Resonant Inductive Power Transfer for Wireless Sensor Network Nodes

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Abstract—This paper presents the experimental study of Wireless Power Transfer through resonant inductive coupling for sensor network nodes. Theoretical background of wireless power transfer is explained in this paper with the help of mathematical models. Both experiment and simulation results are discussed here and a prototype for charging sensor node is proposed. The proposed system can charge the sensor nodes, at distances up to 4 cm effectively.

Index Terms—Wireless power, Inductive coupling, Resonance, magnetic Coupling, Sensor node

1 INTRODUCTION

S
er networks are very important part of modern communication systems. Real time monitoring of different parameters and their collective analysis is effectively done through sensor networks. Sensor nodes are the basic building blocks of a sensor network. Sensor node consists of sensors for the real-time parameter measurements, microcontroller or processor for the processing and controlling, wireless communication modules depending on application and battery for powering the sensor node. Rechargeable batteries are made use of in most of the sensor nodes. When battery drains out, they are replaced with new ones. But there are situations where the battery replacement is difficult [1]. For example in structural health monitoring system, the sensor node is placed inside a column or beam for taking measurements. In those cases if battery drains out, the replacement of battery is very difficult. Similarly in the case of sensors which are placed inside the human body or if the sensors are inside a bridge [2], the battery replacement is a difficult problem.

One way of solving this problem is to increase the capacity of battery. But there are limitations in increasing the capacity of the battery with respect to its size. Another way is to apply the concept of wireless power transfer. When battery drains out, power can be supplied from outside using wireless power transfer, so as to avoid the difficulties in battery replacement. Lot of active research is going on in this area to effectively transfer the power from source to destination without using solid wires.

The evolution of wireless power transfer has a history of more than 100 years. Nikola Tesla was the pioneer in wireless power transfer who was the owner of more than 200 patents. Wardenclyffe tower was designed by Tesla for wireless telephony and also for demonstrating wireless electrical power transmission [3]. In 1960s William C. Brown invented Rectenna which converts microwaves into dc current. He contributed much to the development of microwave power transmission. Wireless power transmission can be mainly classified into radiative and non radiative. Non Radiative is also known near field and radiative is known as far field. Practically, the area surrounding the transmitter within one wavelength is known as near field region. In this region the energy transmitted by the antenna will reside until there is coupling device to couple that energy. Therefore the energy within this region is said to be non-radiative. Non radiative power transfer is further classified into inductive coupling and capacitive coupling. Inductive coupling, the energy associated with transmitter in the near field region is coupled via magnetic fields and in capacitive coupling it is via electric fields. Area after one wavelength is known as far field. Energy in this field is radiative in nature. For power transmission electromagnetic waves are used in this region which includes microwaves [4], ultrasound, x-ray, light etc. In [1], the charging of sensor node via inductive coupling is done where the distance of charging has been reported as 10 mm. In this work magnetic coupling is used as the power transmission method and the distance is increased by employing high frequency transformer at the transmitter.

2 INDUCTIVE COUPLED WIRELESS POWER TRANSFER

Magnetic coupling between two independent coils means the coils affect each other through the magnetic field generated by either of them. In other words the magnetic field generated by one coil is coupled with other coil. In this scenario, in addition to the self-inductance of each coil there exists another inductance known as mutual inductance.

When current flows through an inductor, according to Ampere's law, a magnetic field exists around the inductor. Fig. 1 shows the magnetic field around an inductor carrying current 'i' amperes. The magnetic flux $\phi$ is given by equation (2.1)

$$\phi=\mu_0 NI$$

where $\mu_0$ is the permeability of free space, $A$ is the area of the

Fig. 1.Magnetic field around an inductor
coils, w is the width of coil and N is the number of turns. According to Faraday’s law the magnitude of the voltage induced in the coil V is given by equation (2.2).

\[ V = N \frac{d\phi}{dt} \]  

(2.2)

Equation (2.2) can be re-written as

\[ V = \frac{\mu_0 N^2 A}{\pi w} \frac{dl}{dt} \]  

(2.3)

where L is the self-inductance and is given by equation (2.4)

\[ L = \frac{\mu_0 N^2 A}{w} \]  

(2.4)

When another coil is in close proximity, due to current \( i_1 \) there arises two flux \( \phi_{11} \), which is passing through primary coil and \( \phi_{12} \) passing through secondary coil. Fig. 2 shows the coupling between two inductors. The total flux due to current \( i_1 \) is given by equation (2.5)

\[ \phi_1 = \phi_{11} + \phi_{12} \]  

(2.5)

Voltage \( V_1 \) induced in primary coil

\[ V_1 = \frac{\mu_0 N^2 A}{\pi w} \frac{d\phi_{11}}{dt} \]  

(2.6)

Voltage induced in secondary coil

\[ V_2 = \frac{\mu_0 N^2 A}{\pi w} \frac{d\phi_{12}}{dt} \]  

(2.7)

where ‘M’ is the Mutual-inductance. Mutual inductance depends on the geometry of the coil and number of turns. Coupling coefficient ‘k’ can be defined as the ratio of mutual inductance with the self-inductances of coil as given by equation (2.8).

\[ k = \frac{M}{\sqrt{L_1 L_2}} \]  

(2.8)

The coupling coefficient is zero if there is no coupling between coils. If k=1 it is known as perfect coupling. If k < 0.5, it is known as loose coupling and if k > 0.5 it is known as tight coupling. For the transfer of power in mid field region, the coupling will be loose coupling.

3 ANALYSIS OF COUPLED INDUCTOR CIRCUIT

![Fig. 3. Circuit model of inductive coupled power transfer](image)

Fig. 3 represents a basic circuit of inductive coupled wireless power transfer.

Using mesh analysis and employing dot convention as in Fig. 4

\[ V = i_p R_p + j i_p X_{L_p} - j i_s X_M \]  

(3.1)

\[ j i_p X_M = i_s X_{L_s} + i_p R_s \]  

(3.2)

where \( X_{L_p}, X_M, X_{L_s} \) denotes reactance due to inductances \( L_p, L_s \) and M

Rewriting equation (3.2) with respect to \( i_s \) and substituting in equation (3.1)

\[ V = i_p \left( R_p + j X_{L_p} + \frac{x_{M}}{X_{L_s} + j X_{L_p}} \right) \]  

(3.3)

\[ i_s = i_p \left( \frac{X_{M}}{X_{L_s} + j X_{L_p}} \right) \]  

(3.4)

Solving equation (3.3)

\[ i_p = \frac{V}{R_p + j X_{L_p} + \frac{x_{M}^2}{X_{L_s} + j X_{L_p}}} \]  

(3.5)

\[ V_{L_s} = j i_p X_{M} - j i_s X_{L_s} \]  

(3.6)

\[ V_{L_p} = j i_p X_{L_p} - j i_s X_m \]  

(3.7)

The input impedance seen by the primary coil is given by equation (3.8)

\[ Z_{in} = \frac{V}{i_p} = \left( R_p + j X_{L_p} \right) + \frac{x_{M}^2}{X_{L_s} + j X_{L_p}} \]  

(3.8)

The term \( R_p + j X_{L_p} \) represents the impedance of primary coil where \( R_p \) refers to the effective resistance of the coil. The second term in \( Z_{in} \) represents the results of coupling known as ‘reflected impedance \( Z_{ref} \)’.

\[ Z_{ref} = \frac{x_{M}}{X_{L_s} + j X_{L_p}} \]  

(3.9)

If there exists a load impedance \( Z_l \) series with the secondary
coil, the reflected impedance is given by equation (3.10)

\[ Z_{ref} = \frac{x_m^2}{R_s + jXL_s + Z_L} \] (3.10)

Power delivered to the load \( R_s \) is given by equation (3.11)

\[ P_{R_s} = \frac{1}{2} \frac{1}{Z_L} \] (3.11)

From equation (3.10) it is clear that the reflected impedance depends on reactive component of mutual inductance, load impedance and the coil impedance of secondary.

### 4 Magnetic Coupled Resonance

The efficiency of power transfer can be improved by bringing the concept of resonance into it. Resonance is the phenomenon which occurs in nature in different forms. The power transmitter and receiver are tuned to same frequency so that both can act together as coupled resonators. When two individual resonators are brought together they act as coupled resonators and exchange energy. The main advantage of resonance is that, when resonant coupling, the energy can be transferred from a source coil to a receiver coil, while losing little energy to extraneous off resonant objects [5]. From the previous analysis, we have ‘reflected impedance’ as the impedance seen by the primary coil.

\[ Z_{ref} = \frac{x_m^2}{R_s + jXL_s + Z_L} \] (4.1)

Ideally higher reflected impedance is preferred because at higher frequencies more power can be delivered to load if the reflected impedance is higher. From equation (4.1) it is clear that the reflected impedance depends on the secondary impedance. As secondary impedance decreases, reflected impedance increases. Therefore one way to increase the reflected impedance magnitude is by eliminating the reactance offered by the secondary coil.

In order to eliminate the reactance of secondary coil we can connect a capacitor in series or parallel as shown in fig 5(a) or 5(b). The value of capacitor is to be chosen in such a way that the magnitude of capacitive reactance will be equal to magnitude of the inductive reactance, so as to eliminate the reactance offered by the secondary coil.

The addition of capacitor at secondary coil as shown in Fig 5 will result in resonance. At resonance the magnitude of reflected impedance get increased and thus more power can be delivered to the load. Similarly we can further improve the performance by adding capacitor at the primary side as shown in Fig 6 so that the reactance of primary can be eliminated.

The reflected impedance is then increased to

\[ Z_{ref} = \frac{x_m^2}{R_s + Z_L} \] (4.2)

From equation (4.2) it is clear that the reflected impedance is proportional to the square of the reactance offered by mutual inductance.

### 5 Calculation of Mutual Inductance and Coupling Coefficient Between Circular Coils

The mutual inductance between the two coils can be found out using the Neumann formula [6]. Consider two circular coils. The mutual inductance by coil 1 on coil 2 is given by equation (5.1).

\[ M_{12} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{\phi_1 \phi_2}{d_{s1}d_{s2}} d_{s1} d_{s2} \] (5.1)

where \( d_{s1} \) and \( d_{s2} \) are incremental area of circular sections and \( R_{12} \) is the distance between them. Mutual inductance is a parameter that is completely independent of the current in the circuits. It totally depends on the geometry of the circuits.

The dot product \( d_{s1} \) and \( d_{s2} \) can be rewritten as [7]

\[ M_{12} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} d_{s1} d_{s2} \cos \alpha \] (5.2)

where \( \alpha \) is the angle between the circular cross-sectional areas and is given by \( \alpha = \phi - \phi' \) and \( d_{s1} = a \delta \phi \), \( d_{s2} = b \delta \phi' \) where \( \phi \) and \( \phi' \) are the angle of circular cross section with respect to x axis, ‘a’ and ‘b’ are the radius of the two coils and ‘d’ is the distance between them.

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The coupling coefficient is then defined as in equation (5.8)

\[ L = \frac{\mu_0 \Phi_c}{4\pi} \phi \frac{abc \alpha}{\sqrt{a^2 + b^2 + d^2 - 2 \alpha a}} \, d\phi \, d\phi' \]  

(5.4)

This integral can be calculated using elliptic integral as

\[ M_{12}(m) = \frac{2\pi}{\alpha} \sqrt{ab} \left[ \left( 1 - \frac{m}{2} \right) k(m) - E(m) \right] \]  

(5.5)

where \( k(m) \) and \( E(m) \) represents the elliptic integral of first and second kind respectively and \( m \) is given by

\[ m = \frac{4ab}{(a+b)^2 + R_{12}} \]  

(5.6)

If there are \( N \) turns in coils then the net mutual inductance will be \( M'_{12} \) and is given by equation (5.7)

\[ M'_{12} = N^2 M_{12} \]  

(5.7)

The coupling coefficient is then defined as in equation (5.8)

\[ k = \frac{M'_{12}}{\sqrt{L_1 L_2}} \]  

(5.8)

The self-inductance of a circular coil can be calculated using equation (5.9)

\[ L = N^2 \mu_0 a \log \left( \frac{8a}{r} - 2 \right) \]  

(5.9)

where ‘\( N \)’ is the number of turns of coil, ‘\( a \)’ is the radius of the circular loop and ‘\( r \)’ is the radius of the copper wire.

From equation (5.9) it is clear that the inductance \( L \) is directly proportional to the square of the number of turns, \( N \).

### 6 Mathematical Model of Resonant Wireless Power Transfer

Resonant wireless power transfer can be effectively studied by analyzing the circuit using mathematical models. Following sections (6.1 and 6.2) analyzes the coupled parallel resonators.

#### 6.1 Parallel Circuit Model

![Fig. 7. Inductive power transfer model (Parallel)](image)

Fig 7 represents the model of wireless power transfer through parallel resonators. Here \( L_p \) represents the inductance of coil 1, \( L_s \) the inductance of coil 2, \( C_1 \) the capacitance of capacitor in primary side, \( C_2 \) the capacitance of capacitor in secondary side. \( R_s \) represents impedance of source, \( Z_l \) the impedance of load, \( R_{1s} \) and \( R_{2s} \) the resistances of coil 1 and coil 2, \( V_s \) the supply voltage. \( K \) is the coupling coefficient between the primary and secondary coil. It is assumed that \( L_p = L_s, C_1 = C_2 = C, R_{1s} = R_{2s} = R \).

Direct analysis of parallel circuit is complex and therefore the equivalent circuit model of coupled inductors is used which is shown in Fig 8.

![Fig. 8. Equivalent T circuit of coupled inductors](image)

Equation (6.1.1) shows the relation between coupled inductors and is given by equation (6.1.2)

\[ L_2 = L_p - M, L_1 = M \]  

(6.1.1)

Equation (6.1.2) shows the relation between coupled inductors and its equivalent T circuit.

#### 6.2 Analysis of Parallel Coupled Resonators

Figure 9 represents the equivalent circuit after replacing coupled coils with their equivalent circuit.

![Fig. 9. Equivalent circuit](image)

Let \( I_1, I_2, I_3, I_4 \) be the currents in the loop. By mesh analysis we get the following equations.

\[ V_s = I_1(R_s - jX_c) + I_2X_c \]  

(6.2.1)

\[ 0 = I_1(jX_e + I_2(R + jX_{la} - jX_c - jX_{lc}) - I_3(jX_{lc}) \]  

(6.2.2)

\[ 0 = I_2(-jX_{lc}) + I_3(jX_{lc} + jX_{la} - jX_c + I_4(jX_e) \]  

(6.2.3)

\[ 0 = I_3(jX_e) + (Z_l - jX_c) \]  

(6.2.4)

Let

\[ w = jX_e, x = jX_{la}, y = jX_{la}, z = jX_{lc} \]

Solving the 4 equations, the final result becomes,

\[ I_4 = \frac{V_s}{2aw^2 + aRR_s - aRw + axR_s - axw - awR_s + azR_s - azw} \]  

(6.2.5)

\[ I_3 = \frac{I_4(w - Z_l)}{w} \]  

(6.2.6)

\[ I_2 = \frac{I_4(a)}{z} \]  

(6.2.7)
\[ I_1 = \frac{I_4 (a (R+x-w+z))}{w^2 z} \]  \hspace{1cm} (6.2.8)

where
\[ a = (z+y-w)(w-Z_1) + w^2 \]  \hspace{1cm} (6.2.9)

Output voltage \( V_{out} \) is given by
\[ V_{out} = I_4 Z_i \]  \hspace{1cm} (6.2.10)

Efficiency can be found out by
\[ eff = \frac{P_{load}}{P_{supplied}} = \frac{I_4^2 Z_i}{V_s I_1} \]  \hspace{1cm} (6.2.11)

The efficiency is directly proportional to the square of the current \( I_4 \) where \( I_4 \) depends on the circuit parameters of resonators.

7 EXPERIMENTAL SETUP AND RESULTS

Experimental study was done by using two coils made up of 26 gauge copper wires having 9 numbers of turns each. Each coil is loaded with capacitor to achieve resonance. Both parallel and series resonance are experimented. Function generator is used to give input supply with required resonant frequency. Measurements are taken with the help of Digital Storage Oscilloscope (DSO) and multimeter.

The coil has inductance \( L \) of 32.11\( \mu \)H. Value of capacitor \( C \) used is 6.8nF. The resonant frequency is given by equation (7.1)
\[ f_0 = \frac{1}{2\pi\sqrt{LC}} = 340.57 \text{ kHz} \]  \hspace{1cm} (7.1)

One of the two parallel resonators is made as power transmitter by applying 5V input from the function generator. The frequency of input is kept equal to the resonant frequency given by equation (7.1). The other resonator made as receiver by connecting Light Emitting Diode (LED) as the output load. Fig 10 represents the photograph of the experiment with LED as load.

A 100 ohm resistor is connected across the receiver as load instead of LED and voltage across the load is measured with the help of DSO by varying distance between the transmitter and receiver. Distance varied from 0.5cm to 4cm with a step size of 0.5cm. The results of experiment are given in Table 1 and Table 2. Table 1 shows the experiment and simulation result of output voltage with respect to distance. Table 2 shows the experiment and simulation results of Coupling coefficient and efficiency. Simulation results are obtained by mathematically calculating the quantities with the help of MATLAB software.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Output Voltage (Experiment)</th>
<th>Output Voltage (Simulation)</th>
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<tr>
<td>.5cm</td>
<td>2.21v</td>
<td>2.7475v</td>
</tr>
<tr>
<td>1cm</td>
<td>1.75v</td>
<td>2.0786v</td>
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<tr>
<td>1.5cm</td>
<td>1.32v</td>
<td>1.7009v</td>
</tr>
<tr>
<td>2cm</td>
<td>1.11v</td>
<td>1.4594v</td>
</tr>
<tr>
<td>2.5cm</td>
<td>.993v</td>
<td>1.2969v</td>
</tr>
<tr>
<td>3cm</td>
<td>.986v</td>
<td>1.1858v</td>
</tr>
<tr>
<td>3.5cm</td>
<td>.895v</td>
<td>1.1101v</td>
</tr>
<tr>
<td>4cm</td>
<td>.887v</td>
<td>1.0597v</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Distance</th>
<th>Coupling Coefficient (Experiment)</th>
<th>Coupling Coefficient (Simulation)</th>
<th>Efficiency (%)</th>
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<td>.3199</td>
<td>54.29</td>
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<tr>
<td>1cm</td>
<td>.1750</td>
<td>.2372</td>
<td>30.46</td>
</tr>
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<td>.1320</td>
<td>.1930</td>
<td>20.28</td>
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<td>2cm</td>
<td>.1110</td>
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<tr>
<td>4cm</td>
<td>.0887</td>
<td>.1197</td>
<td>7.84</td>
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</table>

Fig. 10 Photographs of experiment

Fig. 10(a) Experimental setup showing parallel coupled resonators with LED as load.

Fig. 10(b) Experimental setup showing parallel coupled resonators with LED as load.
From Table 2 it is clear that magnitude of coupling coefficient decreases as the distance between the transmitter and receiver increases thus decreases the efficiency of power transfer.

Fig 11 shows the variation of output voltage with respect to the distance. Fig 12 represents the variation of coupling coefficient with respect to the distance. Both experiment and simulation results are combined in these figures. Fig 13 represents the variation of efficiency with respect to distance.

It was observed that output voltage and hence the efficiency decreases with increase in distance.

At the application level, wireless power transfer for charging a battery system was implemented. The maximum possible distance of charging was obtained up to 4 cm. High Frequency Transformer was used to convert the input 230V, 50 Hz supply voltage into 12V, 50 KHz ac supply. This ac voltage is given to circular coils with 9 turns. Receiver is made up with 36 turns. Received voltage is rectified and filtered. After filtering voltage is regulated and can be used to charge battery system. The experimental setup done for charging the battery in mobile phone is shown in Fig 14 and 15.
8 CONCLUSION
The main purpose of our work was to design a device that can provide wireless power to sensor nodes effectively using inductive coupling. Experimental results are in good agreement with simulation results. A wireless power transfer system for charging battery was developed. In this work High frequency transformer has been used at the transmitter with the result that the distance of power transfer has been increased to 4cm.

The wireless power transfer can make remarkable changes in consumer electronics by eliminating the use of messy wires and costly batteries. It is also the best alternative for the reduction of pollution in the world by transmitting power while harvesting it from solar energy. It will be a key enabler for the smart future applications.

9 REFERENCES