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Performance Evaluation of Proportional Navigation Guidance for Low-Maneuvering Targets

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Abstract— A missile system employed with homing guidance can sense and detect the target, guide itself to steer towards the target by generating commands to its own control surfaces. Proportional Navigation (PN) as an effective guidance law is implemented in most of all the homing guidance systems. Since its inception PN is one of the best proven techniques which has been used extensively in past and present homing systems. This paper presents overall survey of the performance evaluation of the Proportional Navigation which is very effective for the missile to intercept the low-maneuvering target. Here two simulation examples are presented. In the first one, even though if the missile is launched in the wrong direction by a heading error of 50 deg, the PN can enable missile to hit the target. The second is to observe how PN was effective to hit a maneuvering target. Also it is explained why linearization of engagement model results in performance projections with consistent accuracy to make it worthwhile for developing design relationships. In order to show how Proportional Navigation work fairly well against maneuvering and constant velocity target, this paper presents a simulation of a two-dimensional missile target engagement geometry in LabVIEW for a point mass missile and target.

Index Terms— Proportional Navigation, Line-of-sight, Heading Error.

1 INTRODUCTION

For a short-to-medium range homing missiles Proportional Navigation (PN) is perhaps the most widely known and used guidance law. A classical proportional navigation simply states if two bodies are closing on each other eventually they will intercept when there is no rotation in the line of sight (LOS) between the two bodies relative to the inertial space. By making the interceptor missile heading proportional to the LOS rate for a non maneuvering target the PN guidance law seeks to null out the LOS rate. The relation can be expressed as follows:

$$a_n = NV_c\,(d\lambda/dt) \qquad (1)$$

Where

a_n = commanded normal acceleration [ft/sec^2] or [m/sec^2]

N = effective navigation ratio

V_c = closing velocity [ft/sec] or [m/sec]

$d\lambda/dt$ = LOS rate measured by the missile seeker [rad/sec]

Depending upon the target maneuver and other system-induced tracking errors the navigation constant (N) will vary based on the missile acceleration requirements. The values on N between 3 and 5 are usually used in order to obtain acceptable miss distance intercept and to minimize the missile acceleration requirement. An onboard seeker is used to measure the line-of-sight angle with respect to fixed space coordinates there by a lateral (or normal) acceleration of the missile is commanded proportional to that line-of-sight rate. To derive the equations representing the proportional navigation consider the missile-target engagement geometry given by Fig. 1.

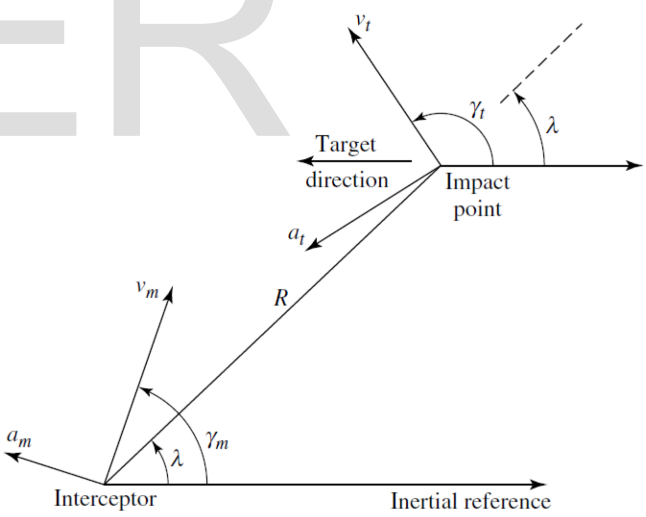

Fig. 1. Interceptor-Target Geometry

During the missile time of flight the missile and target speed is assumed to remain constant. From the Fig. 1 let R be the range between the missile and the target, λ is the line of sight angle rotated from the initial value. Let v_m and v_t be the interceptor velocity and velocity of the target. In the same way let γ_m and γ_t are the missile flight path (or heading) angle and target flight path angle. Also at any time the difference in the normal components of velocity of the target and missile is the rate of rotation of the line of sight and it can be expressed by the equation

$$R\,(d\lambda/dt) = v_t\sin(\gamma_t - \lambda) - v_m\sin(\gamma_m - \lambda) \quad (2)$$

While the velocity component along the line if sight is given by

$$(dR/dt) = v_t\cos(\gamma_t - \lambda) - v_m\cos(\gamma_m - \lambda) \quad (3)$$

The basic differential equation governing the fact that the rate of change of the missile heading γ_m is directly proportional to the rate of change of line-of-sight angle λ is given by

$$d\gamma_m/dt = N(d\lambda/dt) \quad (4)$$

where N is the navigation constant. The complete equations of motion for the system are represented by equations 2, 3 and 4. Therefore R, γ_m and λ, the velocities v_m, v_t are dependent variables and the target's flight path angle γ_t must be known or assumed. Here the usual way of implementing a proportional navigation guidance system is to use the target tracker (or seeker) to measure the line-of-sight rate ($d\lambda/dt$). By differentiating the equation 2 the general form of proportional navigation can be developed and is given by

$$\dot{R}\dot{\lambda} + R\ddot{\lambda} = \gamma_t v_t\cos(\gamma_t - \lambda) - \dot{\gamma}_m v_m\cos(\gamma_m - \lambda) - \dot{\lambda} v_t C - v_m\cos(\gamma_m - \lambda) \quad (5)$$

Substituting Eq. 3 and Eq. 4 in Eq. 5 we get

$$2\dot{R}\dot{\lambda} + R\ddot{\lambda} = \dot{\gamma}_t\cos(\gamma_t - \lambda) - N\dot{\lambda}v_m\cos(\gamma_m - \lambda) \text{ or} \quad (6)$$

$$\ddot{\lambda} + \left(\frac{\dot{\lambda}}{R}\right)[2\dot{R} + N\, v_m\cos(\gamma_m - \lambda)] = (1/R)\,\dot{\gamma}_t\, v_t\cos(\gamma_t - \lambda) \quad (7)$$

From the above derivation the system equation consists of Eq. 3, Eq. 4 and Eq. 5. By investigating the special case where by the target flies a straight line-course that is $d\gamma_t/dt = 0$, we have the inequality

$$N' = -N\{(v_m\cos(\gamma_m - \lambda))/(dR/dt)\} \quad (8)$$

Where N′ is commonly called as *effective navigation ratio*, and missile's closing velocity is $-dR/dt = v_c$. From Eq. 7 we can observe that $R\to 0$, $(d\lambda/dt) \to 0$ when $d^2\lambda/dt^2$ remains finite. Now defining the missile normal acceleration a_n as

$$a_n = v_m(d\gamma_m/dt) \quad (9)$$

Where $d\gamma_m/dt$ is the missile turning rate. Now substituting Eq. 4and Eq. 8 in Eq. 9 we observe

$$a_n = \{-N\dot{R}v_m/\ v_m\cos(\gamma_m - \lambda)\}\left(\frac{d\lambda}{dt}\right) = \{Nv_c/\cos(\gamma_m - \lambda)\}\left(\frac{d\lambda}{dt}\right); \quad (10)$$

Equation 10 is the well-known general classical proportional navigation guidance and is used to generate the guidance commands with the missile velocity and seeker gimbal angle ($\gamma_m - \lambda$).

Now consider non maneuvering target as a special case the LOS rate $d\lambda/dt$ will be investigated by introducing the range R in Eq. 7 from the operator

$$\frac{d}{dt} = \left(\frac{d}{dR}\right)\left(\frac{dR}{dt}\right) \quad (11)$$

At this point by using above operator Eq. 7 becomes

$$R\left(\frac{dR}{dt}\right)\left(\frac{d\dot{\lambda}}{dR}\right) + \dot{\lambda}[2\dot{R} + Nv_m\cos(\gamma_m - \lambda)] = \dot{\gamma}_t\, v_t\cos(\gamma_t - \lambda) \quad (12)$$

Also from Eq. 8

$$N'(dR/dt) = -Nv_m\cos(\gamma_m - \lambda)$$

Now Eq. 12 takes the form

$$R\left(\frac{d\dot{\lambda}}{dt}\right) + \dot{\lambda}(2 - N') = \dot{\gamma}_t(v_t/R)\cos(\gamma_t - \lambda) \quad (13)$$

Upon substituting

$$\xi = \ln(R_0/R) = -\ln(R/R_0) \quad (14)$$

with $R=R_0$(launch) corresponding to $\xi = 0$ and $R=0$ (intercept) corresponding to $\xi = \infty$. Using these one can obtain

$$d\xi = -(R_0/R)(dR/R_0) \text{ and } d/dR = -(1/R)(d/d\xi)$$

With these considerations Eq. 13 becomes

$$d\dot{\lambda}/d\xi + \dot{\lambda}(N' - 2) = -\dot{\gamma}_t(v_t/\dot{R})\cos(\gamma_t - \lambda) \quad (15)$$

If the target is flying on a straight-line course, then $d\dot{\gamma}_t/dt = 0$ and the equation 15 takes the form

$$\dot{\lambda}(\xi) = \dot{\lambda}_0 e^{(2-N')\xi} \quad (16)$$

Substituting Eq. 14 into Eq. 16 yields

$$\dot{\lambda}(R/R_0) = \dot{\lambda}_0\,(R/R_0)\, e^{(N'-2)} \quad (17)$$

From Eq. 17 it can infer that this differential equation tends to zero for the interceptor-target closing, i.e. $(dR(t)/dt)<0$. When $N' = 2$, $d\lambda/dt$ is constant during the flight and a constant target maneuver is required. For $N' > 2$, the acceleration required at intercept is zero which is a highly desirable situation that provides full maneuvering capabilities of the missile at the intercept and this can be achieved by giving early corrections to the heading error. At the beginning of the flight, $d\lambda/dt$ is maximum and for $N' = 3$ it decreased linearly to zero and for $N' > 3$ the value reaches to zero asymptotically. At the final (or intercept) point R=0 the collision course condition $d\lambda(t)/dt = 0$ can be achieved with a vanishing turning rate $d\gamma_m/dt = 0$. If we assume the target is flying from left to right then the plot of $(\dot{\lambda}/\dot{\lambda}_0)$ vs. (R/R_0) is shown by Fig. 2

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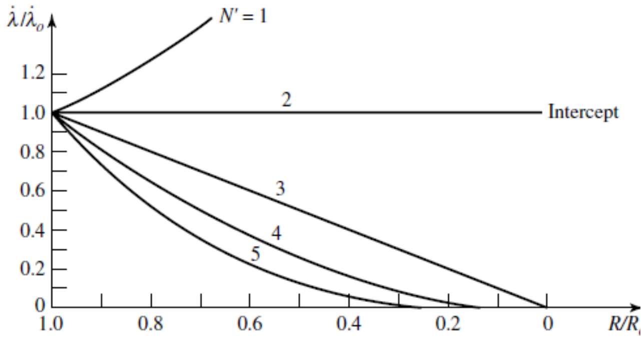

Fig.2 Plot of ($\dot{\lambda}/\dot{\lambda}_0$) vs. ($R/R_0$) with N' as a non maneuvering target parameter.

Now consider the maneuvering target, while estimating LOS rate $d\lambda/dt$, the right-hand side of Eq. 15 remains constant. The solution of Eq. 15 is given by

$$\left(\frac{d\lambda}{dt}\right) = (\gamma_t v_t \cos(\gamma_t - \lambda)/\dot{R}(2 - N'))\{1 - e^{-(N'-2)\xi}\} \quad (18)$$

For eliminating ξ substitute Eq. 14 in Eq. 18 and one can obtain

$$\left(\frac{d\lambda}{dt}\right) = (\gamma_t v_t \cos(\gamma_t - \lambda)/\dot{R}(2 - N'))\{1 - (R/R_0)^{N'-2} \quad (19)$$

The ratio of interceptor missile's normal (or lateral) acceleration to the target's lateral acceleration is given by

$$|a_{nm}/a_{nt}| = |v_m \dot{\gamma}_m / v_t \dot{\gamma}_t| = (v_m/v_t)\, N\, (\lambda/\gamma_t) \quad (20)$$

Using the ratio k as v_m/v_t and Eq. 19, Eq. 20 can be written as

$$|a_{nm}/a_{nt}| = (N'/(N'-2))|\cos(\gamma_t - \lambda)/\cos(\gamma_m - \lambda)|\ \{1-(R/R_0)^{N'-2}\} \quad (21)$$

Making the use of approximation regardless of direction of approach to the target $|\cos(\gamma_t - \lambda)/\cos(\gamma_m - \lambda)| \approx 1$, we can obtain

$$|a_{nm}/a_{nt}| \approx (N'/(N'-2)\{1-(R/R_0)^{N'-2}\} \quad (22)$$

For a maneuvering target with N' as a parameter the ratio of $|a_{nm}/a_{nt}|$ with respect to the relative distance to the target (R/R_0) is shown by Fig. 3

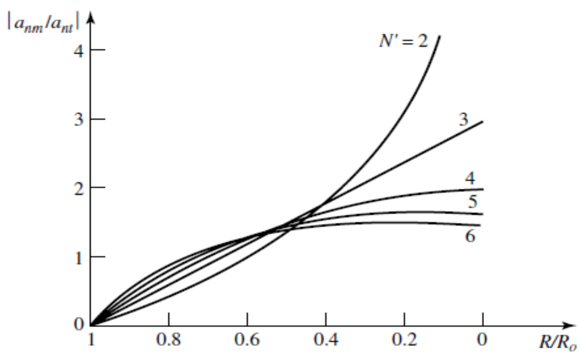

Fig. 3 Plot of $|a_{nm}/a_{nt}|$ vs (R/R_0) for a maneuvering target.

Considering the missile course is from left to right we can observe from Fig. 3 largest acceleration occurs at the end if the flight. The Fig. 3 depicts the fact that a_{nm} grows beyond all limits for $N' = 2$. For evaluating Eq. 22 a special case where $N' = 2$, the following transformation will be applied by transforming the expression $(R/R_0)^{N'-2}$ into

$$a^x = 1 + \frac{\ln a}{1!} + \frac{(\ln a)^2}{2!}x^2 + \cdots \quad (23)$$

Eq. 22 can be obtained in the form

$$|a_{nm}/a_{nt}| = N'\left\{-\frac{\left(\ln\left(\frac{R}{R_0}\right)\right)}{1!} - \frac{\left[\ln\left(\frac{R}{R_0}\right)\right]^2}{2!}\right\}(N'-2) - \cdots \quad (24)$$

Also for $N' = 2$

$$|a_{nm}/a_{nt}| \approx -2\ln\left(\frac{R}{R_0}\right) \quad (25)$$

From Eq. 25 one can understand that as $R \to 0$ the missile acceleration $a_{nm} \to \infty$. From the above analysis we can summarize a launch heading error correction by means of proportional navigation utilizes minimum effective navigation ratio of 2 (i.e $N' = 2$). Also during the terminal portion of the flight the missile maneuvers are at reasonable level where at the earlier in flight the collision course errors are corrected for $N' > 3$.

For various values of effective navigation ratio the family of homing missile trajectories at fixed launching errors is shown by Fig 4. Finally we can conclude as N' increases: heading error decreases, initial acceleration required by the missile is high and acceleration required at the terminal-phase to intercept the target is reduced.

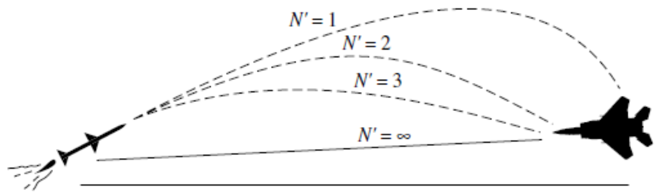

Fig. 4. N' Effecting various missile flight.

2 TWO-DIMENSIONAL MISSILE-TARGET ENGAGEMENT GEOMETRY

The previous section dealt with the theoretical side of proportional navigation and to understand better the working of proportional navigation. Consider point mass missile-target engagement geometry in two dimensional as shown in the Fig. 5.

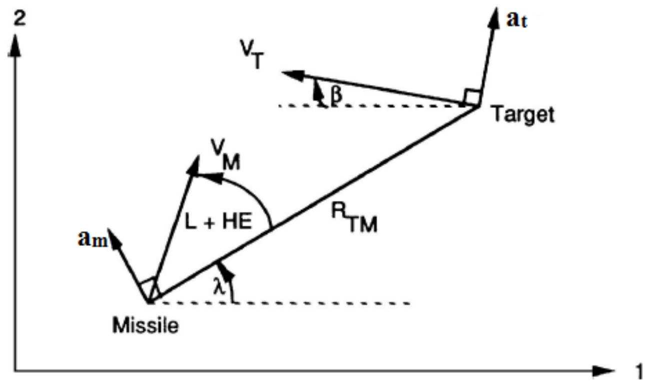

Fig. 5 Two-dimensional missile-target engagement geometry

For simulating this model both missile and target are assumed to travel at constant velocity. From Fig. 5 missile is heading at an angle of L + HE with velocity magnitude v_m with respect to line of sight. Here L is the missile lead angle and theoretically it is the correct angle for the missile to be on a collision triangle and angle HE is known as heading error. Line of sight is the imaginary line connecting the missile and target which makes an angle λ with respect to fixed reference and let R_{TM} be the instantaneous separation between missile and the target. In this simulation we desire to make R_{TM} as small as possible at the expected intercept point. The closest approach point between the missile and the target is known as the miss distance. In this engagement model let a_t be the acceleration magnitude of the target which can maneuver evasively. Also we can express angular velocity of the target as

$$\dot{\beta} = \frac{a_T}{v_T} \qquad (26)$$

2.1 SIMULATION OF TWO-DIMENSIONAL PROPORTIONAL NAVIGATION

The differential equations governing a complete missile-target engagement model represented by Fig. 5 in two dimensions can be found in [7]. Any homing missile incorporated with proportional navigation guidance is not fired at the target but it is fired in a direction to intercept the target. Under a variety of circumstances it is better to simulate and test the properties of guidance algorithm to witness and understand the insight of proportional navigation. Here the initial location of the missile and the target, flight time, speeds and effective navigation ratio are the simulation inputs and the user can vary target maneuver and heading error which are considered as two error sources.

The integration step size for most of the flight is fixed i.e. H=0.01 s and to accurately capture the magnitude of the miss distance the integration step size is made smaller near the end of the flight (H=0.0002 s when R_{TM}<1000ft). The program is terminated when the separation between the target and the missile is minimum, which means closing velocity changes its sign. In order to observe how effective in increasing the navigation ratio causes heading error to be removed more rapidly, a sample case was simulated where the only form of disturbance was a 50-deg Heading error (HEDEG = -50). Fig. 6 shows the sample trajectories for $N^{'} = 4 \; and \; 5$.

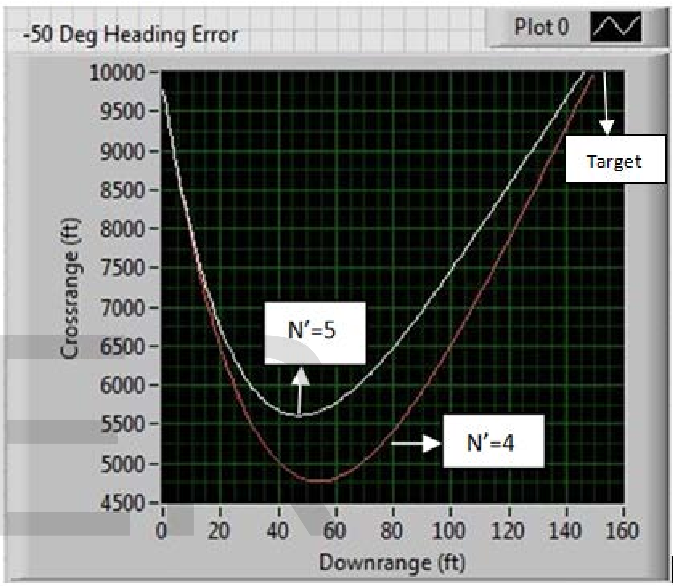

Fig. 6. Initial removing of Heading Error by increasing $N^{'}$

From Fig.6 we can observe because of heading error the missile is flying initially in a wrong direction and consequently the guidance law is forcing the missile to home on the target. A much tighter trajectory can be established with larger effective navigation ratio which removes the initial heading error more rapidly by the missile. At two different instants $N^{'} = 4 \; and \; 5$. proportional navigation guidance law proves that the missile hit the target near zero miss distance. Fig. 7 shows how by increasing the effective navigation ratio can cause more acceleration.

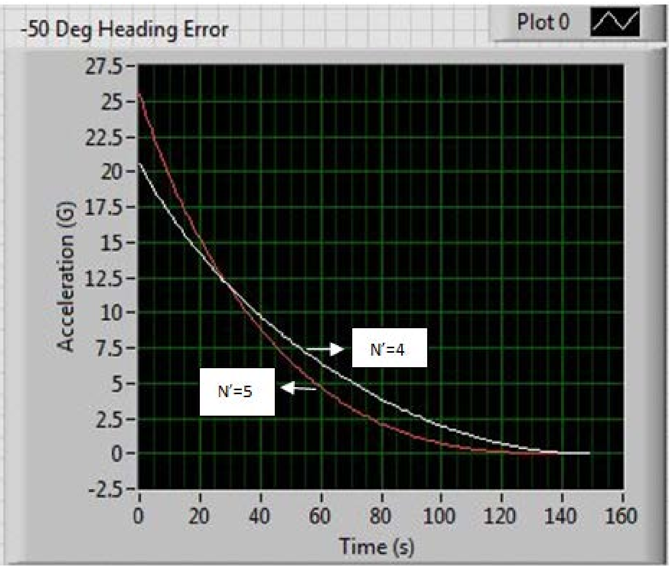

Fig. 7. Higher acceleration at the beginning of flight for different values of $N^{'}$

From the Fig. 7 at two different instants the time histories of missile acceleration are somewhat different. The quicker removal of the initial heading error at $N^{'} = 5$ results in larger acceleration of the missile at the beginning of the flight and lower accelerations at the end of the flight. In both the situations the missile acceleration

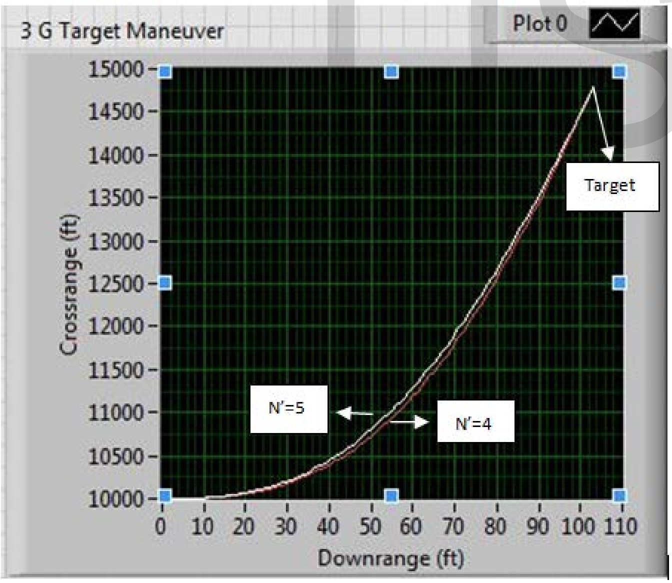

Fig. 8. Against Maneuvering Target

to hit the target and to remove the heading error is monotonically decreasing and becoming zero at the end of the flight. Indeed one of the principles of proportional navigation guidance is to remove the heading error as soon as possible throughout the entire flight. Thus by increasing the effective navigation ratio the missile can take out the initial heading error more rapidly.

In previous simulation analysis the only disturbances was heading error. Now another sample case was run where the only form of disturbance was a 3-g target maneuver. In this case initially the missile and the target are on a collision triangle. Here the target velocity vector is along the line of sight initially and also all of 3 g target acceleration is perpendicular to line of sight. Due to rapid turning of the target the magnitude of the target acceleration which is perpendicular to line of sight diminishes. The simulation results for these missile-target trajectories with effective navigation ratios of 4 and 5are shown in Fig. 8.

We can see from the Fig. 8, a value of $N^{'} = 5$ can lead the missile to intercept the target slightly more than the lower effective navigation ratio and that differentiate the two trajectories. However in both the cases, proportional navigation guidance law can enable the missile to intercept the maneuvering target.

Fig. 9. Higher the $N^{'}$ the less acceleration to hit the target

Fig. 9 depicts the fact that there are significant differences between the maneuvering target acceleration profiles. Even though both acceleration profiles are monotonically increasing, the larger effective navigation ratio causes less acceleration for the missile.

3. LINEARIZATION

In the previous analysis the effectiveness of proportional navigation has been understood from numerical simulation results of two-dimensional engagement simulation. Now in this section to gain more understanding we will linearize the two-dimensional engagement model by defining some relative quantities as shown in Fig. 10.

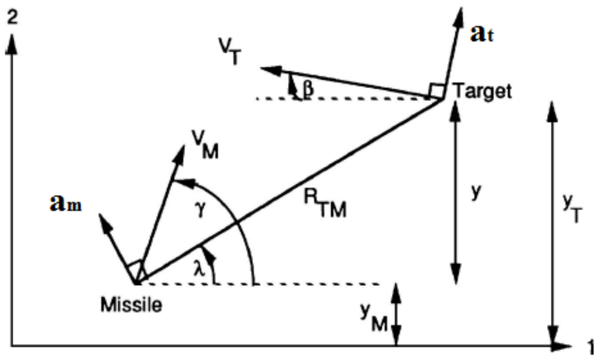

Fig. 10. Intercept model for Linearization.

From the Fig. 10 the difference between missile and the target acceleration which is called relative acceleration can be written as

$$\ddot{y} = a_t cos\beta - a_c cos\lambda \qquad (27)$$

If the cosine terms are approximately unity means if the flight path angles are small the preceding equation can be written as

$$\ddot{y} = a_t - a_c \qquad (28)$$

Using small-angle approximation the line of sight angle can be yielded as

$$\lambda = \frac{y}{R_{TM}} \qquad (29)$$

Closing velocity for a head on case and tail chase case can be approximated as

$$V_C = V_M + V_T \qquad (30)$$

$$V_C = V_M - V_T \qquad (31)$$

With the time varying relationship the range equation can be written as

$$R_{TM} = V_C(t_F - t) \qquad (32)$$

Where t and t_F are the current time and total time of flight of the engagement.

The relative separation between missile and the target y at the end of the flight is taken to be miss distance and is given by

$$Miss = y(t) \qquad (33)$$

3.1 LINEARIZED ENGAGEMENT SIMULATION

After developing the linearized equations for the missile-target geometry the simulation for this engagement will be done in this section to see if the resultant linearized equations give performance projections which are similar to those of nonlinear engagement equations. In simulating the linearized proportional navigation engagement the t_F is the input instead of output.

To verify the model as a reasonable approximation to the nonlinear engagement model, a sample run was made with the only disturbance was a -50-deg heading error. The effective navigation ratio for this case is 4 and the acceleration profiles for both linearized as well as non-linearized engagement models are shown by Fig. 11.

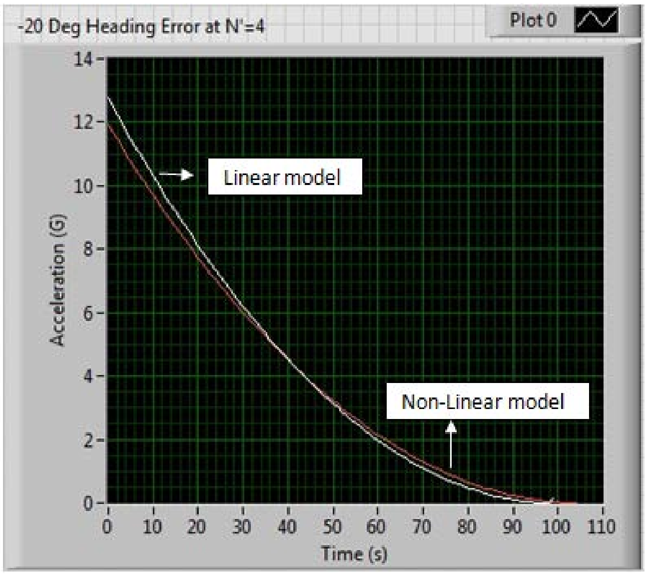

Fig. 11. Comparison of Acceleration Profiles with heading error disturbance

From the Fig. 11 it is clearly understood that resultant acceleration profiles for both models are virtually indistinguishable even for relatively large heading error disturbance.

Another linear engagement model simulation was run for a sample case with a 3-g target maneuver and was shown in Fig. 12.

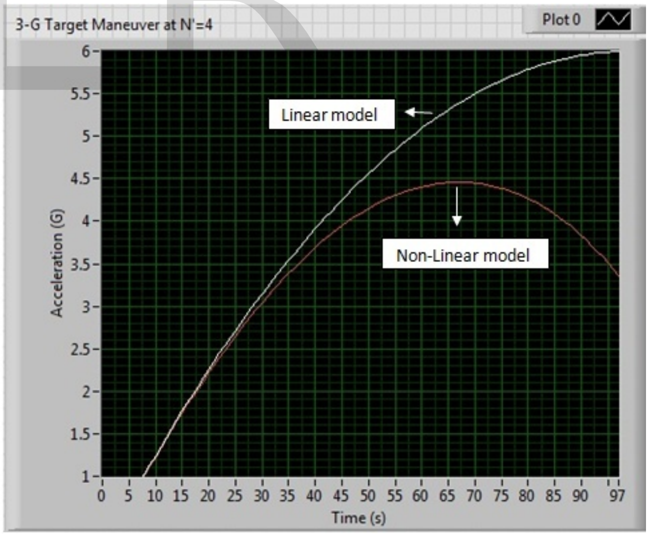

Fig. 12. Comparison of Acceleration Profiles for a 3-g target maneuver.

The discrepancy in the Fig. 12 is that the linear model assumes the magnitude of the target acceleration perpendicular to the line of sight is same and equal to the magnitude of the maneuver. Also the linear model overestimates the missile acceleration due to target maneuver. In reality the component of the acceleration perpendicular to the line of sight decreases because the target is turning. From this analysis its understood that the nonlinear acceleration requirements due to maneuvering target are less than those predicted by the linearized engagement model.

4. CONCLUSION

From the previous simulation results the effectiveness of proportional navigation guidance is very efficient to hit a target even if the missile is launched in wrong direction by 50 deg. Also the guidance law worked for a maneuvering target. In both analyses it requires a certain acceleration levels for a missile to hit a target. They rely on the type of error source and especially on effective navigation ratio. If the missile is not having sufficient acceleration generated by the guidance law, a miss will result. Finally the method of linearization is found to yield performance projections of sufficient accuracy thereby the design relationships can be verified and tested.

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