On Soft Semi-Open Sets

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Abstract—The objective of this paper is to describe the basics of soft semi-open sets and soft semi-closed sets in soft topological spaces by applying the functions of D. Molodstov's soft set theory.

Keywords—soft set, soft topology, soft open sets, soft closed sets, soft semi-open set and soft semi-closed set.

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1 INTRODUCTION

Many researchers followed Molodtov [4] when he introduced soft set theory as a basic mathematical application in describing with the ambiguity of not clearly defined objects. The practical problems in engineering, social science, life science were solved using these mathematical applications. The soft topological spaces and its basic notations were dealt in detail by Shabir and Naz [10].

The equality of two soft sets, subset and superset of Soft set, complement of a soft set, null soft set and absolute soft set with examples were explained by Maji [7]. Current researchers are now dealing with the latest advanced techniques by applying the results in operations research, Riemans integration, Game theory, theory of probability and techniques by applying the results in operations research, Riemans integration, Game theory, theory of probability and arrived at the results for some basic notations of soft set theory. The foundations of the theory of soft topological spaces to open the door for experiments for soft mathematical applications. The soft topological spaces and its practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects. The practical problems in engineering, social science, life science were solved using these clearly defined objects.

The following definitions are essential for the development of the paper.

Definition 2.1. [4] Let $U$ be an initial universe and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$. The pair $(F,E)$ or simply $F$, is called a soft set over $U$, where $F$ is a mapping given by $F:E \rightarrow P(U)$. In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$.

For $e \in U$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $F$. The collection of all soft sets over $U$ and $E$ is denoted by $S(U)$. If $A \subseteq E$, then the pair $(F,A)$ or simply $F_A$, is called a soft set over $U$, where $F$ is a mapping $F:A \rightarrow P(U)$. Note that for $e \notin A$, $F(e) = \emptyset$.

Definition 2.2. [11] The union of two soft sets of $F_B$ and $G_C$ over the common universe $U$, is the soft set $H_B$, where $B$ and $C$ are subsets of the parameter set $E$, $D = B \cap C$ and for all $e \in D$, $H(e) = F(e)$ if $e \in B - C$, $H(e) = G(e)$ if $e \in C - B$ and $H(e) = F(e) \cap G(e)$ if $e \in B \cap C$, we write $F_B \cap G_C = H_D$.

Definition 2.3. [11] The intersection of two soft sets of $F_B$ and $G_C$ over the common universe $U$ is the soft set $H_B$, where $D = B \cap C$ and for all $e \in D$, $H(e) = F(e) \cap G(e)$ if $D = B \cap C$. We write $F_B \cap G_C = H_D$.

Definition 2.4. [11] Let $F_B$ and $G_C$ be soft sets over a common universe set $U$ and $B, C \subseteq E$. Then $F_B$ is a soft subset of $G_C$, denoted by $F_B \subseteq G_C$, if (i) $B \subseteq C$ and (ii) for all $e \in B$, $F(e) = G(e)$. Also, $G_C$, is called the soft super set of $F_B$ and is denoted by $F_B \supseteq G_C$.

Definition 2.5. [11] The soft sets $F_B$ and $G_C$ over a common universe set $U$ are said to be soft equal, if $F_B \subseteq G_C$ and $F_B \supseteq G_C$. Then we write $F_B = G_C$.

Definition 2.6. [11] A soft set $F_B$ over $U$ is called a null soft set denoted by $F_\emptyset$, if for all $e \in B$, $F(e) = \emptyset$.

Definition 2.7. [10] The relative complement of a soft set $F_A$, denoted by $F_A^c$, is defined by the approximate function $F_A^c(e) = \{ e \in U | (e \notin A \cap F(e) \neq \emptyset) \}$. The soft set $F_A^c$ is called the soft complement of $F_A$.
Lemma 2.13. \[12\] Let \( F_A \) be a soft set in \( F \) and \( F_B \) be a soft set in \( F_A \). Then the following hold.

(i) \( \text{int}(F_A) \cap \text{int}(F_B) \subseteq \text{int}(F_A \cap F_B) \).

Lemma 2.14. \[12\] Let \((F_A, \tau)\) be a soft topological space and \( F_B \) and \( F_C \) be a soft set in \( F_A \). Then the following hold.

(i) \( \text{d}(F_B) = \text{d}(F_B) \).

(ii) \( F_B \subseteq F_C \) implies \( \text{d}(F_B) \subseteq \text{d}(F_C) \).

(iii) \( \text{d}(F_B) \cap \text{d}(F_C) \subseteq \text{d}(F_B \cap F_C) \).

(iii) \( \text{d}(F_B) \cup \text{d}(F_C) = \text{d}(F_B \cup F_C) \).

Proposition 2.15. \[2\] Let \((F_A, \tau)\) be a soft topological space over \( F_A \). Then

(i) \( F_\phi \), \( F_\psi \) are soft closed sets in \( F_A \).

(ii) The union of any two soft closed sets is a soft closed set in \( F_A \).

(iii) The intersection of any family of soft closed sets is a soft closed set in \( F_A \).

3. SOFT SEMI-OPEN SETS AND SOFT SEMI-CLOSED SETS:

This section is devoted to the study of soft semi-open sets and soft semi-closed sets.

Definition 3.1. Let \( F_B \) be a soft subset of a soft topological space \((F_A, \tau)\). \( F_B \) is said to be a soft semi-open set if \( F_B \subseteq \text{cl}(F_B) \).

Example 3.2.

Let \( U = \{a, b, c\} \), \( E = \{e_1, e_2, e_3\} \), \( A = \{a_1, e_2\} \subseteq E \).

\( F_A = \{(e_1, \{a\}, \{b, c\}) \), \( (e_2, \{a, b\}) \), \( (e_3, \{a, c\}) \) \). 

\( F_B = \{(e_2, \{a, c\}) \), \( (e_2, \{b, c\}) \), \( (e_1, \{a, c\}) \) \). 

\( F_C = \{(e_2, \{a, b\}) \), \( (e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \) \). 

\( F_D = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_E = \{(e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \) \). 

\( F_F = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_G = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_H = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_I = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_J = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_K = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_L = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_M = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_N = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_O = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_P = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_Q = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_R = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_S = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_T = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_U = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_V = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_W = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_X = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_Y = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

\( F_Z = \{(e_2, \{b, a\}) \), \( (e_2, \{a, c\}) \), \( (e_2, \{b, a\}) \) \). 

Theorem 3.3. Every soft open set in a soft topological space
\((F_A, \bar{r})\) is a soft semi-open set.

**Proof.** The proof follows from the Definition 3.1.

The following Example 3.4 shows that the reverse implication of Theorem 3.3 is not true.

**Example 3.4.** Consider the soft topological space of Example 3.2. Here \(F_{11}, F_{12}, F_{14}, F_{15}\) and \(F_{16}\) are soft semi-open sets but not soft open sets, since \(F_{11}, F_{12}, F_{14}, F_{15}, F_{18} \in \bar{r}\).

Remark: \(F_b\) and \(F_A\) are always soft semi-closed sets and soft semi-open sets.

**Proposition 3.5.** A soft set \(F_b\) in a soft topological space \((F_A, \bar{r})\) is a soft semi-open set if and only if there exists a soft open set \(F_c\) such that \(F_c \subseteq F_b \subseteq cl(F_c)\).

**Proof.** Assume that \(F_b \subseteq cl(int(F_b))\). Then for \(F_c = int(F_b)\), we have \(F_c \subseteq F_b \subseteq cl(F_c)\). Therefore, the condition holds. Conversely, suppose that \(F_c \subseteq F_b \subseteq cl(F_c)\) for some soft open set \(F_c\). Since \(F_c \subseteq int(F_b)\) and so \(cl(F_c) \subseteq cl(F_b)\). Hence \(F_b \subseteq cl(F_c) \subseteq cl(int(F_b))\). Hence \(F_b\) is soft semi-open set.

**Theorem 3.7.** Let \((F_A, \bar{r})\) be a soft topological space and \(\{F_{ba}\} : \alpha \in \Delta\) be a collection of soft semi-open sets in \((F_A, \bar{r})\). Then \(U_{\alpha \in \Delta} F_{ba}\) is also a soft semi-open set.

**Proof.** Let \(\{F_{ba}\} : \alpha \in \Delta\) be a collection of soft semi-open set in \((F_A, \bar{r})\). Then for each \(\alpha \in \Delta\), we have a soft open set \(F_{ca}\) such that \(F_{ca} \subseteq F_{ba} \subseteq cl(F_{ca})\). \(U_{\alpha \in \Delta} F_{ca} \subseteq F_{ba} \subseteq U_{\alpha \in \Delta} cl(F_{ca}) \subseteq U_{\alpha \in \Delta} cl(F_{ca})\).

**Definition 3.8.** A soft set \(F_b\) in a soft topological space \((F_A, \bar{r})\) is said to be a soft semi-closed set, if its relative complement is a soft semi-open set.

**Theorem 3.9.** Every soft closed set in a soft topological space \((F_A, \bar{r})\) is soft semi-closed set.

**Proof.** The proof follows from the Definition 3.8.

The following Example 3.10 shows that the converse implication of Theorem 3.9 is not true.

**Example 3.10.** Consider the soft topological space of Example 3.2. Here \(F_2, F_3, F_5\) and \(F_9\) are soft semi-closed sets but not soft closed sets.

**Theorem 3.11.** \(F_c\) be soft semi-closed in a soft topological space \((F_A, \bar{r})\) if and only if \(int(F_c) \subseteq F_c \subseteq cl(F_c)\) for some soft closed set \(F_c\).

**Proof.** \(F_c\) is soft semi-closed if and only if \(F_c \subseteq cl(F_c)\) and only if there is a soft open set \(F_d\) such that \(F_d \subseteq cl(F_c)\), by Theorem 3.5 if and only if there is a soft open set \(F_d\) such that \(cl(F_d) \subseteq F_c \subseteq F_d\) if and only if there is a soft closed set \(F_d\) such that \(int(F_d) \subseteq F_c \subseteq F_d\) and \(F_e \subseteq F_d\), where \(F_e = F_c\).

**Theorem 3.12.** A soft subset \(F_b\) in a soft topological space \((F_A, \bar{r})\) is soft semi-closed if and only if \(int(cl(F_b)) \subseteq F_b\).

**Proof.** \(F_b\) is soft semi-closed if and only if \(F_b \subseteq cl(F_b)\) and only if \(F_b \subseteq cl(F_b)\) and only if \(F_b \subseteq cl(F_b)\). Hence \(F_b \subseteq cl(F_b)\). This completes the proof.

**Theorem 3.13.** Let \((F_A, \bar{r})\) be a soft topological space and \(\{(F_{ba}) : \alpha \in \Delta\}\) be a collection of soft semi-closed sets in \((F_A, \bar{r})\). Then \(\bigcap_{\alpha \in \Delta} F_{ba}\) is also a soft semi-closed set.

**Proof.** Let \(\{F_{ba}\} : \alpha \in \Delta\) be a collection of soft semi-closed sets in \((F_A, \bar{r})\). Then for each \(\alpha \in \Delta\), we have a soft closed set \(F_{ca}\) such that \(int(F_{ca}) \subseteq F_{ba} \subseteq cl(F_{ca})\). Then \(\bigcap_{\alpha \in \Delta} F_{ca} \subseteq \bigcap_{\alpha \in \Delta} F_{ba} \subseteq \bigcap_{\alpha \in \Delta} cl(F_{ca})\). Because \(\bigcap_{\alpha \in \Delta} F_{ca} \subseteq F_c\) is soft closed by prop.2.15(iii), \(\bigcap_{\alpha \in \Delta} F_{ba}\) is soft semi-closed set.

**Definition 3.14.** Let \((F_A, \bar{r})\) be a soft topological space and let \(F_b\) be a soft set in \(F_A\).

(i) The soft semi-interior of \(F_b\) is the soft set \(\{F_c \subseteq F_b : F_c\) is soft semi-open and \(F_c \subseteq F_b\}\) and is denoted by \(s-int(F_b)\).

(ii) The soft semi-closure of \(F_b\) is the soft set \(\{F_c \subseteq F_b : F_c\) is soft semi-closed and \(F_c \subseteq F_b\}\) and is denoted by \(s-cl(F_b)\).

Clearly, \(s-cl(F_b)\) is the smallest soft semi-closed set containing \(F_b\) and \(s-int(F_b)\) is the largest soft semi-open set contained in \(F_b\). By Theorem 3.7 and 3.13, we have \(s-int(F_b)\) is soft semi-open and \(s-cl(F_b)\) is soft semi-closed set.

**Example 3.15.** Let the soft topological space \((F_A, \bar{r})\) be the same as in Example 3.2, we get \(s-int(F_b) = F_b\).

**Example 3.16.** Let the soft topological space \((F_A, \bar{r})\) be the same as in Example 3.2, we get \(s-cl(F_b) = F_b\).

From the definitions of soft semi-interior and soft semi-closure, we have the following theorem.

**Theorem 3.17.** Let \((F_A, \bar{r})\) be a soft topological space and \(F_b\) be a soft set in \(F_A\). We have \(int(F_b) \subseteq s-int(F_b) \subseteq F_b \subseteq s-cl(F_b)\).
s-cl(F_b) ⊆ s-cl(F_b).

**Proof.** By Theorem 3.3, Theorem 3.9 and Definition 3.14. □

**Theorem 3.18.** Let (F_A, ℰ) be a soft topological space and F_B be a soft set in F_A. Then the following hold.

(i) (s-cl(F_b))' = s-int(F_b)

(ii) (s-int(F_b))' = s-cl(F_b)

**Proof.**

(i) (s-cl(F_b))' = (Ω(F_C : F_C is soft semi-closed and F_C ⊆ F_b))' = 0 (F_C : F_C is soft semi-closed and F_C ⊆ F_b) = 0 (F_C : F_C is soft open and F_C ⊆ F_b) = s-int(F_b).

(ii) (s-int(F_b))' = (0 \ F_C : F_C is soft semi-closed and F_C ⊆ F_b)' = 0 (F_C : F_C is soft semi-closed and F_C ⊆ F_b)' = s-cl(F_b).

**Theorem 3.19.** Let (F_A, ℰ) be a soft topological space and let F_B and F_C be soft sets in F_A. Then the following hold.

(i) s-cl(F_b) = s-cl(F_b) and s-cl(F_A) = F_A.

(ii) F_B is soft semi-closed set if and only if F_B = s-cl(F_b).

(iii) s-cl(s-cl(F_b)) = s-cl(F_b).

(iv) F_B ⊆ F_C implies s-cl(F_B) ⊆ s-cl(F_C).

(v) s-cl(F_B \ F_C) = s-cl(F_B) \ s-cl(F_C).

(vi) s-cl(F_B \ F_C) = s-cl(F_B) \ s-cl(F_C).

**Proof.** (i) is obvious.

(ii) If F_B is soft semi-closed set, then F_B is a soft semi-closed set in F_A which contains F_b. So s-cl(F_b) is the smallest soft semi-closed set containing F_B and F_B = s-cl(F_b). Conversely, suppose that F_B = s-cl(F_b). Since s-cl(F_b) is a soft semi-closed set, F_B is a soft semi-closed set.

(iii) Since s-cl(F_b) is a soft semi-closed set therefore by part (ii)

(iv) Suppose that F_B ⊆ F_C. Then every soft semi-closed super set of F_C will also contain F_B. This means every soft semi-closed super set of F_C is also a soft semi-closed super set of F_B. Hence the soft intersection of soft semi-closed super sets of F_B is contained in the soft intersection of soft semi-closed super sets of F_B. Thus s-cl(F_b) ⊆ s-cl(F_b).

(v) Since F_B \ F_C ⊆ F_B and F_B \ F_C ⊆ F_C, and so by part (iv) s-cl(F_B \ F_C) = s-cl(F_B) \ s-cl(F_C) and s-cl(F_B \ F_C) = s-cl(F_B) \ s-cl(F_C).

Thus s-cl(F_B \ F_C) = s-cl(F_B) \ s-cl(F_C).

(vi) Since F_B \ F_C \ F_B \ F_C \ F_B \ F_C. So by part (iv) F_B \ F_C implies s-cl(F_B \ F_C) = s-cl(F_B \ F_C). Then s-cl(F_b) \ F_c \ F_B \ F_C and s-cl(F_b) \ s-cl(F_b) = s-cl(F_b \ F_C), which is implies s-cl(F_b) \ s-cl(F_b \ F_C) = s-cl(F_b \ F_C). Now, s-cl(F_b) is belong to soft semi-closed set in F_A which implies that s-cl(F_b) is belong to soft semi-closed set in F_A. Then F_B \ F_C \ F_B \ F_C \ F_B \ F_C. Hence s-cl(F_B \ F_C) \ s-cl(F_b) \ s-cl(F_b).

**Theorem 3.20.** Let (F_A, ℰ) be a soft topological space and let F_B and F_C be soft sets in F_A. Then the following hold.

(i) s-int(F_b) = s-int(F_b) and s-int(F_A) = F_A.

(ii) F_B is soft semi-open set if and only if F_B = s-int(F_b).

(iii) s-int(s-int(F_b)) = s-int(F_b).

(iv) F_B ⊆ F_C implies s-int(F_B) ⊆ s-int(F_C).

(v) s-int(F_b) \ s-int(F_C) = s-int(F_b \ F_C).

(vi) s-int(F_b \ F_C) = s-int(F_b) \ s-int(F_C).

**Proof.** (i) is obvious.

(ii) If F_B is soft semi-open set, then F_B is itself a soft semi-open set in F_A which contains F_b. So s-int(F_b) is the largest soft semi-open set contained in F_B and F_B = s-int(F_b). Conversely, suppose that F_B = s-int(F_b). Since s-int(F_b) is a soft semi-open set, F_B is a soft semi-open set in F_A.

(iii) Since s-int(F_b) is a soft semi-open set therefore by part (ii)

(iv) Suppose that F_B ⊆ F_C. Since s-int(F_b) ⊆ F_B ⊆ F_C, s-int(F_b) is a soft semi-open subset of F_C, so by definition of s-int(F_c), s-int(F_b) ⊆ s-int(F_c).

(v) Since F_B ⊆ F_B \ F_C and F_C ⊆ F_B \ F_C, and so by part (iv) s-int(F_b) \ s-int(F_b \ F_C) and s-int(F_b) \ s-int(F_b \ F_C). So that s-int(F_b) \ F_C is a soft semi-open set.

(vi) Since F_B \ F_C \ F_B \ F_C. So by part (iv) F_B \ F_C implies s-int(F_b) \ s-int(F_b \ F_C). Then s-int(F_b) \ s-int(F_b \ F_C) and s-int(F_b) \ s-int(F_b \ F_C), which is implies s-int(F_b) \ s-int(F_b \ F_C) \ s-int(F_b \ F_C). Now, s-int(F_b) is belong to soft semi-open set in F_A which implies that s-int(F_b) is belong to soft semi-open set in F_A. Then F_B \ F_C \ F_B \ F_C \ F_B \ F_C. Hence s-int(F_b) \ s-int(F_b) \ s-int(F_b).

**Theorem 3.21.** Let (F_A, ℰ) be a soft topological space and let F_B be soft sets in F_A. Then the following hold.

(i) s-cl(cl(F_b)) = s-cl(F_b) \ s-cl(F_b).

(ii) s-int(int(F_b)) = s-int(F_b) \ s-int(F_b).

**Proof.** (i) Let (F_A, ℰ) be a soft topological space and because int(F_b) is soft open set, we have int(F_b) is soft semi-open set by Theorem 3.3. So we can get s-int(int(F_b)) = int(F_b) by Theorem 3.19(ii). By Theorem 3.17, we have int(F_b) ⊆ s-int(F_b) ⊆ F_B, then we can get int(F_b) ⊆ s-int(F_b) ⊆ int(F_b) and so int(F_b) = s-int(F_b). This completes the proof.
(ii) Because \( \text{cl}(F_B) \) is soft closed set, we have \( \text{cl}(F_B) \) is soft semi closed set by Theorem 3.9. So we can get s-
\( \text{int}(\text{int}(F_B)) = \text{int}(F_B) \) by Theorem 3.18(ii). By Theorem
3.17, we have \( F_B \subseteq s\text{-cl}(F_B) \subseteq \text{cl}(F_B) \), then we can get
\( \text{cl}(F_B) \subseteq \text{cl}(s\text{-cl}(F_B)) \subseteq \text{cl}(F_B) \) and so \( \text{cl}(s\text{-cl}(F_B)) = \text{cl}(F_B) \).
This completes the proof. □

**Theorem 3.22.** Let \((F_A, \tau)\) be a soft topological space and let \( F_B \)
be soft sets in \( F_A \). Then the following are equivalent.
(i) \( F_B \) is soft semi closed set.
(ii) \( \text{int}(\text{cl}(F_B)) \subseteq F_B \).
(iii) \( \text{cl}(\text{int}(F_B)) \subseteq F_B \).
(iv) \( F_B^\circ \) is soft semi-open set.

**Proof.** (i) ⇒ (ii) If \( F_B \) is soft semi-closed set, then there exist
soft closed set \( F_C \) such that \( \text{int}(F_C) \subseteq F_B \subseteq F_C = \text{int}(\text{cl}(F_C)) \subseteq F_B \).
By the property of interior, we have
\( \text{int}(\text{cl}(F_B)) \subseteq \text{int}(F_B) \subseteq F_B \).

(ii)⇒(iii) \( \text{int}(\text{cl}(F_B)) \subseteq F_B \Rightarrow F_B^\circ \subseteq \text{int}(\text{cl}(F_B)) = \text{cl}(\text{int}(F_B)) \subseteq F_B \).

(iii)⇒(iv) \( F_C = \text{int}(F_B^\circ) \) is an soft open set such that \( \text{int}(F_B^\circ) \subseteq F_B \subseteq \text{cl}(\text{int}(F_B^\circ)) \), hence \( F_B^\circ \) is soft semi-open set.

(iv)⇒(i) As \( F_B^\circ \) is soft semi-open set, there exists an soft open
set \( F_C \) such that \( F_C \subseteq F_B \subseteq \text{cl}(F_C) \Rightarrow \text{cl}(F_C) \subseteq F_B \) is a soft closed set such
that \( F_B \subseteq F_C \) and \( F_B \subseteq \text{cl}(F_C) \Rightarrow F_B \subseteq F_B \). Hence \( F_B \) is soft
semi-closed set. □

**REFERENCES**


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