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Monte Carlo Markov Chain algorithm: Application of a digital technique in the assessment of classroom psycho-social covariates of Physics achievement

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Abstract

This study investigated the application of Monte Carlo Markov Chain (MCMC) algorithm in the assessment of psycho-social covariates of achievement in Physics. The need for integration of MCMC algorithm and data during analysis of random variables is to ensure negligible standard error (SE) of estimation. Higher SE has been the bane of research results in most studies, hence the need for this study. The study adopted a fully Bayesian experimental design. The experimental nature of the design was in terms of manipulating the variances and error terms in the model until convergence was reached. The real population for the study was 4246 Physics students. The real sample for the study consisted of 206 SS3 Physics students sampled using multi-stage sampling from Igbo-Etiti Local Government Area. The instrument used to collect data was the Physics classroom environment questionnaire (PCEQ) and students' result pro-forma. The estimates of internal consistency (using Cronbach's alpha) of PCEQ ranged from .67 to .78 for its clusters while the overall estimate was .74. The data collected were analyzed using regression weights, trace plots and deviance information criterion values. The results indicated that: (i) the direct effects on Physics achievement included the paths from student-student interaction, teacher-student interaction, communication, order and organization. Satisfaction and task orientation had weak positive direct effects on the criterion. Involvement, teachers' control and innovations had direct negative effects on the criterion variable. (ii) the MCMC sample at models' convergence was 58031 (iii) the sub-models were invariant.

KEY WORDS: Monte Carlo Markov Chain, Psycho-social covariates, classroom and achievement.

Introduction

The contributions of Physics to solving myriads of socio-economic and political problems of any country cannot be over-emphasized. For instance, the development of the global positioning system (GPS) is rooted in the theory of relativity and the concept of atomic clocks. The navigation system has improved so much because of GPS in terms of its accuracy, speed, graphics and ease of use since its market debut, and has gained popularity resulting in skyrocketing demand (Ben, 2010). In some countries, Physics–based industries including photonics and nano-technology contribute meaningfully to a country's manufacturing sector specifically and the economy at large (The United Kingdom Institute of Physics, 2003). The knowledge of Physics is also important to be able to fit well in an ever changing society due to the advances made in science and technology. The knowledge of electric power is important especially to anyone who makes use of electricity to avoid electrocution, wastage of electric power and to minimize expenses in replacing burnt electrical appliances. Also, the knowledge of radiation Physics is necessary for users of telecommunications devices including computers and phones, to minimize over-exposure to electromagnetic radiations which pose health-hazards.

The knowledge of Physics is needed to boost the scientific literacy of the citizens of any nation. No wonder why the Australian national research council (2001) reported that Physics Education should have a goal of producing the society with broad scientific literacy at all levels, to be able to overcome the challenges posed to the environment and the society. The challenges which occasioned the change in the science education policy both at the global and local levels according to Weiman and Perkins (2006) included:

- Society now faces critical global issues that are fundamentally technical in nature. For example, climate change, genetic modification and energy supply. Only a far more scientifically and technologically literate citizens can make wise decisions on such issues.
- Modern economies are so heavily based on technology that having a better understanding of science and technology, and better problem solving skills will enhance a person's career aspirations, almost independent of occupation.
- A modern economy can thrive only if it has workforce with high level of technical understanding and skills (p.36).

The knowledge of Physics is needed in the construction of satellites, drones, cameras, videos, missiles, etc which can be used for surveillance purposes. The dreaded *boko haram* insurgence, militancy activities in the Niger-Delta and other terrorist groups in Nigeria can be controlled by using these technologies, which depend on the principles of Physics for their making and applications. It is clear from the above points that the knowledge of science of which is an integral part of is needed by all and sundry, if the course of socio-economic and political growth of any economy is to be sustained.

In recognition of the importance of science education, the Federal Government of Nigeria through its various policy statements including the science, technology and innovation policy made through the Federal Ministry of Science and Technology, FMS & T (2012) outlined the following policy frameworks for science, technology and innovation (ST&I) strategies at the primary and secondary school levels:

> (i).Encouraging relevant stakeholders to provide students in primary and secondary schools, as well as technical colleges with broad-based curricula comprising relevant scientific knowledge and vocational skills. (ii). Promoting broad-based curricular comprising relevant scientific and technological skills for schools and colleges…… (ix). Promoting inventions and innovations that address immediate local needs (p.31).

However, despite the importance of Physics for the well being of individuals and the society, the Nigerian secondary school Physics students have consistently achieved poorly in Physics external examinations. The registrars' reports for external examination bodies in Nigeria including National Business and Technical Examinations Board (NABTEB) (2010), West African Examinations Council (2012) and National Examinations Council (NECO) (2017) attest to the fact the students' achievements in Physics in recent times have been poor. Right from the 1970's to the present day, Nworgu (2016) pointed out that consistent poor achievement in the sciences (Physics included) has been a characteristic feature of pedagogical failure in Nigeria. Physics Education researchers including Nworgu (2016) have partly attributed students' poor achievements in the sciences to poor assessment practices and psycho-social variables nested within the Physics classroom (Abuh, 2014). Nworgu further observed that the use of instruments of poor psychometric qualities to collect educational data, including students' outcomes (like achievement) has been a common practice among science teachers. Such a practice has its attendant consequences. The use of instruments with poor psychometric qualities to collect data produces fake data. Also, the analysis of fake data produces fake results. Poor assessment practice can also manifest in the use of easy and conventional non-Bayesian technique of parameter estimation, including the maximum likelihood, which has relatively higher estimation errors, when compared to Bayesian estimation (Nworgu & Nnadi, 2017). Maximum likelihood is dependent on asymptotic distribution assumptions. The result got from maximum likelihood estimation is strictly based on the data and sample-size submitted for analysis. It does not simulate the data either the sample-size. Therefore, inferring the results got from non-Bayesian estimations (maximum likelihood included) to the wider target population becomes a relatively higher error-laden inference and therefore lacks transparency. Conventional parameter estimation techniques also known as the frequentist techniques are widely used because of their ease of use (Sharma, 2017). Sharma further reported that the conventional technique of parameter estimation fails when it is used to analyze complex, random and multi-dimensional relationships and integrals. Unfortunately for the conventional technique of parameter estimation, educational data are becoming complex, random and multi-dimensional in nature and therefore require a paradigm shift in estimation technique from the conventional (non-Bayesian) to Bayesian. The higher standard error of estimation common in the frequentist estimation technique is overcome by using Bayesian data analysis, which adopts Monte Carlo Markov Chain (MCMC) computer algorithms. In support for a technology-enabled assessment, the Department of Education, United States of America (n.d) has observed that it reduces time, resources and provides a more complete and nuanced picture of students' needs, interests and abilities than can traditional assessment. This means that the recent advances which culminated in the birth of MCMC was designed to produce a reliable (almost zero standard error) estimate. In MCMC the stochastic dependence of the Markov chain reduces the standard error of estimation (Geyer, 1990). Therefore, the reduction of the standard error of estimation arising from the use of MCMC increases the reliability of posterior parameter estimates. MCMC and Bayesian statistics are two independent disciplines, the former being a method to sample from a distribution, while the latter is a theory to interpret observed data (Sharma,2017). Sharma further reported that and when the two disciplines are combined, the effect is so dramatic and powerful that it revolutionized data analysis in most disciplines of science and arts alike. The Bayesian theorem, propounded by Rev Thomas Bayes (1763 and published post-humously by Price) was used during the second world-war at Bletchley park, United Kingdom to crack the German enigma code. In support for a paradigm shift from non-Bayesian to Bayesian inference for testing hypothesis, the American Statistical Association, ASA (2016) issued a warning on the misuse of statistical p-value by researchers. ASA reported that p-value:

> (i).can indicate how incompatible the data are with specified statistical model.(ii).does not measure the probability that the studied hypothesis is true, or the probability that the data were produced by random chance alone.(iii). Scientific conclusions and business or policy decisions should not be based only on whether a p-value passes a specified threshold.(iv). Proper inference requires full reporting and transparency.(v).or statistical significance, does not measure the size of an effect or the importance of a result.(vi).does not provide a good measure of evidence regarding a model or hypothesis (p.8-11).

The lesson that can be learnt from the above quotation is that the statistical p-value which hitherto has become a conventional statistic for testing hypothesis is a frequentist (non-Bayesian) parameter approximation. It lacks the credibility of being generalized unto the target population because its value is sample-dependent. Therefore, the need has arisen for an inference system which is capable to provide a reliable measure of evidence in respect of a specified target population.

In Bayesian inference, the frequentist approach is not completely thrown away. Rather the collected data and the prior probability distribution (which represents the results of previous data analysis) are blended together to form the posterior distribution with the aim of heightening the objectivity level of the posterior estimates through randomization (Nnadi, 2017).

The psycho-social climate within the Physics classroom relate to those psychological and socially related variables within the Physics classroom. Herbert (2004) reported that psycho-social classroom climate influenced learning outcomes. The psycho-social variables of Physics classroom environment including involvement, student-student interaction, teacher-student interaction, satisfaction, task orientation, order and organization, teacher control and innovation meaningfully influenced achievement in Physics (Abuh, 2017). However, the effect decompositions of the psycho-social variables on Physics achievement has not been previously sought using Bayesian estimation, specifically the Hamiltonian algorithm of MCMC. It is on this premise that the researchers deemed it fit to develop and test-run a structural model of psycho-social variables nested within the Physics classroom using the stated Bayesian MCMC estimation algorithm and data to determine their direct effects on Physics achievement. Put in question form, what are the direct effects of Physics classroom environment sub-scales on Physics achievement?

Purpose of the study

The study was designed to: (i) determine the direct effects of the Physics classroom environment sub-scales on Physics achievement in the most meaningful model (ii) determine the MCMC sample used in the computation of the posterior estimates in the model and (iii) determine if any significant difference existed between the constrained and unconstrained models' deviance information criterion values based on students' gender.

Research Questions

Two research questions guided the study. They included (i) What are the direct effects of the Physics classroom environment sub-scales on Physics achievement in the most meaningful model? (ii). What is the MCMC sample used in the computation of the posterior estimates in the model?

Hypothesis

The study was also guided by one null hypothesis tested at 95% confidence interval:

H_{o1}: There is no significant difference between the constrained and unconstrained models' deviance information criterion values based on students' gender.

Research Method

The design of the study was a fully Bayesian experimental design. The experimental nature of the design was in terms of manipulating the variables in the model by imposing some constraints on the model's parameters until a working model was produced. The population for the study consisted of four thousand, two hundred and forty six senior secondary three Physics students nested within 208 (Igbo-Etiti,54; Nsukka, 98 & Uzo-Uwani,56) public secondary schools in Nsukka Education zone of Enugu state (Ministry of Education Enugu, 2010). The sample for the study consisted of 206 SS3 Physics students from Igbo-Etiti Local Government Area. The sampling technique adopted was multi-stage. Stage one involved using purposive sampling to sample Igbo-Etiti Local Government Area in Nsukka education zone of Enugu state. The reason for purposively sampling Igbo-Etiti was because of convenience. Stage two involved the use of simple random sampling, specifically balloting with replacement to sample 5 schools out of 54 public secondary schools in the area. Stage three involved the use of purposive sampling to sample SS3 class out of other classes in the sampled schools. SS3 class was used because their external examinations in Physics were set by statutory examination bodies in Nigeria and as such were standardized. Finally, in school that had more than one stream of science class, simple random sampling, specifically balloting with replacement was used to sample only one intact class. The instrument used to collect data in this study included Physics classroom environment questionnaire (PCEQ). PCEQ consisted of 9 sub-scales and 36 manifest variables. The original instrument, Physics classroom environment scale questionnaire (PCESQ) was adopted from Abuh (2014). It consisted of 9 sub-scales with 41 manifest variables. 5 manifest variables were dropped on the basis of having poor psychometric qualities. The sub-scales included: innovation, student-student interaction, teacher-student interaction, satisfaction and task orientation. Others included competition, order and organization, teachers' control and innovation. The internal consistency of the original questionnaire (PCESQ) ranged between .46 to .72 whereas the Chronbach's alpha reliability for the whole questionnaire items was .60. The alpha values of PCEQ ranged between .67 to .78, while the overall alpha was .74 after the trial test. Also, the confirmatory factor analysis result of the exploratory factor analysis, obtained using maximum likelihood estimation indicated that the correlation between the sub-scales of PCEQ ranged between .119 to .693. The values indicated that the sub-scales of PCEQ positively correlated, indicating that they measured same underlying construct (Physics classroom environment). The Physics students' 2016/2017 senior secondary school results (WAEC and NECO) formed the achievement part of the study.

MCMC Experimental Procedures

The MCMC experimental procedures adopted in this study had two phases: preliminary and main phases. The preliminary phase involved the use of Physics classroom environment scale questionnaire (PCESQ with 9 latent and 41 manifest variables) by two research assistants to collect data from Physics students nested within two schools (1 urban and 1 rural) located outside the population for the study. The research assistants also collected the Physics students' 2016/2017 WAEC and NECO results from the official school records when they were released. Initially, one hundred copies of PCESQ were given to the students during the trial testing period. However, due to unengaged responses, more than 10% of missing data in the dataset and seizure or withholding of candidates' SSCE results, the trial sample thinned down to 54. The data collected were exposed to exploratory factor analysis (EFA). The principal component method was adopted in extracting the factors, in addition to the use of rotated factor solution as the display format to ensure sufficient correlation of the factors. Factors were extracted based on ten fixed latent factors at 50 iterations. The nine latent variables originally present in PCESQ were extracted. However, only 36 manifest variables loaded distinctly on one latent variable at a time. So, the 5 manifest variables in PCESQ whose coefficients were either below the set limit of .35 or that loaded on more than one latent variable at a time were not included in the main simulation experiment.

Phase two of the experimental procedure involved model specification, evaluation, identification and modification. Model specification adopted involved the use of symbols, arrows and curves to represent the manifest, latent and error terms; directional effects and correlation/covariance matrix of the exogenous variables. The measurement model was first specified before structural model specification. Model constraints including configural invariance (the same factor loading in each sub-group model was equated to 1), metric invariance (configural invariance + setting all the other factor loadings in each subgroup to be equal) and structural invariance (metric invariance + setting all the covariance curves to be equal across the subgroups) testing were done to determine the equivalence of male and female sub-group models in producing a unified model. The data were loaded and the models' parameters were initially estimated using maximum likelihood (ML) to help diagnose estimation problems. The structural model did not run despite that it was identified (degrees of freedom of model was greater than zero). The variance of the error term (e37) on Physics achievement in the structural model had a negative value of -.222 and therefore was constrained to a positive value of .02 as part of modification exercise and the model ran.

However, the correlation indices between the latent variables were very low. In addition, some factor loadings had outrageous linear coefficients (above 1). To have a properly identified model, the variances of all the exogenous latent variables were constrained to 1, allowing all the factor loadings within each latent variable to freely vary. The model was run the third time using ML, and all the coefficients of the directional effects were reasonable (had linear coefficients ranging from -1 through 0 to 1). The final estimation of the structural model was done using Bayesian MCMC algorithm, specifically Hamiltonian. Hamiltonian algorithm was chosen because it saved computational time relative to *random-walk* algorithm. During the MCMC sampling process of both the posterior parameter values and the MCMC sample-size, the prior distribution of each parameter was set to normal probability distribution. The reason was to reduce the model parameters' computation complexities as the data collected were tested to be multivariate normal. However, the model did not run using Bayesian estimation. An error massage: waiting to accept a transition before beginning burn-in cropped-up. The implication of the massage was that the maximum likelihood estimation was inadmissible on the prior tab of the Bayesian SEM window. The problem was attributed to the long run of the rejected MCMC candidates at the beginning of estimation cycle. So, the software (Amos version 22) discarded every sample until it first accepted the first MCMC candidate before the Bayesian estimation could run. The admissibility test box was therefore unchecked and the model was run again. It ran.

Results

The results are presented according to the formulated research questions and hypotheses that guided the study.

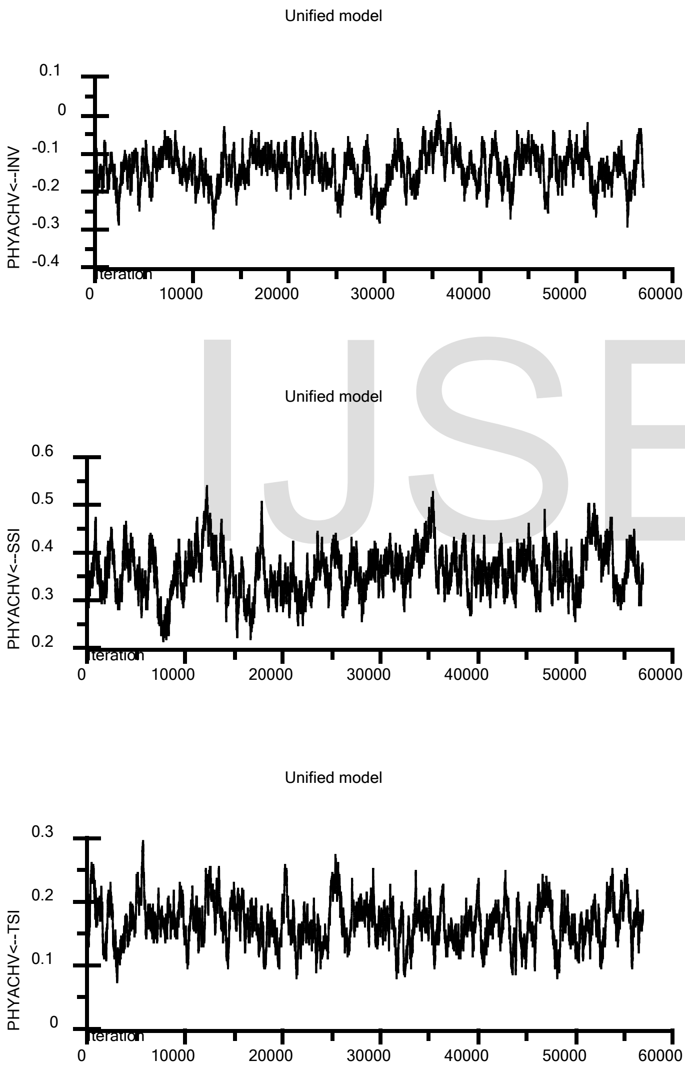
Research question 1 (RQ1) sought information on the direct effects of the Physics classroom environment sub-scales on Physics achievement in the most meaningful model. The data presented in Table 1 were used to answer RQ1.

Table 1: Direct effects of the Physics classroom environment sub-scales on Physics achievement in the most meaningful model

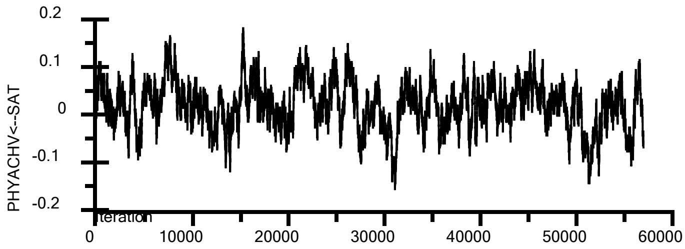
Regression weight	Mean of path	Standard Error	Weighted path coefficient	Interpretation
PHYACHV<--INV	-.15	.01	-.16	Negative
PHYACHV<--SSI	.36	.00	.36	Positive
PHYACHV<--TSI	.16	.00	.16	Positive
PHYACHV<--SAT	.02	.00	.02	Weakly positive
PHYACHV<--TAS	.03	.00	.03	Weakly positive
PHYACHV<--COM	.11	.00	.11	Positive
PHYACHV<--ODO	.21	.00	.21	Positive
PHYACHV<--TCO	-.06	.01	-.07	Negative
PHYACHV<--INN	-.32	.00	-.32	Negative

From Table 1, (extracted from Appendix A, p.) the positive direct effects on Physics achievement in the model included the paths from student-student interaction, SSI (.36), order and organization, ODO (.21), teacher-student interaction, TSI (.16) and competitions, COM (.11). However, satisfaction, SAT (.02) and task orientation, TAS (.03) have weak positive direct effect on Physics achievement. The negative direct effects on Physics achievement in the model included teachers' control, TCO (-.06), involvement, INV (-.15) and innovations, INN (-.32).

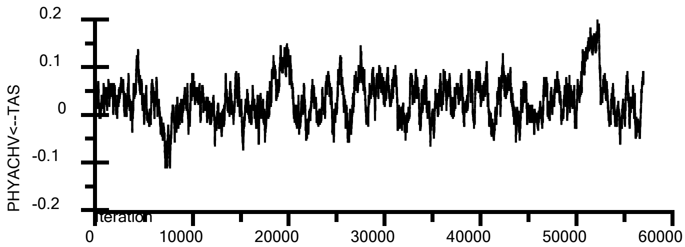
Research question 2 (RQ2) sought information on the MCMC sample used in the computation of the posterior estimates in the model. The trace plots in figure 2 were used to answer RQ2.



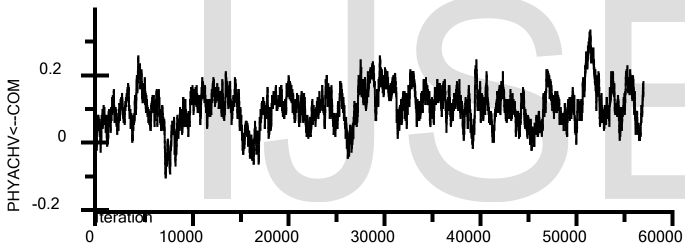
Unified model



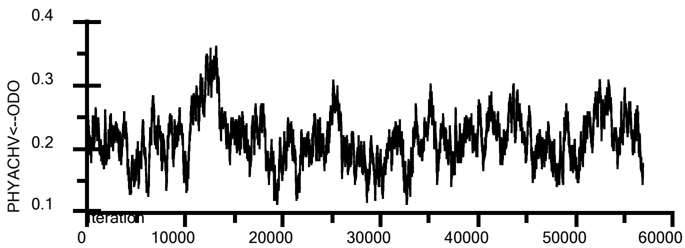
Unified model

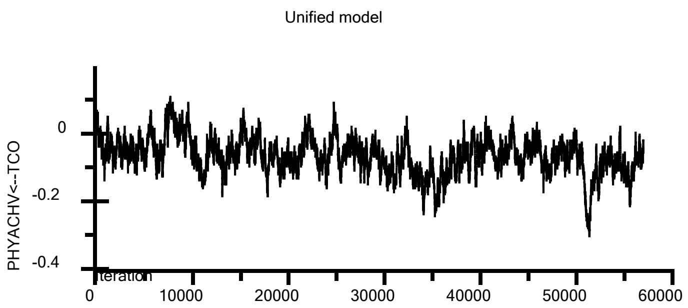


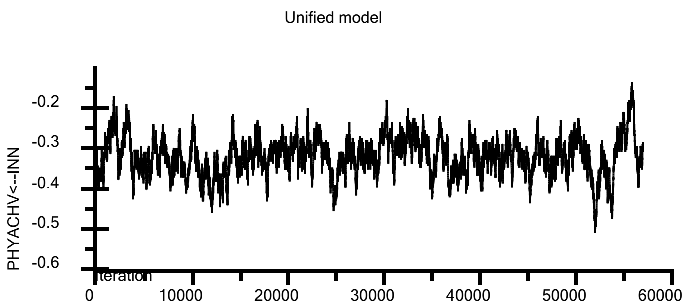
Unified model



Unified model







The trace plot shows the path coefficient of each sub-scale of Physics classroom environment at its point of convergence. It is plotted on the vertical axis, whereas the iteration (MCMC sample) used in the computation of the posterior parameter estimates for the unified model is represented on the horizontal axis. For the paths from involvement to Physics achievement with convergence of -.15 on PHYACHV<--INV axis, the iteration level was 58,031. The same iteration level of 58,031 was reached for other paths to Physics achievement including student-student interaction with convergence of .36, teacher-student interaction (.16), satisfaction (.02), task orientation (.03), competition (.11), order and organization (.21), teachers' control (-.06) and innovation (-.32). The iteration level of 58,031 for each estimate in the model provided empirical evidence regarding the number of MCMC sample used for calculating the posterior estimates.

Hypothesis 1 (H_o1) sought to determine if any significant difference existed between the constrained and unconstrained models' deviance information criterion values based on students' gender. The data presented in Table 2 were used to test the hypothesis.

Table 2: Deviance information criterion (DIC) values of the constrained and unconstrained male and female models tested at 95% confidence interval.

Gender	**Constrained Model's DIC**	**Unconstrained Model's DIC**	**Differences**		**Interpretation**
			(a-d)	**(c-b)**	
Male	167.34 **(a)**	121.31**(b)**		56.03	Not significantly different
Female	177.34 **(c)**	111.24 **(d)**	56.10		

From Table 2, the sub-model for male Physics students had a constrained model's DIC value of 167.34 while the female sub-model had a DIC value of 177.34. For the unconstrained models, the male sub-model had a DIC value of 121.31 whereas the female sub-model had a DIC value of 111.24. The difference between constrained and unconstrained male and female models is 56.10, while the difference between the constrained and unconstrained female and male sub-models is 56.03. This implies that the male and female sub-models are invariant and their combination to produce a unified model is statistically reasonable.

Discussion of Findings

The results in Table 1 provided answer to the research question 1. Out of the nine sub-scales of Physics classroom environment questionnaire, the highest positive direct effect on the criterion variable was the path from student-student interaction. The next sub-scales in decreasing magnitude of meaningful paths included order and organization, teacher-student interaction and competitions. However, teacher satisfaction and task orientation had weak and positive direct effects on the criterion variable. Other variables including teachers' control, involvement and innovations had negative direct effects on the criterion variable. Teachers' control had the highest negative direct effect on Physics achievement. Involvement followed and innovation was the least. The weighted path coefficient from involvement and teachers' control to Physics achievement varied slightly by -.01 each. All the other weighted paths did not vary from the estimated path coefficients. The implication for the result is that MCMC based approach to parameter estimation provides reliable estimates due to the fact that it records very low standard error of estimates. From the Table 1, it becomes apparent that the level of students satisfaction and task orientation in the population is low and needs improvement. Other clusters of Physics classroom environment scale that needs serious intervention included the ones with negative direct effects in the model. They are paths from involvement, teachers' control and innovation. However the statuesque should be maintained for student-student interaction, teacher-student interaction, communication and order and organization within the Physics classroom.

The trace plots in figure 2 provided answer to the research question 2. The size of MCMC sample that was utilized in the computation of the posterior estimates, which corresponded to the iteration level was fifty-eight thousand and thirty one. The target population for the study was four thousand, two hundred and forty six senior secondary three Physics students. Since the MCMC sample is larger than the target population, the result of this study can be generalized on the target population. Hence, the technique of data analysis, which combines data and prior distribution in the estimation of the posterior values of parameter estimates is used to overcome sample-size induced error in making generalization to the target population.

From Table 2, the difference between the constrained male and unconstrained female models was positive. Also, the difference between the constrained female and unconstrained male models was positive. This shows that the sub-group models were equivalent. This result is in line to the recommendation made by Zang, Hamagami,Wang, Nesselroade, and Grimm

(2007). The authors noted that sub-group models' equivalence was achieved when the DIC of the constrained model was higher than the DIC value of the unconstrained model.

Recommendations

Based on the findings of this study the following recommendations are made. Stakeholders in the Education sector should work towards improving the level of students' involvement, satisfaction, task orientation, teachers control and innovations within the Physics classroom. MCMC method of data analysis should be employed by researchers in Physics Education to ensure negligible standard error of parameter estimation. The invariant model should be used by the government to make policies concerning Physics Education.

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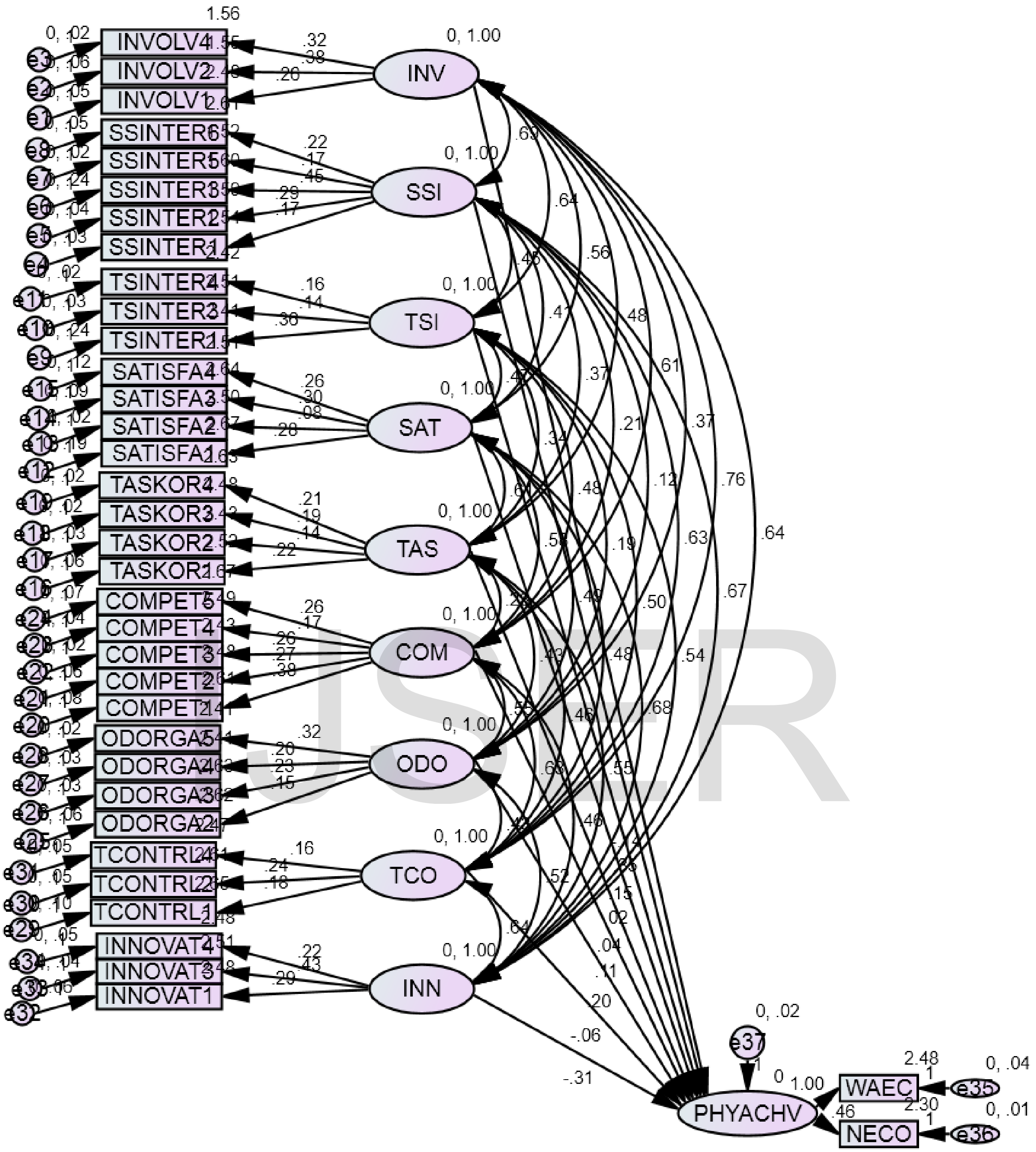
Appendix A: Maximum Likelihood Estimation Trial Result



KEY: INV-involvement, SSI-student-student interaction, TSI-teacher-student interaction, SAT-satisfaction, TAS-task orientation, COM-competition, ODO-order and organization, TCO-teachers' control, INN-innovations and PHYACHV-physics achievement.

Appendix B: Full Posterior Summaries of Estimates.

	Mean	S.E.	S.D.	C.S.	Median	95% Lower bound	95% Upper bound	Skew ness	Kurtosis	Min	Max	Name
Regression weights												
INVOLV1<--INV	0.26	0.00	0.02	1.00	0.26	0.22	0.30	0.22	-0.10	0.20	0.33	
INVOLV2<--INV	0.38	0.00	0.03	1.00	0.38	0.33	0.43	0.29	-0.05	0.29	0.47	
INVOLV4<--INV	0.32	0.00	0.02	1.00	0.32	0.29	0.36	0.31	0.03	0.26	0.40	
SSINTER1<--SSI	0.17	0.00	0.01	1.00	0.17	0.14	0.20	0.09	-0.45	0.13	0.22	
SSINTER2<--SSI	0.29	0.00	0.02	1.00	0.29	0.25	0.34	0.14	0.14	0.22	0.36	
SSINTER3<--SSI	0.45	0.01	0.05	1.01	0.45	0.36	0.55	0.02	0.00	0.29	0.61	
SSINTER5<--SSI	0.18	0.00	0.01	1.01	0.18	0.15	0.21	0.28	0.17	0.13	0.23	
SSINTER6<--SSI	0.22	0.00	0.02	1.00	0.22	0.18	0.26	0.14	-0.03	0.16	0.29	
TSINTER1<--TSI	0.36	0.00	0.04	1.00	0.36	0.27	0.45	0.07	0.04	0.20	0.52	
TSINTER3<--TSI	0.15	0.00	0.02	1.00	0.15	0.11	0.18	-0.02	0.15	0.08	0.21	
TSINTER4<--TSI	0.16	0.00	0.02	1.00	0.16	0.13	0.19	0.05	0.26	0.10	0.23	
SATISFA1<--SAT	0.28	0.00	0.04	1.00	0.28	0.20	0.36	-0.02	0.16	0.15	0.41	
SATISFA2<--SAT	0.08	0.00	0.01	1.00	0.08	0.06	0.11	-0.05	-0.31	0.05	0.12	
SATISFA3<--SAT	0.30	0.00	0.03	1.00	0.30	0.25	0.37	0.26	-0.12	0.20	0.40	
SATISFA4<--SAT	0.26	0.00	0.03	1.01	0.26	0.20	0.32	0.02	0.24	0.15	0.37	
TASKOR1<--TAS	0.23	0.00	0.02	1.01	0.23	0.19	0.28	0.14	-0.09	0.16	0.32	
TASKOR2<--TAS	0.14	0.00	0.02	1.01	0.14	0.12	0.18	0.19	-0.27	0.09	0.19	
TASKOR3<--TAS	0.20	0.00	0.02	1.00	0.20	0.17	0.23	0.06	-0.12	0.15	0.26	
TASKOR4<--TAS	0.22	0.00	0.02	1.00	0.22	0.19	0.25	0.14	0.04	0.17	0.27	
COMPET1<--COM	0.38	0.00	0.03	1.00	0.38	0.33	0.45	0.29	0.07	0.29	0.49	
COMPET2<--COM	0.27	0.00	0.02	1.01	0.27	0.23	0.32	0.12	0.06	0.20	0.35	
COMPET3<--COM	0.26	0.00	0.02	1.01	0.26	0.23	0.29	0.07	0.00	0.21	0.31	
COMPET4<--COM	0.17	0.00	0.02	1.01	0.17	0.14	0.21	0.40	0.58	0.12	0.25	
COMPET5<--COM	0.27	0.00	0.03	1.01	0.27	0.22	0.32	0.24	1.05	0.18	0.38	
ODORGA2<--ODO	0.15	0.00	0.02	1.01	0.15	0.11	0.19	0.11	-0.33	0.09	0.22	
ODORGA3<--ODO	0.23	0.00	0.02	1.01	0.23	0.20	0.27	0.32	0.12	0.18	0.30	
ODORGA4<--ODO	0.20	0.00	0.01	1.00	0.20	0.17	0.23	0.23	-0.07	0.16	0.25	
ODORGA5<--ODO	0.33	0.00	0.02	1.00	0.33	0.29	0.36	0.03	-0.22	0.26	0.39	
TCONTRL1<--TCO	0.18	0.00	0.03	1.00	0.18	0.13	0.24	0.14	-0.04	0.09	0.29	
TCONTRL2<--TCO	0.24	0.00	0.03	1.00	0.24	0.19	0.29	0.21	-0.02	0.16	0.32	
TCONTRL4<--TCO	0.16	0.00	0.02	1.00	0.16	0.12	0.20	0.32	-0.04	0.10	0.24	
INNOVAT1<--INN	0.29	0.00	0.02	1.00	0.29	0.25	0.34	0.12	0.40	0.22	0.39	
INNOVAT3<--INN	0.42	0.00	0.03	1.00	0.42	0.37	0.48	0.07	-0.29	0.34	0.51	
INNOVAT4<--INN	0.22	0.00	0.02	1.01	0.22	0.19	0.26	-0.07	-0.11	0.16	0.29	
NECO←PHYACHV	0.44	0.01	0.05	1.01	0.44	0.35	0.57	0.67	1.30	0.29	0.65	

PHYACHV<--INV	-0.15	0.01	0.05	1.00	-0.14	-0.27	-0.06	-0.91	2.26	-0.46	0.01
PHYACHV<--SSI	0.36	0.00	0.05	1.00	0.36	0.26	0.49	0.52	0.84	0.17	0.58
PHYACHV<--TSI	0.16	0.00	0.04	1.00	0.16	0.09	0.24	0.23	0.36	0.03	0.32
PHYACHV<--SAT	0.02	0.00	0.05	1.00	0.02	-0.08	0.11	-0.15	0.21	-0.16	0.18
PHYACHV<--TAS	0.03	0.00	0.04	1.00	0.03	-0.04	0.12	0.36	0.55	-0.11	0.20
PHYACHV<--COM	0.11	0.00	0.05	1.00	0.11	0.00	0.22	0.28	1.10	-0.11	0.34
PHYACHV<--ODO	0.21	0.00	0.04	1.00	0.21	0.14	0.30	0.60	0.88	0.10	0.39
PHYACHV<--TCO	-0.06	0.01	0.06	1.01	-0.06	-0.19	0.04	-0.65	1.56	-0.33	0.12
PHYACHV<--INN	-0.32	0.00	0.05	1.00	-0.32	-0.43	-0.23	-0.59	1.71	-0.62	-0.14

Intercepts

INVOLV1	2.46	0.00	0.02	1.00	2.47	2.42	2.51	0.00	-0.26	2.39	2.55
INVOLV2	1.53	0.00	0.03	1.01	1.53	1.47	1.59	0.06	0.05	1.43	1.64
INVOLV4	1.54	0.00	0.03	1.01	1.55	1.49	1.59	-0.21	-0.06	1.46	1.63
SSINTER1	1.53	0.00	0.02	1.01	1.53	1.50	1.57	0.00	-0.35	1.48	1.59
SSINTER2	1.58	0.00	0.03	1.00	1.58	1.53	1.63	-0.02	0.18	1.49	1.66
SSINTER3	1.58	0.00	0.05	1.00	1.58	1.48	1.67	-0.04	-0.31	1.40	1.72
SSINTER5	1.51	0.00	0.02	1.00	1.51	1.48	1.54	-0.02	0.14	1.45	1.57
SSINTER6	2.60	0.00	0.02	1.00	2.60	2.55	2.64	-0.18	-0.12	2.52	2.68
TSINTER1	2.39	0.00	0.04	1.01	2.39	2.30	2.48	-0.06	0.01	2.24	2.53
TSINTER3	2.50	0.00	0.02	1.01	2.50	2.47	2.54	-0.13	-0.21	2.45	2.55
TSINTER4	2.41	0.00	0.02	1.01	2.41	2.38	2.44	-0.23	0.54	2.34	2.47
SATISFA1	2.65	0.00	0.04	1.00	2.65	2.58	2.72	-0.07	-0.25	2.52	2.77
SATISFA2	2.50	0.00	0.01	1.00	2.50	2.47	2.52	0.00	-0.08	2.45	2.53
SATISFA3	2.62	0.00	0.03	1.00	2.62	2.57	2.68	0.23	0.02	2.54	2.73
SATISFA4	2.49	0.00	0.03	1.00	2.49	2.43	2.55	-0.32	0.60	2.36	2.58
TASKOR1	2.51	0.00	0.02	1.00	2.51	2.46	2.55	0.03	-0.23	2.43	2.58
TASKOR2	2.42	0.00	0.01	1.01	2.42	2.39	2.45	0.02	-0.05	2.38	2.47
TASKOR3	2.47	0.00	0.02	1.00	2.47	2.44	2.50	-0.18	-0.18	2.41	2.53
TASKOR4	2.62	0.00	0.02	1.00	2.62	2.58	2.65	-0.06	-0.04	2.55	2.68
COMPET1	2.59	0.00	0.04	1.00	2.59	2.52	2.66	0.15	0.07	2.47	2.71
COMPET2	2.47	0.00	0.03	1.00	2.47	2.42	2.52	0.21	-0.27	2.40	2.55
COMPET3	2.42	0.00	0.02	1.00	2.42	2.37	2.46	0.01	-0.09	2.35	2.49
COMPET4	2.48	0.00	0.02	1.00	2.48	2.45	2.52	0.30	0.01	2.42	2.55
COMPET5	2.65	0.00	0.03	1.00	2.65	2.60	2.70	0.09	-0.08	2.57	2.74
ODORGA2	2.61	0.00	0.02	1.00	2.61	2.57	2.65	0.21	0.08	2.55	2.69
ODORGA3	2.62	0.00	0.02	1.00	2.62	2.58	2.66	0.11	0.13	2.55	2.71
ODORGA4	2.40	0.00	0.02	1.00	2.40	2.36	2.44	0.09	-0.10	2.34	2.46
ODORGA5	2.40	0.00	0.03	1.00	2.40	2.35	2.45	0.02	-0.12	2.32	2.49
TCONTRL1	2.63	0.00	0.03	1.00	2.63	2.58	2.68	-0.11	0.17	2.54	2.72

TCONTRL2	2.60	0.00	0.03	1.00	2.60	2.55	2.64	-0.06	-0.21	2.51	2.68
TCONTRL4	2.46	0.00	0.02	1.00	2.46	2.42	2.50	0.13	0.11	2.40	2.54
INNOVAT1	2.47	0.00	0.03	1.00	2.47	2.41	2.52	-0.08	-0.16	2.37	2.56
INNOVAT3	2.49	0.00	0.03	1.00	2.49	2.43	2.55	-0.01	-0.02	2.39	2.59
INNOVAT4	2.47	0.00	0.02	1.01	2.47	2.42	2.52	-0.04	-0.13	2.39	2.55
WAEC	2.47	0.00	0.03	1.00	2.47	2.42	2.52	0.00	0.00	2.37	2.55
NECO	2.30	0.00	0.01	1.00	2.29	2.27	2.32	0.22	-0.10	2.26	2.34

Covariances

INV<->SSI	0.66	0.00	0.05	1.00	0.66	0.55	0.74	-0.42	-0.03	0.48	0.81
INV<->TSI	0.61	0.01	0.06	1.01	0.61	0.50	0.73	0.04	0.02	0.41	0.82
INV<->SAT	0.50	0.01	0.08	1.01	0.51	0.34	0.64	-0.32	-0.03	0.21	0.73
INV<->TAS	0.43	0.01	0.07	1.00	0.43	0.29	0.56	-0.33	0.45	0.13	0.63
INV<->COM	0.57	0.01	0.05	1.01	0.57	0.46	0.67	-0.27	-0.06	0.39	0.74
INV<->ODO	0.33	0.01	0.08	1.00	0.34	0.17	0.47	-0.25	0.14	0.07	0.57
INV<->TCO	0.72	0.01	0.06	1.00	0.72	0.59	0.84	-0.07	0.63	0.47	0.99
INN<->INV	0.59	0.01	0.06	1.01	0.59	0.47	0.69	-0.31	0.05	0.38	0.75
SSI<->TSI	0.41	0.01	0.08	1.00	0.41	0.24	0.58	0.00	0.36	0.09	0.74
SSI<->SAT	0.38	0.01	0.09	1.01	0.38	0.20	0.54	-0.13	-0.11	0.09	0.64
SSI<->TAS	0.35	0.01	0.08	1.01	0.35	0.19	0.49	-0.33	0.15	0.06	0.56
SSI<->COM	0.16	0.01	0.07	1.00	0.16	0.01	0.30	-0.01	-0.11	-0.07	0.41
SSI<->ODO	0.09	0.01	0.08	1.00	0.09	-0.08	0.24	-0.20	0.12	-0.18	0.33
SSI<->TCO	0.60	0.01	0.07	1.01	0.60	0.45	0.74	-0.16	0.15	0.32	0.84
INN<->SSI	0.64	0.01	0.05	1.01	0.64	0.54	0.73	-0.10	-0.11	0.46	0.79
TSI<->SAT	0.42	0.01	0.09	1.01	0.43	0.24	0.58	-0.24	-0.03	0.14	0.71
TSI<->TAS	0.32	0.01	0.08	1.00	0.32	0.15	0.46	-0.13	0.21	0.03	0.63
TSI<->COM	0.44	0.01	0.08	1.01	0.45	0.29	0.58	-0.16	-0.26	0.19	0.65
TSI<->ODO	0.15	0.01	0.09	1.01	0.15	-0.03	0.32	0.07	-0.03	-0.17	0.50
TSI<->TCO	0.46	0.01	0.09	1.00	0.46	0.30	0.63	-0.02	-0.51	0.19	0.70
INN<->TSI	0.51	0.01	0.07	1.01	0.50	0.38	0.64	0.08	-0.48	0.31	0.73
SAT<->TAS	0.59	0.01	0.07	1.00	0.59	0.43	0.71	-0.37	-0.01	0.33	0.79
SAT<->COM	0.55	0.01	0.07	1.01	0.56	0.41	0.68	-0.20	-0.28	0.32	0.77
SAT<->ODO	0.48	0.01	0.07	1.01	0.48	0.33	0.60	-0.20	-0.15	0.24	0.71
SAT<->TCO	0.44	0.01	0.09	1.00	0.44	0.28	0.61	0.00	-0.31	0.12	0.74
INN<->SAT	0.65	0.01	0.06	1.01	0.65	0.51	0.76	-0.40	0.12	0.39	0.83
TAS<->COM	0.18	0.01	0.08	1.01	0.18	0.02	0.34	-0.10	-0.09	-0.11	0.45
TAS<->ODO	0.40	0.01	0.08	1.01	0.40	0.24	0.53	-0.41	0.10	0.11	0.59
TAS<->TCO	0.44	0.01	0.08	1.01	0.44	0.27	0.59	-0.23	-0.20	0.17	0.70
INN<->TAS	0.52	0.00	0.06	1.00	0.52	0.41	0.64	-0.11	0.07	0.24	0.71
COM<->ODO	0.52	0.01	0.06	1.01	0.53	0.40	0.63	-0.25	0.02	0.29	0.71

COM<->TCO	0.60	0.01	0.07	1.01	0.60	0.46	0.72	-0.29	-0.13	0.35	0.79
INN<->COM	0.41	0.01	0.07	1.01	0.42	0.28	0.54	-0.14	-0.21	0.18	0.61
ODO<->TCO	0.40	0.01	0.09	1.01	0.41	0.21	0.56	-0.31	-0.24	0.11	0.66
INN<->ODO	0.50	0.01	0.06	1.00	0.50	0.37	0.62	-0.14	-0.18	0.30	0.71
INN<->TCO	0.61	0.01	0.08	1.00	0.62	0.44	0.74	-0.33	0.16	0.36	0.87

Variances

e1	0.05	0.00	0.01	1.00	0.05	0.04	0.06	0.45	0.01	0.03	0.07
e2	0.06	0.00	0.01	1.00	0.06	0.05	0.08	0.24	0.01	0.04	0.09
e4	0.03	0.00	0.00	1.00	0.03	0.02	0.04	0.43	0.27	0.02	0.04
e6	0.26	0.00	0.03	1.01	0.25	0.21	0.33	0.62	0.72	0.17	0.39
e7	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.37	-0.14	0.02	0.03
e8	0.06	0.00	0.01	1.00	0.06	0.05	0.07	0.12	0.02	0.04	0.08
e10	0.03	0.00	0.00	1.00	0.03	0.03	0.04	0.16	0.07	0.02	0.05
e11	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.24	-0.21	0.01	0.04
e12	0.20	0.00	0.02	1.00	0.19	0.16	0.25	0.60	0.60	0.14	0.29
e13	0.02	0.00	0.00	1.00	0.02	0.02	0.02	0.25	-0.25	0.01	0.03
e15	0.13	0.00	0.01	1.00	0.13	0.10	0.16	0.07	0.00	0.08	0.18
e16	0.06	0.00	0.01	1.00	0.06	0.05	0.07	0.34	0.31	0.04	0.09
e17	0.03	0.00	0.00	1.00	0.03	0.02	0.03	-0.06	0.13	0.02	0.04
e18	0.02	0.00	0.00	1.00	0.02	0.01	0.03	0.20	0.14	0.01	0.03
e19	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.15	-0.08	0.01	0.04
e20	0.09	0.00	0.01	1.01	0.09	0.07	0.11	0.59	0.59	0.06	0.13
e21	0.06	0.00	0.01	1.00	0.06	0.05	0.07	0.37	0.16	0.04	0.09
e22	0.02	0.00	0.00	1.01	0.02	0.01	0.03	0.32	0.06	0.01	0.03
e23	0.04	0.00	0.00	1.01	0.04	0.03	0.05	0.38	0.22	0.03	0.06
e24	0.07	0.00	0.01	1.00	0.07	0.06	0.09	0.29	0.37	0.05	0.10
e25	0.06	0.00	0.01	1.01	0.06	0.05	0.07	0.52	0.49	0.04	0.08
e26	0.03	0.00	0.00	1.01	0.03	0.02	0.03	0.69	0.97	0.02	0.04
e27	0.03	0.00	0.00	1.00	0.03	0.02	0.04	0.56	0.51	0.02	0.04
e28	0.02	0.00	0.00	1.00	0.02	0.01	0.03	0.23	0.31	0.01	0.04
e29	0.11	0.00	0.01	1.00	0.11	0.09	0.13	0.63	0.56	0.07	0.16
e30	0.05	0.00	0.01	1.00	0.05	0.04	0.07	0.22	-0.04	0.03	0.09
e31	0.05	0.00	0.01	1.01	0.05	0.04	0.06	0.41	0.13	0.03	0.07
e32	0.06	0.00	0.01	1.01	0.06	0.05	0.08	0.48	0.24	0.04	0.10
e33	0.04	0.00	0.01	1.01	0.04	0.03	0.06	-0.08	0.01	0.01	0.07
e34	0.06	0.00	0.01	1.01	0.05	0.04	0.07	0.45	0.06	0.04	0.08
e3	0.02	0.00	0.00	1.00	0.02	0.01	0.03	0.16	0.21	0.01	0.03
e9	0.25	0.00	0.03	1.00	0.25	0.19	0.30	0.06	-0.08	0.16	0.34
e5	0.05	0.00	0.01	1.01	0.05	0.04	0.06	0.61	0.23	0.03	0.07

e14	0.10	0.00	0.01	1.01	0.10	0.07	0.12	0.03	-0.22	0.06	0.14
e35	0.03	0.00	0.01	1.01	0.03	0.01	0.06	0.44	0.67	0.00	0.07
e36	0.01	0.00	0.00	1.01	0.01	0.00	0.01	-0.47	0.63	0.00	0.02