PAPR Reduction Performance for LTE OFDM Systems with Different Techniques

Mamdouh Gouda, Khaled Ali Shehata, Mohamed Hussien

Abstract — Orthogonal Frequency Division Multiplexing (OFDM) is one of the Strong candidate for Transmission of high data rate due to Multicarrier Modulation. One of the challenging issue of OFDM is its high Peak to Average Power Ratio (PAPR) which cause large number of sub-carriers, that make restrictions for practical applications. This Paper discusses different PAPR Reduction techniques in OFDM. The Selective Mapping, The DFT-spreading technique it is known as the Single Carrier-Frequency Division Multiple Access (SC-FDMA), which is adopted for uplink transmission in the 3rd Generation Partnership Project (3GPP) LTE standard and Partial Transmit sequence Technique is used in this paper. In this paper, we mainly investigate the PAPR reduction performance using PTS, this method is sub-entities of phase rotation scheme. A new algorithm using PTS technique, We also show that a SCFDMA system with Interleaved-FDMA or Localized-FDMA performs better than Orthogonal- FDMA in the uplink direction where transmitter power efficiency is of great importance. This shows better PAPR reduction compared to the existing algorithms is proposed. Results are verified using MATLAB software.

Index Terms — Orthogonal frequency division multiplexing (OFDM), partial transmits sequences (PTS), Localized-frequency-division-multiple-access (LFDMA), Interleaved-frequency-division multiple-access (IFDMA), peak-to-average power ratio (PAPR), single carrier frequency division multiple access (SC-FDMA), Longterm-evolution (LTE), Selective Mapping (SLM).

1 Introduction

Long Term Evolution (LTE) is standardized by the 3rd Generation Partnership Project (3GPP) as an evolution of the 3G systems to meet the requirements of increasing the data rates, high mobility and low latency over a bandwidth of up to 20 MH. Researchers have been trying for the next evolutionary fourth generation (4G) communication systems to provide a comprehensive and secure IP solution where voice, data, and multimedia can be offered to users at "anytime, anywhere" with higher data rates than previous generations [1]. Multiple input multiple outputs (MIMO) and (OFDM) modulation have therefore been adopted due to their superior performance. These developing modulation used in LTE which promise to become the key for high-speed wireless communication technologies and combining them can provide wireless industry evolution from 3G to 4G systems. In OFDM systems which are uses MIMO state that, the output is the superposition of multiple sub-carriers in this case, instantaneous power outputs increases and may demand higher powers than the mean power of the system since the phases of these carriers are the same. OFDM is multicarrier multiplexing access Technique for Transmitting Large data over Radio waves. One of the major drawbacks of OFDM signals is its large PAPR, power [16]. And to reduce the PAPR, many techniques have been proposed, such as clipping, coding, PTS, selected mapping (SLM), interleaving[17][18], nonlinear companding transforms[14][19], hadamard transforms[20]. These schemes can mainly be categorized into signal scrambling techniques, such as PTS, and signal distortion techniques such as clipping, companding techniques, in this Paper all techniques which can be used to reduce PAPR in OFDM system are listed.

OFDMA is a broadband multicarrier modulation scheme where SC-FDMA is a single carrier modulation scheme. Research on multicarrier transmission started to be an interesting research area [2]-[3]. OFDM modulation scheme leads to better performance than a single carrier scheme over wireless channels. OFDM uses a large number of orthogonal, narrowband sub-carrier that is transmitted simultaneously in parallel however; high PAPR becomes an issue that limits the uplink performance more than the downlink due to the low power processing terminals. SC-FDMA adds additional advantage of low PAPR compared to OFDM making it appropriate for uplink transmission.

In our previous work, we analyzed a low complexity PTS algorithm which is introduced to determine sub-optimal weighting factor for each sub block instead of conducting an ergodic search so as to reduce the calculation complexity significantly. This sub-optimal algorithm gives a better approach to the real conditions in engineering practice by providing a compromise between the PAPR reduction performance and computational complexity [24]. The outline for the paper is as follows: After system Structure, which is presented in Section II, definition of PAPR and its reduction techniques focusing on signal scrambling techniques - especially (PTS) and (SLM)- and DFT spreading technique are investigated in section III, comprehensive analysis are conducted in terms of all possible influencing factors on PAPR reduction performance and some research findings are
reported based on the simulation results in section IV. Finally, conclusions of this paper are presented in section V.

2. Basic Structure of OFDMA and SC-FDMA Systems

Fig. 1 shows a block diagram of OFDM transceiver. Each symbol can transmit up to 4000 bits. Serial to parallel (S/P) converts the input data allowing transmission in each OFDM symbol. The modulation scheme and number of subcarriers determine the data allocated to each symbol. The Inverse Fourier Transform (IFFT) transforms the signal to the time domain for transmission and reduces the amount of calculations dramatically. The cyclic prefix prevents inter-symbol interference (ISI) in which the discrete-time signal \( \{X[n]\} \) after applying IFFT at the transmitter can be expressed as:

\[
X[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} kn}
\]

For a sequence of QPSK or QAM-modulated data symbols \( \{X[k]\} \). In other words, \( X[n] \) is given by adding the \( N \) different time-domain signals \( e^{j \frac{2\pi}{N} kn} \), each of which corresponds to the different orthogonal subcarrier. The one is modulated with data symbol \( X[k] \). Fig. 2(a) shows the individual time domain Quadrature Phase Shift Keying (QPSK) modulated subcarrier signals for \( N=8 \). The PAPR worsens as the number of subcarriers increases.

The PAPR characteristics of the OFDM signal shown in Fig 2(b) which includes the distributions of \( x[n] \), as well as the imaginary and real parts of \( x[n] \) follow a Gaussian distribution while \( x[t] \) follow a Rayleigh distribution for \( N=16 \).

3. System Model

One of the major disadvantages of OFDM systems is that the OFDM signal has high (PAPR), and to deal with this problem many typical techniques have been proposed. Each technique is different from the other in its complexity and performance, the PAPR reduction techniques are listed in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1: CLASSIFICATION OF PAPR TECHNIQUES</th>
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<tbody>
<tr>
<td>PAPR techniques</td>
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<tr>
<td>Block coding</td>
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<tr>
<td>Sub block coding</td>
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<tr>
<td>Selective mapping</td>
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<tr>
<td>Partial transmit sequence</td>
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<tr>
<td>Interleaving</td>
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<tr>
<td>Linear block coding</td>
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<tr>
<td>Tone reservation</td>
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<td>Tone injection</td>
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</table>

In this paper DFT spreading technique and symbol-scrambling techniques -especially partial transmit sequences and selective mapping- are focused on.

3.1 Partial Transmit Sequences Technique

PTS is one of the most important methods that is used to reduce PAPR in the OFDM system. And it can be presented in two main steps. First, by dividing the original OFDM signal into a number of sub-blocks. Secondly, adding the phase rotated sub-blocks to develop a number of candidate signals to pick the one with smallest PAPR for transmission. There is another way that can also be used to express PTS method by multiplying the original OFDM signal with a number of phase sequences [13].
PTS technique partitions and input data block of N symbols into V disjoint sub-blocks as follows:

$$X = [X^0 X^1 X^2, \ldots, X^{V-1}]^T$$  (4)

where $X^i$ the subblocks that are consecutively located and are also of equal size, scrambling is applied to each subblock [11] which rotating its phase independently in the PTS technique as in Fig.4. Then each partitioned subblock is multiplied by a corresponding complex phase factor $b^v = e^{j\theta_v}$ where $v = 1,2,\ldots,V$, subsequently taking its IFFT to yield:

$$X = \text{IFFT}\{\sum_{v=1}^{V} b^v X^v\} = \sum_{v=1}^{V} b^v X^v$$  (5)

Where $X^v$ is referred to as PTS. The phase vector is chosen so that the PAPR can be minimized [5], which is shown as:

$$[b^{-1}, \ldots, b^V] = \arg \min_{\{b^{-1}, \ldots, b^V\}} \max_{n=0}^{N-1} |\sum_{v=1}^{V} b^v X^v(n)|$$  (6)

Figure 3 shows that the number of computations in this suboptimal combination algorithm is $V$, which is much fewer than that required by the original PTS technique which make $(V \ll V^N)$. Then the corresponding time-domain signal with the lowest PAPR vector can be expressed as:

$$\tilde{X} = \sum_{v=1}^{V} b^v X^v$$  (7)

Fig. 3 Block diagram of PTS technique for PAPR reduction

3.1 Selective Mapping Technique

Selected mapping (SLM) is a promising PAPR reduction technique of OFDM system. The main idea of SLM technique is to generate a number of OFDM symbols as candidates and then select the one with the lowest PAPR for actual transmission. From a number of different data blocks (independent phase sequences) that have the same information at the transmitter, block diagram of SLM scheme is demonstrated in Fig. 4 [22].

It generates the set of favourable blocks at the transmitter end which represent the original information and then chooses the most favourable block for transmission as proposed in [21]. Here the input block given by $X=[X(0),X(1),\ldots,X(N-1)]$ is multiplied with U different phase sequences $P^u = [P^u_0, P^u_1, \ldots, P^u_{N-1}]^T$ to produce a modified data block given by:

$$X^u = [X^u[1], X^u[2], \ldots, X^u[N-1]]^T$$  (8)

The IFFT of U independent sequences are taken to produce the time domain sequences shown in equation (8) among which the one with the lowest PAPR is selected for transmission. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the proper design of the phase sequences [11]. In order to recover the original symbol vector in the receiver, the transmitter must send the index information about the phase sequences which is known as Side information. The implementation of SLM technique requires U IFFT operations where

$$\tilde{u} = \arg \min_{u=1,2,\ldots, U} \max_{n=0,1,\ldots,N-1} |X^u[n]|$$  (9)

3.3 Discrete Fourier Transform spreading Technique

The DFT-spreading technique is to spread the input signal with DFT, which can be subsequently taken into IFFT. This can reduce the PAPR of OFDM signal to the level of single-carrier transmission. This technique is particularly useful for mobile terminals in uplink transmission. It is known as the Single Carrier FDMA (SC-FDMA), which is adopted for uplink transmission in the 3GPP LTE standard [25]-[26].

Fig. 4. Block diagram of SLM technique for PAPR reduction
Figure 5 shows a block diagram of the uplink transmitter with the DFT-spreading technique. Here the input data $X[m]$ is DFT-spread to generate $X[i]$ and then, allocated as:

$$\hat{x}[k] = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{S}{M} k} & k = S, m_1, m_2, \ldots, M - 1 \\ \text{zero} & \text{otherwise} \end{cases}$$

(10)

The IFFT output sequence $\hat{x}[n]$ with $n = (M \cdot s) + m$ for $s = 0, 1, 2, \ldots, S-1$ and $m = 0, 1, 2, \ldots, M - 1$ can be expressed as:

$$\hat{x}[n] = \hat{x}[S m + s] = \frac{1}{N} \sum_{k=0}^{N-1} \hat{x}[k] e^{j2\pi \frac{n}{M} k}$$

$$= \frac{1}{S} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{sm}{SM}}$$

(11)

For $s = 0$, the previous equation can be expressed as:

$$\hat{x}[n] = \hat{x}[S m + s] = \frac{1}{S} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{m_1}{M} k}$$

$$= \frac{1}{S} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{m_1}{M} k}$$

(12)

For $s \neq 0$, equation (12) can be expressed as:

$$\hat{x}[n] = \hat{x}[S m + s] = \frac{1}{S} (A) \hat{x}[m]$$

(13)

The following equations represent the CCDF of OFDM signals. Let $Z_{\text{max}}$ denote the crest factor ($Z_{\text{max}} = \max_{n=0,1,\ldots,N-1} Z[n]$). Now, the cumulative distribution function (CDF) of $Z_{\text{max}}$ is given as:

$$F_{Z_{\text{max}}}(z) = P(\text{Z}_{\text{max}} < z)$$

$$= P(\text{Z}_{n} < z) \cdot P(\text{Z}_{n-1} < z)$$

$$= (1 - e^{\frac{z}{\alpha}})^N$$

(15)

where $\text{Z}_{n} < z = \int_{0}^{z} f_{X}(x) \, dx$, $n = 0, 1, 2, \ldots N-1$. In order to find the probability that the crest factor (CF) exceeds $z$, we consider the following CCDF as:

$$F_{Z_{\text{max}}}(z) = P(\text{Z}_{\text{max}} > z)$$

$$= 1 - F_{Z_{\text{max}}}(z)$$

$$= 1 - (1 - e^{\frac{z}{\alpha}})^N$$

(16)

Since Equations (15) and (16) are derived under the assumption that $N$ samples are independent and $N$ is sufficiently large, they do not hold for the band limited or oversampled signals. It is due to the fact that a sampled signal does not necessarily contain the maximum point of the original continuous-time signal, the following simplified CCDF will be used as:

$$F_{Z}(z) = (1 - e^{\frac{z}{\alpha}})^N$$

(17)

where $\alpha$ has to be determined by fitting the theoretical CDF into the actual one [10]. Using simulation results, it has been shown that $\alpha = 2.8$ is appropriate for sufficiently large $N$ (i.e.); In general for PTS technique, the selection of the phase factors $b_v^m$ is limited to a set of elements to reduce the search complexity [4].

**4. Simulation Results**

SLM and PTS algorithms are two typical non-distortion techniques for reducing PAPR in OFDM system. In order to have error-free demodulation in the receiving end, side
Start

Partition the input data block into V sub-blocks as in Equation (4).

Set all the phase factors $b^v = 1$ for $v = 1:V$, find PAPR of Equation (5), and set it as $PAPR_{\text{min}}$ with $b^v = -1$.

Generate all possible combinations of weighting factor set in PTS method.

Applying PTS algorithm with the number of generated OFDM symbols equals to 10000.

Calculate and plot complementary cumulative distribution function (CCDF) of different PAPR.

If $PAPR > PAPR_{\text{min}}$, switch $b^v$ back to 1. Otherwise, update $PAPR_{\text{min}} = PAPR$.

If $v < V$, increment $v$ by one and go back, exit this process with the set of optimal phase factors $b^v$.

Yes

end

Fig 8 Flow chart of PAPR reduction performances in PTS method

The PTS technique requires $N\cdot \text{IFFT}$ operations for each data block and $\lceil \log_2 W^V \rceil$ bits of side information. In fact, there are three different kinds of the sub-block partitioning schemes: adjacent, interleaved, and pseudo-random.[7]. As discussed above, the PTS technique suffers from the complexity of searching for the optimum set of phase vector, especially when the number of subblock increases. In the literature [8] [9], various schemes have been proposed to reduce this complexity. Sub-block partition scheme in PTS algorithms is typically non-distortion techniques for reduction of PAPR in OFDM system [12]-[10]. The parameter used for calculation of PAPR are illustrated in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values used</th>
</tr>
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<tbody>
<tr>
<td>Number of sub-carriers (N)</td>
<td>16, 128</td>
</tr>
<tr>
<td>Oversampling factor (OF)</td>
<td>8</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>QAM</td>
</tr>
<tr>
<td>Number of sub-blocks used in PTS methods (V)</td>
<td>2, 4, 8, 16, 32</td>
</tr>
<tr>
<td>Total number of combinations or IFFT for weighting factor 1 and 2</td>
<td>256</td>
</tr>
<tr>
<td>Number of generated OFDM signal</td>
<td>10000</td>
</tr>
</tbody>
</table>

Figure 9 shows the CCDF of PAPR for a 16-QAM/OFDMA system using PTS technique as the number of subblock varies. It is seen that the PAPR performance improves as the number of sub blocks increases with $V = 1, 2, 4, 8, 16$.

As shown in Fig. 10 the blue curve represent the CCDF of OFDM signals using adjacent partition scheme, and remaining plotted curves in the graph are based on the pseudo-random partition scheme. As we can see from the graph, for each $V$ system performance of pseudo-random partition is superior by 0.5dB (at minimum) to the one based on adjacent partition.
SLM method [23] applies scrambling rotation to all sub-carriers, each carrier, adopting QPSK constellation mapping, and weighting factor being \( \theta \in \pm 1, \pm j \); The flow chart used for PAPR reduction technique is given in Fig. 12.

![Flow Chart](image1.png)

**Fig 12 Flow Chart of PAPR Reduction Performances in SLM Method**

Figure 13 shows the CCDF as a function of PAPR distribution when SLM method is used with 64 numbers of subcarrier which generate 10000 OFDM symbol.

![CCDF of SLM Proposed Scheme](image2.png)

**Fig 13** PAPR reduction performances of SLM proposed algorithm.

In the following simulation results, we compared different allocation schemes of SC-FDMA systems and their PAPR. These types of allocation schemes are subject to inter symbol

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As shown in Fig. 11. The original curve has PAPR equals to 10.5 dB. After applying to the proposed algorithms, the value was significantly reduced to 5.6 dB. This proves that the algorithm gives better results which is superior performance in PAPR reduction.

![Fig. 11](image3.png)

**Fig. 11** PAPR reduction performances of PTS proposed algorithm.

After discussing the simulation results for PTS technique, there are varying parameter which impact the PAPR reduction performance which .The number of sub-blocks V, the number of possible phase value W which affect the complexity strongly.

In particular SLM technique whole set of signal represent the same signal but form it most favorable signal is chosen related to PAPR transmitted. The side information must be transmitted with the chosen signal. This technique is probabilistic based will not remove the peaks but prevent it from frequently generation. This scheme is very reliable but main drawback that is side information must be transmitted along with chosen signal.
interference when the signal suffers from sever multipath propagation. In SC-FDMA this type of interference can be substantial and usually an adaptive frequency domain equalizer is placed at the base station. This type of arrangement makes sense in the uplink of cellular systems due to the additional benefit that SC-FDMA adds in terms of PAPR. In this type of arrangement, i.e., single carrier system the burden of linear amplification in portable terminals is shifted to the base station at the cost of complex signal processing, that is frequency domain equalization.

Figure 14 show the performance of PAPR while the number of subcarriers is 256 and the number of subcarriers assigned to each unit or mobile device is 64. This simulation helps in evaluating the performance of PAPR with different mapping schemes and modulation techniques. In LFDMA each user transmission is localized in the frequency domain where in the DFDMA each user transmission is spread over the entire frequency band making it less sensitive to frequency errors and diversifies frequency.

Our results show the effect of using DFT spreading technique to reduce PAPR for OFDMA, LFDMA and OFDMA, A comparison is shown in Fig. 12. Utilizing different modulation schemes. The reduction in PAPR is significant when DFT is used. For example, Fig. 12 where OFDMA, LFDMA and IFDMA have the values of 3.9 dB, 8.8 dB and 11.3 dB, respectively. The reduction of PAPR in IFDMA utilizing the DFT-spreading technique compared to OFDMA with the use of DFT is 7.4 dB. Hence SC-FDMA systems with IFDMA and LFDMA perform better than OFDMA in the uplink transmission. Although IFDMA performs better than OFDMA and LFDMA, LFDMA is preferred due to the fact that assigning subcarriers over the whole band of IFDMA is complicated while LFDMA doesn’t require the insertion of pilots of guard bands.

The PAPR reduction techniques discussed in this paper namely PTS, SLM and DFT spreading technique using IFDMA and LFDMA have been simulated to compare their performance characteristics.

Figure 15 shows different results of PAPR reduction techniques, these techniques are the DFT technique which is categorized into IFDMA and LFDMA, and Signal scrambling techniques especially PTS and SLM. The figure shows that in case of using IFDMA, the best result is achieved for PAPR reduction giving only 3.9 dB. However, the PTS and SLM techniques which gave 5.7 dB and 6.2 dB respectively results in better PAPR reduction than the LFDMA in DFT technique which gave 8.7 dB.

Also Fig.14 show that when the single carrier is mapped either by LFDMA or DFDMA, it outperforms OFDMA due to the fact that in an uplink transmission, mobile terminals work differently then a base station in terms of power amplification. In the uplink transmission PAPR is more of a significant problem then on the downlink due to the type and capability of the amplifiers used in base station and mobile devices. For instance, when a mobile circuit’s amplifier operates in the nonlinear region due to PAPR, the mobile devise would consume more power and become less power efficient whereas base stations do not suffer from this consequence. Therefore, OFDM works better in the downlink transmission in terms of PAPR.
5. CONCLUSION

In this paper, the concept of PAPR in OFDM signals is discussed. The PAPR reduction techniques like Selected mapping(SLM), Partial Transmit sequence(PTS) and DFT spreading technique using Localized FDMA have been investigated. The Simulation results show that as the PAPR reduces with these techniques, they can be used in OFDM transmitter effectively.

In PTS technique as the number of subblocks increases, the PAPR decreases. In Localized FDMA(LFDMA) technique, even though the PAPR increases for 64 QAM, it can be applied for systems demanding high data rates.LFDMA technique is applied in Uplink transmission for 3GPP LTE systems.

This paper provides an overview of Multiple-Input-Multiple-Output (MIMO) technology and Orthogonal-Frequency-Division-Multiplexing (OFDM). The focus of this paper is to investigate one of the bottleneck problems that exist in OFDM wireless communication system. The purpose of this paper was to reduce the High (PAPR) of OFDM signals.

Using the proposed PTS and SLM algorithms, this was successfully achieved. The main contributions in this paper are listed below: The comprehensive research and comparison are put forward for a variety of currently promising PAPR reduction methods quoted in the literature in this research area. Among these different proposals, this paper mainly focused on the signal scrambling technology and DFT-spreading technique, which verified the theoretical analysis by observing the MATLAB simulation results. At the same time, some meaningful guidance and conclusions were obtained through the comparative analysis of these simulation results as well. A series of detailed simulations were conducted and results were obtained of this scheme for PAPR reduction in a complex system. Using the above methods, observing that the CCDF of OFDM signals were improved compared to other literature searches.

REFERENCES