On Supra I-open Sets and Supra I-continuous functions

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Abstract—In this paper we introduce and investigate a new class of sets and functions between topological spaces called supra I-open sets and supra I-continuous functions respectively.

Keywords—Supra I-open set; Supra I-continuous functions; Supra topological spaces.

1 INTRODUCTION

In 1983, A.S.Mashhour et al. [6] introduced the supra topological spaces and studied S-continuous maps and S*-continuous maps. In 1987, M.E.Abd El- Monsef et al. [3] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions. This class of sets contained in the class of β-open sets [1] and contains all semiopen sets [2] and all pre-open sets [6]. In 2008, R.Devi et al [5] introduced and studies a class of sets and maps between topological spaces called supra α-open and supra α*-continuous maps, respectively. Now, we introduce the concept of supra I-open set, and supra I-continuous functions and investigate several properties for these classes of maps.

1.1 Definition
A sub family T* of X is said to be a supra topology on X if,
(i) X, ∅ ϵ T*,(ii) if A ϵ T*, ∀ i ϵ J, then U Ai ϵ T*

(x, τ*) is called supra topological space. The elements of τ* are called supra open sets in (x, τ*) and complements of a supra open set is called a supra closed set.

1.2 Definition
The supra closure 0 a set A is denoted by supra cl(A) and defined as supra cl(A) = ∩ {B : B is a supra closed and A ⊆ B}. The supra interior of a set A is denoted by Supra int(A), and defined as supra int(A) = ∪ {B : B is a τ supra open and A ⊆ B}.

1.3 Definition
Let (x, τ) be a topological space and τ* be a supra topology on x. We call τ a supra topology associated with τ if τ ⊆ τ*.

2 SUPRA I-OPEN SETS

In this section, we introduce a new class of generalized open sets called supra I-open sets and study some of their properties.

2.1 Definition
Let (x, τ*) be a supra topological space. A set A is called supra I-open set if A ⊆ Supra int (supra cl(A)). The complement of a supra I-open set is called a supra I-closed set.

2.2 Theorem
Every supra open set is supra I-open set.
Proof.
Let A be a supra open set in (x, τ*) since, A ⊆ supra cl(A), then supra int(A) ⊆ supra int (supra cl(supra int (A))). Hence A ⊆ supra int (supra cl (supra int (A))).
The converse of the above theorem need not be true as shown by the following examples.

2.3 Example
Let (x, τ*) be a supra topological space. Where X = {a, b, c} and τ* = {x, φ, {a}, {a, b}, {b, c}}. Here, {a, c} is a supra I-open set, but it is not a supra open.

2.4 Theorem
In [3], the author proved that every supra I-open set is supra semi-open. (Instead of example 3.2 [3] ) shows the converse need not be true.

2.5 Example
Let (x, τ*) be a supra topological space. Where X = {a, b, c, d} and τ* = {x, φ, {a}, {a, b}, {b, c}}. Here {b, c} is a supra semi-open set, but not a supra I-open.

2.6 Theorem

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Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\tau'\) be a supra topology on \(X\). A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is called a supra \(\tau\)-continuous functions if the inverse image of each open set in \(Y\) is a supra \(\tau\)-open set in \(X\).

3.2 Theorem

Every continuous function is supra \(\tau\)-continuous functions.

Proof.

Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be a continuous function. Therefore \(f^{-1}(A)\) is a open set in \(X\) for each open set \(A\) in \(Y\). But, \(\tau'\) is associated with \(\tau\). That is \(\tau' \subseteq \tau\). This implies \(f^{-1}(A)\) is a supra \(\tau\)-open set in \(X\). Hence \(f\) is a \(\tau\)-continuous functions.

The converse of the above theorem is not true as shown in the following example.

3.3 Example

Let \(X = \{a, b, c\}\) and \(\tau = \{x, \phi, \{a, b\}, \{b, c\}\}\) be a topology on \(X\). The supra topology \(\tau'\) is defined as follows, \(\tau' = \{x, \phi, \{a, b\}, \{b, c\}\}\). Let \(f : (X, \tau) \rightarrow (X, \tau)\) be a function defined as follows \(f(a) = a, f(b) = c, f(c) = b\). The inverse image of the open set \(\{a, b\}\) is not an open set but if a supra \(\tau\)-open. Then \(f\) is supra \(\tau\)-continuous but it is not continuous.

3.4 Theorem

Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces. Let \(f\) be a function from \(X\) into \(Y\). Let \(\tau'\) be associated supra topology with \(\tau\). Then the following are equivalent.

a) \(f\) is \(\tau\)-continuous.

b) The inverse image of closed set in \(Y\) in supra \(\tau\)-closed set in \(X\).

c) Supra \(\tau\) closure of \(f^{-1}(A)\) is \(f^{-1}\) of \(\tau\) closure of \(A\) for every set \(A\) in \(Y\).

d) \(f\) is \(\tau\)-continuous functions.

e) \(f^{-1}(\text{int}(B))\) is supra \(\tau\)-closed set for every set \(B\) in \(Y\).

Proof.

(a) \(\Rightarrow\) (b): Let \(A\) be a closed set in \(Y\), then \(Y - A\) is open in \(Y\). Thus, \(f^{-1}(X - A) = X - f^{-1}(A)\) in supra \(\tau\)-open in \(X\). It follows that \(f^{-1}(A)\) in a supra \(\tau\)-closed set of \(X\).

(b) \(\Rightarrow\) (c): Let \(A\) be any subset of \(X\). Since \(\tau\) closure of \(A\) is closed in \(Y\), then it follows that \(f^{-1}(\tau\text{ closure of } A)\) is supra \(\tau\)-closed in \(X\). Therefore \(\tau\) closure of \(A\) = supra \(\tau\) closure of \(\tau\) closure of \(A\) \(\subseteq\) supra \(\tau\) closure of \(f^{-1}(A)\).
(c) $\Rightarrow$ (d): Let A be any subset of X. By (c) we obtain, $f^{-1}(\text{cl}(f(A))) \supseteq \text{supra I cl}(f(A))$ and hence $f(\text{supra I cl}(A)) \subseteq \text{cl}(f(A))$.

(d) $\Rightarrow$ (e): Let $f(\text{supra I cl}(A)) \subseteq \text{cl}(f(A))$ for every set A in X. Then supra I cl(A) $\subseteq f^{-1}(\text{cl}(f(A)))$, X-supra I cl(A) $\supseteq X - f^{-1}(\text{cl}(f(A)))$ and supra I int(X - A) $\supseteq f^{-1}(\text{int}(f(A)))$. Then supra I int(f^{-1}(B)) $\supseteq f^{-1}(\text{int}(B))$. Therefore $f^{-1}(\text{int}(B)) \subseteq \text{supra I int}(f^{-1}(B))$, for every B in Y.

(e) $\Rightarrow$ (a): Let A be a open set in Y. Therefore $f^{-1}(\text{int}(A)) \subseteq \text{supra I int}(A)$, hence $f^{-1}(A) \subseteq \text{supra I int}(f^{-1}(A))$. But by other hand we know that, supra I int(f^{-1}(A)) $\subseteq f^{-1}(A)$. Then $f^{-1}(A) = \text{supra I int}(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a supra I-open set.

3.5 Theorem

If a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is supra I-continuous and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ is continuous map, then $g f: (X, \tau) \rightarrow (Z, \gamma)$ is supra I-continuous.

**Proof.** Obvious.

3.6 Theorem

Let (x, $\tau$) and (y, $\sigma$) be topological space. Let $\tau'$ and $\sigma'$ be associated supra topologies with $\tau$ and $\sigma$ respectively. Then $f: (X, \tau) \rightarrow (Y, \sigma)$ is supra I-continuous map, if one of the following holds.

1. $f^{-1}(\text{supra I int}(B)) \subseteq \text{int}(f^{-1}(B))$ for every set B in Y.
2. $\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{supra I cl}(B))$ for every set B in Y.
3. $f(\text{cl}(A)) \subseteq \text{supra I cl}(f^{-1}(A))$ for any set A in X.

**Proof.**

Let B be any open set of Y, if condition (1) is satisfied, then $f^{-1}(\text{supra I int}(B)) \subseteq \text{int}(f^{-1}(B))$. We get $f^{-1}(B) \subseteq \text{int}(f^{-1}(B))$. Therefore $f^{-1}(B)$ is supra open set. Every supra open set is supra I-open set. Hence $f$ is a supra I-continuous map.

If condition (2) is satisfied, then we can easily prove that $f$ is a supra I-continuous map.

If condition (3) is satisfied and B is any open set of Y. then $f^{-1}(B)$ is a set in X and $f(\text{cl}(f^{-1}(B))) \subseteq \text{supra I cl}(f^{-1}(B))$. This implies $f(\text{cl}(f^{-1}(B))) \subseteq \text{supra I cl}(A)$. This is nothing but conditions (2). Hence $f$ is a supra I-continuous map.

**REFERENCES**


