PAPR Reduction Based on Precoding Techniques with Companding in OFDM Systems

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Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is a technique which is used in modern wireless communication systems because of high data rate and high frequency spectral efficiency. The high Peak to Average Power Ratio (PAPR) of the transmitted signals is an important problem in any OFDM system. The high peaks decrease the high power amplifier (HPA) efficiency in OFDM transmitter and hence overall performance of the system. In this paper, we review and analysis some pre-coding techniques, called Zadoff-Chu Transform (ZCT) and Walash-Hadamard transform (WHT) with the μ-law companding to reduce the PAPR of the OFDM signals. Simulation results show that this technique have better PAPR gain than conventional OFDM.

Keywords-OFDM, PAPR reduction, Zadoff-Chu Transform (ZCT), and Walash-Hadamard transform (WHT), Companding.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technology for wireless communications. It offers the considerable high data rate transmission, high spectral efficiency; immunity to the frequency selective fading channels multipath delay spread tolerance, and high power efficiency, [1]. OFDM is used in systems such as: wireless local area networks and WiMAX, digital audio broadcasting (DAB), digital video broadcasting (DVB), [2-6]. One of the major drawbacks of OFDM is the high Peak to Average Power Ratio (PAPR) value of the transmitted signals. This problem comes from the nature of the modulation itself, when multiple sub-carriers are added together to form the transmitted signal. On the other hand, when the numbers of subcarriers are increased with the same phases, then the OFDM signals will have a large envelope fluctuation which leads to high PAPR values, due to this High PAPR system require significant back-off of the average operating power of an RF power amplifier if the signal is to be linearly amplified.

In recent years, a lot of PAPR reduction techniques have been proposed such as: block coding [15], interleaving [16], selected mapping (SLM) [17], partial transmit sequences (PTS) [7,18], phase optimization [19], tone reservation (TR), tone injection (TI) [20], pre-coding techniques [21]. These techniques reduces PAPR parameter by decreasing the instantaneous peak power values of OFDM signal which caused by Inverse Fast Fourier Transform (IFFT).

In this paper, we use pre-coding techniques for PAPR reduction in OFDM systems, which do not need any side information to be transmitted from the transmitter to the receiver. We investigate ZCT and WHT pre-coding method with μ-law companding. In this we use the ZCT and WHT transform coding on the modulated data stream, than we apply the ‘N’ point IFFT into the pre-coded data. Finally, we apply the companding technique then OFDM signal will be obtained and then PAPR parameter will be calculated.

2. PAPR IN OFDM SYSTEM

In this paper, we consider an OFDM system with M-QAM modulation and take a total of N baseband modulated symbols per OFDM block. Figure (1) illustrates that in an OFDM system first of all, the serial input baseband modulated symbols are converted to parallel stream. The output data vector is assumed as: X=[X1, X2, …, XN]T and Xi is i-th data symbol. Than OFDM signal is calculated using the ‘N’ point IFFT. Oversampling is recommended for the better approximation of the PAPR of the continuous-time OFDM signals, the. It can be implemented by computing an LN-point IFFT of the data block with (L-1) N zero-paddings and the oversampled IFFT output can be expressed as:
\[
x[n] = \frac{1}{\sqrt{L}} \sum_{k=0}^{LN-1} X_k e^{j\frac{2\pi}{LN} kn}, \quad n = 0, 1, \ldots, LN-1
\]

We are considering the oversample factor as ‘L’. The PAPR parameter in any OFDM symbols can be calculated by using the following equation:

\[
PAPR = \frac{\max_{0 \leq n \leq LN-1} |x[n]|}{E[|x[n]|^2]}
\]

Where \(E[.\] denotes expected value.

Figure 1. A simple block diagram of an OFDM system.

When \(N\) is large, the distribution of the output time vector converges to Gaussian due to Central Limit Theorem. Hence, Complementary Cumulative Distribution Function (CCDF) can be written as:

\[
P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N
\]

In order to obtain sufficient transmission power most of the radio systems employ High Power Amplifiers (HPA) in the transmitters for the achievement of the maximum output power efficiency and for this reason High Power Amplifiers (HPA) are usually operated at or near the saturation region where its input output characteristic is non-linear. But this non-linearity makes the HPA very sensitive to the variations of the signal amplitude. Due to the non-linearity of the HPA reduces the spectral efficiency of the OFDM transmitter. Which leads to the variations of the instantaneous power of the OFDM signals and their will be high value of PAPR. Because, the HPA will introduce inter-modulation between the different subcarriers and as a result additional interference and the reduction of the spectral efficiency will occur. If High Power Amplifier (HPA) are operate in its linear region for zero distortion of the OFDM signal, then the HPA with a large dynamic range are require. But these amplifiers are very expensive. Thus, if we reduce the PAPR of the OFDM signal by manipulating the various PAPR reduction techniques on the digital data than we are reducing the cost of OFDM system and reducing the complexity of A/D and D/A converters.

PROPOSED TECHNIQUE:
For the PAPR reduction the proposed technique is shown in Figure (2) and the principles of Precoding techniques and Companding will be studied in the following section.

\[
\text{Figure 2. A simple block diagram of an OFDM system with pre-coding and companding.}
\]

3. PRECODING TECHNIQUES

The following pre-coding techniques are used to obtain a signal with lower PAPR than in the case of conventional OFDM without pre-coding techniques and to reduce the interference produced by multiple users in the OFDM system. The PAPR reduction techniques must compensate the nonlinearity of the HPA. Some pre-coding techniques are presented in the following.

3.1 Zadoff-Chu matrix transform (ZCT)

Zadoff-Chu are complex-valued sequences, when these sequences are applied to the radio signal and gives an electromagnetic signal with constant amplitude. These sequences are class of polyphase sequences having optimum correlation properties and having an ideal periodic autocorrelation and constant magnitude. When cyclically the shifted version of the sequence is imposed on a signal result in zero cross-correlation. Zadoff-Chu matrix transform pre-coding has been introduced in [11,23] which can be reduces the PAPR parameter of any OFDM systems. This technique uses Zadoff-Chu sequences of length ‘L’ which can be define as equation (1).
\[ z(k) = \begin{cases} \frac{2\pi}{L} \left( \frac{k^2 + qk}{2} \right) & \text{for } L \text{ even} \\ \frac{2\pi}{L} \left( \frac{(k+1)^2 + qk}{2} \right) & \text{for } L \text{ odd} \end{cases} \]  

(4)

Where \( k=0,1,2,\ldots,L-1 \), ‘q’ and ‘r’ are any integer relatively prime to ‘L’. Using these sequences the Zadoff-Chu matrix can be defined as ‘N×N’ matrix by equation (5).

To become any row of the Z matrix orthogonal to each other, it must be set \( q=0 \) and \( r=1 \).

\[ R = \left[ r_{m,l} \right] = \left[ \begin{array}{cccc} r_{0,0} & r_{0,1} & \cdots & r_{0,N-1} \\ r_{1,0} & r_{1,1} & \cdots & r_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N-1,0} & r_{N-1,1} & \cdots & r_{N-1,N-1} \end{array} \right] \]  

(5)

where \( r_{m,l} = \frac{k^2 + rql}{N} \), \( k = mN + l \), \( m,l = 0,1,2,\ldots,N-1 \)

Pre-coding of the baseband modulated data can be done using equation (6) as \( Y=RX \).

\[ Y_m = \sum_{m=0}^{N-1} r_{m,l} X_l \quad m=0,1,2,3,\ldots,N-1 \]  

(6)

In the equation (6), \( r_{m,l} \) means the \( m \)th row and \( l \)th column of pre-coder matrix. Therefore, the complex baseband OFDM signal with \( N \) subcarriers with ZCT pre-coding is given by:

\[ x_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \left( e^{j2\pi mn/N} \right) \left( e^{j2\pi mN/N} \right) e^{j\left( \frac{2\pi mq}{N} \right)} \]  

(7)

3.2 Walsh-Hadamard Transform

The Walsh-Hadamard Transform (WHT) is a non sinusoidal and it is an orthogonal technique which decomposes a signal into set of basic functions. These functions are called Walsh functions. The hadamard transform scheme reduce the occurrence of the high peaks comparing the conventional OFDM system. We used the hadamard transform because it reduce the autocorrelation of the input sequence to reduce the peak to average power (PAPR) of OFDM signal. It also not requires to send side information to the receiver. The FWHT for a signal \( x \) of length \( N \) are defined as:

\[ y_n = \frac{1}{N} \sum_{i=0}^{N-1} x_i \text{WAL}(n,i) \]

where \( i = 0,1,\ldots,N-1 \) and WAL \((n,i)\) are Walsh functions.

4. COMPAANDING TECHNIQUE

We used the compander in order to reduce PAPR of OFDM signal because compander amplify the small signal and increase the average power of the OFDM signal and reduce the high peak value. In general a compander consists of a compressor and an expander. The compressor is a simple logarithm computation block. The inverse System of a compressor is called expander. In this paper, the compession is applied at the transmitter end, after the IFFT process and the expansion is applied at the receiver end prior to the FFT process. These two types are the \( \mu \)-law and the A-law compander [8-10].

4.1 \( \mu \)-law Compander

The \( \mu \)-law compander employs the logarithmic function at the transmitting side. In general a \( \mu \)-law compression characteristic is expressed as:

\[ Y = \frac{\log(1 + \mu |x|)}{\log(1 + \mu)} \text{sgn}(x) \]

where \( \mu \) is the \( \mu \)-law parameter which controls the amount of compression, \( x \) is the input signal and \( V \) is the maximum value of \( x \). For normalized input signal: \( |x| \leq 1 \), then the characteristics becomes:

\[ Y = \frac{\log(1 + \mu |x|)}{\log(1 + \mu)} \text{sgn}(x) \]

5. SIMULATION RESULTS

In this section, we present the results of computer simulations used to evaluate PAPR reduction capability proposed scheme. The channel was modeled as additive white Gaussian noise (AWGN). In the simulation, an OFDM system with a sub-carrier of \( N = 64 \) and M-QAM=4, 16 modulation is used. We can evaluate the performance of the PAPR reduction scheme using the complementary cumulative distribution (CCDF) of the PAPR of the OFDM signal at different values of the \( \mu \) parameter of \( \mu \)-law compander.

5.1 CCDF Performance

The cumulative distribution function (CDF) is one of the most widely used parameters, which is used to measure the efficiency of and PAPR technique. By using this parameter we can evaluate the performance of PAPR using the cumulative distribution of PAPR of OFDM signal. We used the complementary CDF
(CCDF) function in stead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold value.

\[ P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \]

Figure 3: CCDF of PAPR parameter ZCT-OFDM and WHT-OFDM with MQAM=4 and N=64

Figure 4: CCDF of PAPR parameter ZCT-OFDM and WHT-OFDM with MQAM=16 and N=64

Figure 5: CCDF of PAPR parameter ZCT-Companding OFDM and WHT-Companding OFDM with MQAM=4 and N=64, with \( \mu = 10 \)

Figure 6: CCDF of PAPR parameter ZCT-Companding OFDM and WHT-Companding OFDM with MQAM=4 and N=64, with \( \mu = 20 \)

Figure 7: CCDF of PAPR parameter ZCT-Companding OFDM and WHT-Companding OFDM with MQAM=4 and N=64, with \( \mu = 50 \)
The pre-coding was realized in the transmitter before the computation of IFFT. The Figures 3, 4 represent the PAPR distribution obtained for pre-coding techniques, Zadoff-Chu sequences (ZCT), WHT with 4, 16 QAM and the Figures 5, 6, 7, 8 and 9 represent the PAPR distribution obtained for pre-coding techniques with μ-law companding. We can observe in Figure 3 that at CCDF=10^{-2}, the PAPR is reduced by approximately 6.5dB for Zadoff-Chu Transform, by 8.5dB for WHT pre-coding based OFDM systems. When we use these pre-coding techniques with companding, we can observe in Figure 9 that at CCDF=10^{-2}, the PAPR is reduced by approximately 4.1dB for Zadoff-Chu Transform (ZCT), by 4.5dB for WHT pre-coding.

6. CONCLUSION

In this paper, we analyzed the performance in terms of PAPR in OFDMA by using some pre-coding techniques along with μ-law companding. These pre-coding techniques produced the lower PAPR as compared to conventional OFDM system. Furthermore ZCT is better than WHT because it produced the lowest PAPR than WHT. The pre-coding techniques do not introduced any signal degradation representing the better PAPR reduction methods than other techniques because they do not require any side information to be sent to the receiver and do not require any power increment, complex optimization. Mu-law companding further reduces PAPR of OFDM signal and as we increasing the μ value PAPR reduces. The obtained results approved that the proposed method have get the better results than conventional OFDM.

As a further research topic, we intend to include the selected mapping (SLM) and Discrete Wavelet Transform (DWT) between the pre-coding techniques.

REFERENCES


[23] I. Baig and V. Jeoti, “PAPR Reduction in OFDM Systems: Zadoff-Chu Matrix Transform Based Pre/Post-Coding Techniques”, the 2nd International Conference on Computational Intelligence,

BIBLIOGRAPHY

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