SER analysis for the best relay selection with interferences for DF

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Abstract—In this paper, we analyze the performance of a new cooperative diversity scheme based on best relay selection with interferences at relay nodes, only relay node can be affected by interference. For the selection scenario, only a best relay that maximizes the end-to-end received signal-to-interference and noise ratio (SINR) at the destination is selected to forward the information. Through statistical analyses, we establish a closed-form expression for the symbol error rate (SER), by deriving the analytical expressions for the probability density function (PDF), the cumulative density function (CDF), and the moment generating function (MGF) of the instantaneous received signal to interference and noise ratio (SINR) from relay selection. At relay nodes we use selection combiner (SC), which only process one of the diversity branches that has the highest SINR, alternatively, at the destination we used maximum ratio combiner (MRC) to combine the transmitted signals from the source and that one from the best selected relay, we show that our new scheme optimizes well the available resources (two channel links: the direct and the best relay links). Simulations are given to validate the analytical results with different number of relays and different interference power.

Index Terms—Wireless network, Best relay selection, Decode-and-Forward, symbol error rate, Multiple interferences.

1 INTRODUCTION

Cooperative diversity obtained without implementing multiple antennas on small communication terminals. By using virtual antennas (relays) between source and destination the full diversity will occur. Cooperative diversity is a cooperative with virtual antennas for improving, network channel capacities. In cooperation communication, the surrounding users act as relays to help in forwarding information to the destination to achieve full diversity[1]. Cooperative diversity can be implemented by helping of the Intermediate nodes called relays. So, relaying strategy concept has become an interesting field of research and has been widely investigated [2]. Therefore, one of the most advantages of using it in wireless networks is that the event of having all relay nodes goes down is very low. So the destination can receive multiple copies of the initial information sent from the source by applying some relaying protocols such as the amplify-and-forward (AF), decode-and-forward (DF)[3]. In amplify-and-forward protocol, the relay station amplifies the received signal from the source and forwards to the destination, whereas in Decode-and-Forward (DF) protocol, the relay forwards the information to the destination after decoding it. At the destination, the receiver detects the total information from the combination of transmitted signals from the source and that one from the relay nodes, using one of the combination methods[4].

Nevertheless, relay selection is employed to work in cooperative systems to make improvements to the multi-relay networks in which n+1 signals are required for a number of relays n [5]. And as a consequence in a multi-relay nodes technique, a bandwidth penalty might be observed caused by the presence of n+1 time slots. Therefore, relay selection goes obviously to be a highly interesting field which requires to be taken into account with an aim to fill the gap of investigation in wireless cooperative communication systems by efficiently using the channel resources. Within this paper, only two time slots are required to help the transmission scheme: one for the direct link and one for the best selected relay link.

Multiple Relay based communication is very susceptible to interference due to the fact the interference influences equally the received signal at the relay, as well as the received signal at the destination. Interference is a major infection in wireless cooperative communication systems. Multiple relay networks exploit a number of intermediate nodes known as relays to allow connection between source and destination; hence, the probability of interference affect the system is very high. On the other hand, the performance gains of cooperative systems are affected by multiuser interference, because the incoming signals can interfere with adjacent cells, especially in urban scenarios with many users and cells close to each other. That is why many researchers consider the effect of interference in cooperative network using relay selection, [6-9]. Minghua Xia and Sonia Aissa[10] investigated the performance of cooperative with AF protocol, With the effect of interference powers at primary
users and the Nakagami-\(m\) fading parameter of the interference channels. The authors in [11-13] found that the relays can decode and forward or amplify and forward the information if channel’s coefficient is above or below certain threshold. The authors in [11] proposed a decode-and-forward (DF) relay selection scheme using max-min criterion for an interference-limited multiple relaying network, they derived bit error rate (BER) and outage probability under the effect of interference. J. B. Si et al. [12], proposed a threshold-based relay selection protocol for wireless relaying networks with interference to choose one best relay to forward the information, in which the interference associated with the chosen relay is below. Amplify-and-forward strategy for interference limited networks in [13], is considered, in [14] the authors investigated the interference aware relay assignment using a heuristic algorithm (IRA). Work in [15] proposed three cooperative algorithms to obtain suboptimal solutions for end-to-end sum-rate maximization challenge in a multiple-antenna amplify-and-forward (AF) relay interference channel.

In this paper, we consider a multiple relaying networks in which, only the best relay that can be used to maximize the end-to-end received SINR in terms of (SER) furthermore we assumed that the relay can decode the transmitted information perfectly which mean that the source relay link couldn’t be affected by noise or interferences. The perfect channel side information (CSI) available at the receiver is considered. Assuming also that main channel gains are known to the transmitter, the signals can be transmitted through orthogonal channels by using some common access techniques such as TDMA, FDMA or CDMA. For a simple analysis, we use the TDMA system.

The rest of this paper is organized as follows: Section 2 a system model of best relay using DF protocol is presented. Section 3 discusses SINR analysis and show how we can select the best relay after making some derivation of PDF, and CDF. Asymptotic symbol error rate (SER) and MGF are analyzed in Section 4. Simulation results and Conclusion are provided in Section 5 and 6 respectively.

### 2 SYSTEM MODEL

As in fig. 1, our cooperative relay network system consisting of one source node \(S\) and \(N\) cooperative relays \(R_i (i =1,2,...,N)\) with \(L\) interferences at relay nodes and one destination \(D\). The channels from \(S\) to \(R_i\) and from \(R_i\) to \(D\) are statistically mutually independent, and identically distributed (i.i.d.). The system works under Rayleigh fading channel (any two nodes in the network is subject to Rayleigh fading) and additive white Gaussian noise (AWGN) \(N_0\).

The transmission scheme can be divided into two phases. In phase 1, the source broadcasts the information to the \(N\) relay nodes and the destination simultaneously. So the received signals at the \(i^{th}\) relay (considering the effect of interferences), and the destination (D) are respectively given by:

\[
Y_{sd} = \sqrt{P_s} h_{sd} x + n_{sd} \tag{1}
\]

\[
Y_{sr} = \sqrt{P_s} h_{sr} x + \sqrt{P_s} h_{sr} x' + n_{sr} \tag{2}
\]

Where:

- \(Y_{sd}\) and \(Y_{sr}\) represent the received signal at the destination and \(i^{th}\) relay respectively and \(h_{sd}\), \(h_{sr}\) are the fading channel coefficient between source \(S\) to destination \(D\) and between source and \(i^{th}\) relay, \(n_{sd}\) and \(n_{sr}\) denote the additive white Gaussian noise (AWGN) with variance \(N_0\), \(x\) and \(P_s\) are transmitted information symbol and transmitted source power (\(P_s\) normalized to unity), on the other hand \(x'\) modeled for interfered cluster as transmitted information symbol with fading channel coefficient \(h_{sr}\).

And in part 2, the selected best relay \(r_i\) forwards the signal received from Source to Destination as follows:

\[
Y_{rd} = \sqrt{P_i} h_{rd} x + \sqrt{P_i} h_{rd} x' + n_{rd} \tag{3}
\]
Where, \( \bar{x} \) and \( \bar{x}' \) are the decoded information at the relay \( r_i \) for both desired and interfered signals respectively.

### 3 SINR ANALYSIS AND SELECTION SCHEME

#### 3.1 SINR Analysis

From (2) and (3) the signal to interference and noise ratio for \( i \thinspace \)th relay link can be written as:

\[
\gamma_{SINR} = \frac{\max_{i} P_i |h_{di}|^2}{\sum_{i=1}^{L} P_i |h_{di}|^2 + N_0} = \frac{\max_{i} \gamma_{rd}}{\sum_{i=1}^{L} \gamma_{ir} + 1}
\]

Where \( \gamma_{i} \) represent the SNR of the \( i \thinspace \)th relay link, and \( \gamma_{ir} \) is interference to noise ratio (INR) of interference link, (the 1 in denominator is value of \( N_0 \) which normalized as unity).

For analytical tractability in deriving the distribution in (5) at high signal to noise ratio \( \text{SNR} \to \infty \) the one in denominator of relays link is ignored and thus (5) approximated to:

\[
\gamma_{SINR} = \frac{\max_{i} \gamma_{rd}}{\sum_{i=1}^{L} \gamma_{ir}}
\]

#### 3.2 Max Relay Selection Analysis

For source \( i \thinspace \)th relay link we use Selection combiner (SC), since SC make processing for one of the diversity branches that has the highest SINR. Consider the combiner works under Rayleigh fading with an i.i.d branches. Therefore the instantaneous SINR for \( i \) branch has the following pdf:

\[
p(\gamma_{i}) = \frac{1}{\gamma_{i}} e^{-\gamma_{i}/\gamma_{i}} = \frac{1}{(\gamma_{SINR})} e^{-\gamma_{i}/(\gamma_{SINR})}
\]

Where, \( \gamma_{i}/\gamma_{i} \) is the average SINR at every diversity branch, by assuming that all branches have the same average SINR then, the received SINR at the combiner can be written as[16, 17]:

\[
\gamma_{sc} = \max \gamma_{i} = \max \{\gamma_{0}, \gamma_{1}, \ldots, \gamma_{L-1}\}
\]

Therefore assuming that the channels are independent and identically distributed, we can express the cumulative density function (CDF) of \( \gamma_{sc} \) as:

\[
F_{\gamma_{sc}}(x) = P\left(\max_{i=2, \ldots, L} \gamma_{i} < x\right) = \prod_{i=1}^{N} P\{\gamma_{i} < x\} = \left(F_{\gamma_{i}}(x)\right)^{N}
\]

Where, \( F_{\gamma_{i}}(x) \) represent the CDF of \( \gamma_{i} \). According to our system, \( \gamma_{i} \) is an exponential random variable. So its CDF can be:

\[
F_{\gamma_{i}}(x) = 1 - e^{-x/\gamma_{i}}
\]

Equation (12) is also an outage probability for \( i \) branch. Consequently substituting (12) into (10), the CDF of \( \gamma_{sc} \) will be:
\[ F_{\gamma_w}(\gamma) = \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}ight)^N = \left(1 - e^{-\lambda_{SINR}}\right)^N \]  

\[ = \sum_{m=0}^{N} \binom{N}{m} (-1)^m e^{-m\lambda_{SINR}} \]  

(13)

Where \( \bar{\gamma} \) when all branches are equal, and \( \binom{N}{m} = \frac{N!}{m!(N-m)!} \) represent the binomial coefficient. By differentiating (13) with respect to \( \gamma \) we can obtain the following pdf:

\[ \frac{dF_{\gamma_w}(\gamma)}{d\gamma} = f_{\gamma_w}(\gamma) = \frac{N}{\gamma} \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^{N-1} e^{-\frac{\gamma}{\bar{\gamma}}} \]  

(14)

\[ = \sum_{m=1}^{N} \lambda_{SINR}^{m} \binom{N}{m} (-1)^{m-1} e^{-m\lambda_{SINR}} \]  

The average SNR of the combiner from (13) can be written as follows[18, 19]:

\[ \bar{\gamma}_c = \int_{0}^{\infty} \gamma f_{\gamma_w}(\gamma) d\gamma \]  

(15)

**4 MGF AND SER PERFORMANCES ANALYSIS**

MGF is a random variable that gives an alternative method of representing a PDF through a specific function of a single variable [20]. Using the definition of the MGF, we can define \( \gamma_c \) given by equation (14) as:

\[ M_{\gamma_w}(w) = \int_{0}^{\infty} e^{-w\gamma} f_{\gamma_w}(\gamma) d\gamma \]  

(16)

So substituting equation (14) into equation (16) and computing the above mentioned integral we finally obtain the expression of the moment generating function for the best relay as follows:

\[ M_{\gamma_c}(w) = \sum_{m=1}^{N} \binom{N}{m} m(-1)^{m-1} \frac{\lambda_{SINR}^{m}}{w + \lambda_{SINR}} \]  

(17)

From the above formulation a conditional closed model expression for the SER with M-PSK modulation is mentioned as follows[21]:

\[ P_{\text{SER}} = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{2}} \text{MGF}\left(\frac{-b}{\sin^2(\theta)}\right) d\theta \]  

(18)

Using equation (18), the total SER from relay selection and source relay link can be expressed as:

\[ P_{\text{SER}_{\text{total}}} = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{2}} M_{\gamma}(w) M_{\gamma}(w) d\theta \]  

(19)

Substituting the different moment generating functions by their values, hence the resulting expression will be:

\[ P_{\text{SER}_{\text{total}}} = \frac{1}{\pi} \sum_{m=1}^{N} \binom{N}{m} (-1)^{m-1} \int_{0}^{\frac{(M-1)\pi}{2}} (a)(b) d\theta \]  

(20)

Where \( a = \left(\frac{\sin^2(\theta)}{\sin^2(\theta) + x}\right) \), \( b = \left(\frac{\sin^2(\theta)}{m \sin^2(\theta) + y}\right) \) and \( x = \frac{b}{\lambda_{\gamma}} \), \( y = \frac{b}{m\lambda_{SINR}} \).

The expression in (20) represents the obtained SER from relay selection and can be calculated by using some partial fraction expansions as:

\[ P_{\text{SER}_{\text{total}}} = \frac{1}{\pi} \sum_{m=1}^{N} \binom{N}{m} (-1)^{m-1} \left(\frac{A_m}{x - y} + B_m\frac{y}{y_m}\right) \]  

(21)

Where, \( A_m = \frac{\sin^2(\theta)}{\sin^2(\theta) + x} \), \( B_m = 1-A_m \)

And:

\[ Q(c) = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{2}} \left(\frac{\sin^2(\theta)}{\sin^2(\theta) + c}\right) d\theta = \frac{(M-1)\pi}{2} \int_{0}^{\frac{(M-1)\pi}{2}} \frac{d\theta}{\alpha + \beta \cos(\theta)} \]  

(22)

\[ = \frac{2c}{\pi(\alpha + \beta)} \tan^{-1}\left(\frac{\alpha - \beta}{\alpha + \beta} \tan\left(\frac{\theta}{2}\right)\right)^{\theta(\sim)2(M-1)\pi/2} \]  

Where, \( \alpha = 1 + 2c \) and \( \beta = -1 \)

Proof: see appendix.

**5 SIMULATION RESULTS**

Within this section, we present numerical proof that confirms simulation results. Fig. 2 shows the representation of symbol error rate using QPK modulation for analytical and simulation curves compared with best relay when no interference as we did in [22] when the number of relays and interferences equal to 2 and the \( \lambda_{SINR} = 5 \) it can be seen that the result between the curves are closely matched, additionally, we can easily conclude that SINR based on best relay demonstrates visible gain over best relay based no interferences. Taking into consideration the information is
usually transferred through best relay selection path using DF and the direct link.

This work analyzed the performances of a cooperative communication networks by considering the effect of interference then we compare it with the case of no interferences. The best relay selection using DF protocol is considered. After selecting the best relay using selection combiner (SC) that maximizes received SINR form source-relay link, a maximal ratio combiner (MRC) is used to combine best relay signal and the signal from the direct link. Through statistical analyses, we derived the expression of the received SINR at the destination and established a closed-form expression for the symbol error probability of the system under studied.

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**Appendix: proof equation (22)**

\[
Q(c) = \frac{1}{\pi} \int_0^{\pi} \left(\frac{\sin^2 \theta}{\sin^2 \theta + c}\right) d\theta 
\]

\[
= \frac{1}{\pi} \int_0^{\pi} \left(\frac{1 - c}{\sin^2 \theta + c}\right) d\theta 
\]

\[
= \frac{1}{\pi} \int_0^{\pi} \left(\frac{2c}{1 - \cos 2\theta + 2c}\right) d\theta 
\]

\[
= \frac{c}{\pi} \int_0^{\pi} \left(\frac{1}{1 + 2c - \cos \theta}\right) d\theta 
\]

**References**


