Optimum and Quasi Optimum Adaptive Planar Arrays Design Using Evolutionary Algorithms

Marian Farouk Mikhail, Mohamed S. El-Mahallawy, Mohamed A. Aboul-Dahab

Abstract— In this paper, a design procedure for the adaptive beamforming planar arrays under multiple constrains using the evolutionary algorithms, namely the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is introduced. Both algorithms seek optimum values of weight coefficients and inter element spacing. The constraints are divided into two categories, one deals with the array parameters, and the other deals with the evolutionary algorithm. The constraints dealing with array parameters are the first null beam width, the first side lobe level, and a null imposed at certain direction, whereas those of the evolutionary algorithms deal with the weights of the cost functions, the limits of array weight coefficients and the spacing between array elements. For the ease of implementation, quasi-optimum weights are devised which are quantized values of the optimum weights. For this purpose, an additional constraint is imposed to the cost function of the algorithms. Computer simulations have been carried out to evaluate the performance of using GA and PSO algorithms in the design of the planar array and to investigate the effect of the used quasi optimum values on the planar array performance. The evaluations indicate that the PSO superior performance over that of the GA in both optimum and quasi optimum arrays.

Index Terms— Evolutionary Algorithms, GA, PSO, Planar array, Quazi optimum array, Weight Coefficients, Inter element Spacing.

1 INTRODUCTION

Nowadays, adaptive array antennas play an important role in improving signal quality in the wireless communications by keeping the main beam and imposing nulls in the directions of interfering signals [1]. To reach optimum weight coefficients of array elements adaptively, a variety of algorithms were devised, amongst of which are the evolutionary algorithms [2], [3]. In evolutionary algorithm, a fitness function is formed to match a required design criterion, and relevant optimum weights are reached via variation and selection operation [2], [3], [4]. Two of the mostly used evolutionary algorithms in the design of adaptive antenna arrays are the Genetic Algorithm (GA) and particle swarm optimization (PSO) algorithm [5]. In [6], the GA was utilized to seek optimum weights linked to antenna elements that result in the reduction of side lobe level.

The GA was used for the synthesis of antenna array radiation pattern in adaptive beam forming [7]. In [5],[8] the design of non uniformly spaced linear antenna arrays using PSO algorithm was presented for the purpose of reaching a desired radiation pattern while improving the performance of these arrays in terms of side lobe levels. Another approach was presented in [2], where the synthesis of linear antenna array using PSO algorithm was carried out to reach optimum amplitude excitations for performance improvement in the sense of minimum side lobe level (SLL) and null control with periodic spacing between the elements.

Due to the usage of adaptive planar array in a variety of applications such as tracking radars, search radars, remote sensing and communication systems, and due to additional variables which can be used to control and shape the array pattern, a lot of research have been developed to improve the performance of these arrays [9]. In [3], planar array synthesis using Chebyshev’s method and GA was proposed to control the amplitude and phase of signals of array elements for the purpose of placing nulls at the directions of the interfering sources and placing the main beam in the direction of the desired signal. Another approach was devised using the GA to seek the optimum weight coefficients of the array elements to achieve a minimum SLL with narrower beam width. The results were compared with synthesized pattern using Gaussian, Kaiser, Hamming and Blackman weights coefficients where a significant improvement had been achieved [4]. The PSO had been utilized to control Signal-to-Interference-plus-Noise Ratio and to find the set of weights that configure a rectangular array to effectively maximize the power towards a desired direction and avoid direction of interferences [10].

A comparison between the GA and the PSO for the design of linear and planar antennas arrays with uniformly spaced elements for the purpose of side lobe reduction and main beam width constraints had been carried in [11]. The weights adjustment via amplitude only and amplitude plus phase had been investigated. The results showed that an appreciable reduction of the order of the first SLL that had been attained in comparison with the case of uniform arrays. Also, the results showed that PSO with adaptive scheme had a better performance than GA due to its simplicity in implementation and minor computing time. [11]

In this paper a design procedure for an optimum adaptive beamforming planar arrays is introduced by changing the array weight coefficients and the spacing between array elements with multiple constrains using the GA and PSO algorithms. These constrains deal with the array parameters (which are the first null beam width, the first SLL, and a null imposed at certain direction) and with the evolutionary algo-
rithms (the weights of the cost function, the limits of array weight coefficients and limits of the spacing between array elements). For the ease of implementation, quasi-optimum weights are chosen, where an additional constraint to the optimization cost function is also imposed.

This paper is organized as follows. In section 2, problem formulation will be discussed. In section 3, a brief description of the utilized optimization techniques, namely the GA and PSO is given. The optimum and quasi optimum arrays are described in sections 4 and 5 respectively. Computer simulation results and discussions are given in section 6, and the paper is concluded in section 7.

2 PROBLEM FORMULATION

For a planar array composed of even number of elements \((2M\times 2N)\) symmetrically placed over along the x-z plane, as shown in Fig.1. The planar array factor is given by [12], [13]

\[
AF = 4 \sum_{m=1}^{M} \sum_{n=1}^{N} a_m * a_n * \cos((2m-1)u) * \cos((2n-1)v)
\]

where

\[
u = \frac{\pi}{\lambda} x_m \sin \theta \cos \phi \quad \text{and} \quad v = \frac{\pi}{\lambda} z_n \cos \theta
\]

\[
AF(u,v) = 4 \sum_{m=1}^{M} \sum_{n=1}^{N} a_m * a_n * \cos((2m-1)u) * \cos((2n-1)v)
\]

where \(a_m, a_n\) are the amplitude excitations, \(x_m\) is the position of the \(m\)th element and \(z_n\) is the position of the \(n\)th element.

It is required to design this array in such a way that the following objectives are achieved:

1. A null to be imposed at certain direction.
2. First null beam width to be kept unchanged with respect to uniform array.
3. First SLL to be reduced below that of a uniform array.

These objectives represent constraints that are imposed on the design procedure of the array. In this respect, a cost function (CF) is developed to be minimized. An expression for the cost function will contain three additive terms inspired from [1] as

\[
CF = C_1 \frac{|AF(\text{null})|}{|AF(\text{max})|} + C_2 \frac{H * (Q - \delta)}{(\delta)} + C_3 \left(\frac{FNBW_{\text{computed}} - FNBW_{a_n,a_m=1}}{FNBW_{a_n,a_m=1}}\right)
\]

where AF(null), is the value of the designed array factor at the particular null position, \(AF_{\text{max}}\) is the maximum value of the array factor, \(Q\) is the SLL of the desired array in dB at peak point \(\theta_k\) & \(\phi_k\), \(\delta\) is the desired value of the SLL in dB of the uniform array, \(FNBW_{\text{computed}}\) is the computed first null beam width of the designed array, the value of \(FNBW_{a_n,a_m=1}\) of the uniform array, \(C_1, C_2, C_3\) are weighting coefficients used to control the relative importance of each term of Equation (2) and \(H\) is defined as

\[
H = \begin{cases} 1 & (Q - \delta) > 0 \\ 0 & (Q - \delta) \leq 0 \end{cases}
\]

where the side lobes whose peaks exceed the threshold \(\delta\) must be suppressed. The solution of the optimization problem will yield the required weight coefficients (amplitudes) as well as the inter-element spacing in the dimensions of the array.

3 EVOLUTIONARY ALGORITHMS

3.1. Genetic Algorithm Optimization (GA)

It is based on principle of the evolution of the natural species introduced by Charles Darwin [14]. The optimization process in the (GA) is based upon the following procedure [15]:

1. Creating an initial random population of weights of elements and inter distance between elements.
2. Evaluation of population based on the fitness function of (2).
3. Some values (chromosomes) are selected as a parent by a selection technique (roulette-wheel-tournament).
4. Offspring and Mutation can be generated from selected parents.
5. The process will continues until the termination condition is achieved.

3.2. Particle Swarm Optimization (PSO)

Unlike GAs, the PSO is based upon the cooperation among the individuals rather than their competition. Moreover, it is easier to calibrate and to control the parameters of the PSO over the GA [16].

In PSO, the optimization process proceeds as follows[17]:

1. Each particle is initialized with a random position and velocity.
2. Each particle is then evaluated for fitness value of (2).
3. Each time a fitness value is calculated, it is compared against the previous best fitness value of the particle and the previous best fitness value of the whole swarm, and the personal best (pbest) and global best positions (gbest) are updated where appropriate.
4. The process is repeated until a stopping criterion is met.

4. OPTIMUM ARRAY

The far field pattern of an array is controlled by many factors. In addition to the geometrical configuration of the overall array, there are the relative displacement between elements, excitation amplitudes of individual elements, excitation phase of individual elements, and far field pattern of the individual elements [5]. In this paper, inter-spacing between elements, excitation amplitude of individual elements will be optimized.
to achieve the required first null beam width, lower value of the first SLL and a null that is imposed at certain direction.

The weighting coefficients $C_1$, $C_2$ and $C_3$ of the cost function are selected according to there relative importance of the imposed constraint. In the proposed design, we emphasize on the superiority of the imposed null as well as the first null beamwidth. For this reason, the ratio of the coefficients is taken to be 1:3:3 respectively. In order to obtain reasonable values for the optimum weight coefficients of array elements, as well as optimum inter element distances of the array, some constraints need to be imposed on the cost function. Moreover, appropriate values have to be selected for the initiation of the optimization process. The following is a proposed constraint on the weight coefficients of array elements:

$$0.5 \leq w_{ij} \leq 1 \quad i=1,2,\ldots M, j=1,2,\ldots N$$

As far as the inter element distances are concerned, the following constraint is proposed

$$0.25\lambda \leq d_x, d_z \leq 0.5\lambda$$

The upper bound is necessary to avoid the presence of grating lobes in the visible range of the array. It is worth mentioning that the above constraints are applied to both E-plane pattern and H-plane pattern of the array.

5. QUASI OPTIMUM ARRAY

The optimum values of the weights and distances can take any value within the imposed bounds. For ease of implementation of the array, it is more reasonable to have a number of discrete, rather than continuous values of the optimum values. For this reason, the optimum weight coefficients are approximated to two discrete values, namely 0.5 and 1.0. As far as the inter element distances are concerned, three discrete values are selected, namely 0.3\(\lambda\), 0.4\(\lambda\)and 0.5\(\lambda\). Actually, the approximated values will not yield the required optimum array performance. We shall rather have a quasi optimum array. The approximated values of the weights and inter element distances are derived from the optimum weights that are obtained from using either the GA or the PSO algorithm. The performance of the quasi optimum arrays is investigated in the next section to have some insight on the limitations and tolerances in their performances.

6. RESULTS AND DISCUSSION

The case of a planar array composed of 12 x 12 elements is considered. The array is placed symmetrically on x-z plane, and is assumed to be uniform in its initial condition with weight coefficients to be unity and inter element distances in both directions to be 0.5 \(\lambda\). The constraints imposed on the array are as follows:

- First null beamwidth (FNBW) = 0.334 radians (same as that of the uniform array)
- Deep nulls to appear at angles \(\theta_n = \varphi_n =54.43^\circ\) (the angles at of the 3rd side lobe in the uniform array) and to be less than -40dB.
- A reduction of the first SLL (at angles \(\theta_{1L} = \varphi_{1L} =76.2^\circ\)) below that of the uniform array.

To investigate the performance of the optimum and quasi optimum planar arrays, a computer simulation for the optimization problem has been developed using MATLAB 14 software package, where the results of 25 optimization runs using the GA and PSO algorithms have been obtained. PSO & GA parameters are empirically chosen to achieve the best results in our simulation. The following are the parameters utilized in the simulation process with either GA or PSO algorithm.

The maximum and minimum values of the FNBW, imposed null and first SLL in both x-y and y-z planes in the designed of optimum and quasi optimum arrays using the GA and the PSO algorithms are shown in Table 2. It is clear from the results in the case of optimum array, that PSO gives nearly the same FNBW as in the imposed constraint rather than the case of GA. The null imposed at the required angles is deeper in the case of PSO than that of the case of GA. However, a considerable reduction in the level of the first SLL has been achieved in the case of the PSO than that in the case of GA. In the case of quasi optimum array, the values of the FNBW are not so far from the imposed value. The level of the imposed null is not so deep, and the first SLL in some runs are higher than that of the imposed constraint.

### Table 1: GA and PSO Parameters

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>Population size</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Generations No.</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Weights upper limit</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Weights lower limit</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Inter elements distances upper limit</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Inter elements distances lower limit</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Crossover rate</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Mutation rate</td>
<td>0.01</td>
</tr>
<tr>
<td>PSO</td>
<td>Particles swarm</td>
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</tr>
<tr>
<td></td>
<td>Iterations No.</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Weights upper limit</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Weights lower limit</td>
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</tr>
<tr>
<td></td>
<td>Inter elements distances upper limit</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Inter elements distances lower limit</td>
<td>0.25</td>
</tr>
</tbody>
</table>

### Table 2: Maximum and minimum values of FNBW, null depth and first SLL

<table>
<thead>
<tr>
<th>plane</th>
<th>Algorithm constraint</th>
<th>FNBW(_{\text{max}})(rad)</th>
<th>FNBW(_{\text{min}})(rad)</th>
<th>(\theta_{1L})(rad)</th>
<th>(\varphi_{1L})(rad)</th>
<th>(\text{Null level}_{\text{max}})(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-z</td>
<td>PSO</td>
<td>0.347</td>
<td>0.340</td>
<td>0.360</td>
<td>0.360</td>
<td>-122.950</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>0.340</td>
<td>0.340</td>
<td>0.340</td>
<td>0.340</td>
<td>-98.231</td>
</tr>
<tr>
<td></td>
<td>PSO</td>
<td>0.347</td>
<td>0.340</td>
<td>0.354</td>
<td>0.360</td>
<td>-22.965</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>0.340</td>
<td>0.340</td>
<td>0.340</td>
<td>0.340</td>
<td>-43.186</td>
</tr>
<tr>
<td>x-y</td>
<td>PSO</td>
<td>0.347</td>
<td>0.340</td>
<td>0.360</td>
<td>0.360</td>
<td>-122.950</td>
</tr>
<tr>
<td></td>
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<td>-43.186</td>
</tr>
</tbody>
</table>

**Note:** The values of FNBW, null depth and first SLL are obtained using both GA and PSO algorithms. The results show that PSO gives slightly better performance than GA in terms of FNBW and null depth.
The stability of the designed performance of the optimum and quasi optimum planar arrays are investigated by monitoring the average values of the 25 optimization runs that have been carried out for both cases of PSO and GA. The deviations from the average values are also monitored for both cases. Table 3 illustrates the average values of the FNBW, null depth and first SLL as well as the deviations from the average. It is clear that the FNBW in both optimum and quasi optimum arrays have been achieved with satisfactorily percentage of deviation using both algorithms. The first SLL constraint has been achieved in the optimum array using both algorithms although the percentage deviation is relatively high. It is notable that the PSO has a superior performance since its percentage deviation is lower than that of the GA algorithm. As for the quasi optimum array, the first SLL constraint has been marginally achieved, but the percentage deviation is considerably high. As far as the deep null is concerned, the optimum array has satisfactorily achieved the required constraint although the percentage deviation is considerably high. However, the quasi optimum array has not achieved the required constraint, in addition to the remarkable high deviation percentage.

The array factor using best optimized and quasi optimized values of weight coefficients as well as inter element distances are shown in Fig. 2, Fig. 3 for the x-y and y-z planes for the case of PSO algorithm. It is clear from these figures that the array factors for the optimum and quasi optimum arrays are very close to each other except at the location of the imposed null where the quasi optimum array has not achieved this constraint.

<table>
<thead>
<tr>
<th>Constraint Parameters plane</th>
<th>Average and max deviation values</th>
<th>PSO</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimum Array</td>
<td>Quasi Optimum Array</td>
<td>Optimum Array</td>
</tr>
<tr>
<td>FNBW</td>
<td>y-z</td>
<td>0.342</td>
<td>0.347</td>
</tr>
<tr>
<td></td>
<td>x-y</td>
<td>0.344</td>
<td>0.346</td>
</tr>
</tbody>
</table>

Fig. 4, Fig.5 illustrate the array factors using best optimized and quasi optimized values of the weight coefficients and inter element distances for the x-y and y-z planes for the optimum and quasi arrays for the case of the GA algorithm. It is clear from these figures that the array factors for the optimum and quasi optimum arrays are far from each other except at the location of the main lobe.
7. CONCLUSION

The design of optimum planar array under certain imposed constraints is presented in this paper. The optimum values of the weigh coefficients as well as the optimum inter element distances have been achieved using two evolutionary algorithms, namely the particle swarm optimization (PSO) and the genetic algorithm (GA). A quasi optimum array has been devised in both cases by rounding the weight coefficients and inter element distances to a limited number of discrete values. This approach has a practical advantage since it is possible to have certain preset values for the weight coefficients and the inter element distances. The simulation results illustrated that the optimum arrays have achieved the imposed constraints. However, the performance of quasi optimum arrays has been found partially satisfactory. It is worth noting that use of PSO has resulted in superior performance over that of the GA in both optimum and quasi optimum arrays.

REFERENCES


