be expected for a stream of this size based on physical reasoning. Hence, it can be concluded that the computation of flood using SUH method by following the [4] over-estimates peaks values to larger extent, at least for the region considered. Since the procedure is being widely practiced in railways and elsewhere, it becomes essential to verify this proposition mean of an extensive further study.

VI. SUMMARY AND CONCLUSION

A. Summary
Flood assessment of three sub-catchments near Thrissur Corporation was done using SUH and GIUH methods.

B. Conclusion
Based on the study following conclusions were made.
(i) The computation of flood using SUH method by following [4] over-estimates peaks values to larger extent, at least for the region considered.
(ii) Since the procedure is being widely practiced in railways and elsewhere, it becomes essential to verify this proposition by means of an extensive study.

References