Modelling a Landing Gear Device as a Double Inverted Pendulum by Simulating Bird’s Legs

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Abstract—Landing gear devices are very important components of an unmanned aerial vehicle (UAV), especially when it is landing, taking-off and taxiing. In this paper, a new concept of landing gear device has been introduced that simulates a bird’s legs and talons. First, the simple and double inverted pendulums were analysed to find out how they are related to a landing gear apparatus for an aircraft. Stability is a very important issue when landing, so the landing gear device is modelled as an inverted pendulum. The simple and the double inverted pendulums have been used in determining the stability of the landing gear devices. The triple inverted pendulum has also been introduced to some extent. The concept of modelling the landing gear device as an inverted pendulum by simulating bird’s leg and talon is one of the feasible and promising approaches to create new type of landing device.

Index Terms—Bird’s leg, Double inverted pendulum, Landing gear device, Modelling, UAV

1 INTRODUCTION

Landing gear is the undercarriage of aircrafts or spacecrafts. Being a major component, it supports the aircraft when it is on the ground and enabling it to taxi, take-off, and land. The design of landing gear in normal tends to have quite some interference with the design of the structure of the aircraft [1]. In spite of this, it is one of the critical subsystems of the aircraft which needs to be designed with a minimum of weight, minimum volume, high performance gear designers and practitioners. Over the years, with the development of advanced technologies, better equipment and techniques have been created to achieve the challenges that are met in design and development of landing gears [2].

New types of landing gear devices are still being developed for better safety and higher efficiency. Since aircrafts have been developed after birds, their landing gears also followed nearly the same idea but in a different way. Birds have a special way of landing after a flight and sleeping on a branch without falling with the help of their legs and talon. Creating a mechanism which follows nearly the same features as a bird’s leg is quite challenging. This paper introduces a new landing gear device which follows more closely to the bird’s leg and talon, and a mathematical model of inverted pendulum is established to help design of the landing gear device.

2 LITERATURE REVIEW ON CURRENT STRUCTURE OF LANDING GEARS

Designing and development of a landing gear is a relatively complex process. The type of gear used on an aircraft depends on the design of the aircraft and in its intended uses. Aircrafts are usually equipped with wheels to facilitate operations to and from hard surfaces such as the runways of airports but some aircrafts are equipped with skis to operate on surfaces of frozen lakes and snowy areas and some that carry operations in water surrounded environment have pontoon-type landing gear [3]. The following section introduces briefly some of the landing gear devices of aircrafts and UAV that are used more often.

2.1 Landing Gear Device

With the current development in aircraft landing gears, three basic arrangements are being employed among many others; tail gear, which is also called as the tail-wheel landing gear, tandem landing gear, which is also called the bicycle-type landing gear, and tricycle-type landing gear [4], they are shown in Fig. 1.
2.2 UAV Aspect

The landing gear devices for UAV differ from sizes, shapes, characteristics and configurations depending on the types of work they are being built for and the places of their operations [5]. Landing skids have been used to certain extent but they are not effective everywhere. Fig. 2 shows a quadcopter drone with its landing skids. Monolithic Shape Metal Alloys (SMA) springs were once used to develop a landing gear device which did give positive results when experimenting [6].

Samuel Baker [7] designed a passive landing gear device which can allow the UAV to land on uneven or unknown terrains. The landing gear was designed by using the principle of mechanical differential which means between two components that are mechanically isolated, there is a shared loading. This gave the system more than one degree of freedom and many solutions for a given state. Fig. 3 shows the prototype of the landing gear.

With current fast development of technologies, UAV technology is also advancing swiftly, especially for military purposes. A mechanism, developed by Tedrake et al, allows a
fixed-wing UAV to perch on power-lines with the help of the magnetic field of the power-lines [8]. According to the research, the UAV will use on-board equipment such as magnetometers together with an inertial measurement unit for localisation of the UAV relative to the power-line. This can help the UAV to recharge its batteries at the same time with the help of the magnetic field. Cutkosky et al developed a mechanism by which an UAV can perch on a wall [9]. The UAV is fitted with an ultrasonic sensor to set off a pitch-up action when it is flying towards the wall. The UAV makes contact with the wall through the spines attached to it by engaging the roughness on the surface of the wall. The UAV is fitted with a non-linear suspension to absorb the kinetic energy while having the spines attached to the wall.

The UAV cannot stand on a cable wire or branch independently of any other systems. The aim is to design a perching mechanism for the UAV that is can use for the perching action, at the same time as a landing device. The UAV with this landing gear device can at that time decrease its thrust and recharge its batteries using photovoltaic effect.

3 DEVELOPMENT OF THE NEW MODEL OF THE LANDING GEAR

Bird’s perching is an interesting idea in a way that it is a really effective one. Incorporating perching like a bird in UAV will be a great step towards innovation. A bird can land anywhere it wants because the anatomy of its leg permits it to do so, whether it is on a branch or rocky areas due to its under-actuated leg.

3.1 The Working Principle of Bird’s Perching

A bird usually sits on a branch or cable for relaxing itself. Because of this relaxation while sitting, the weight of its body causes the knee and the ankle joints to bend. This causes the Flexor Digitorum Longus and Flexor Hallucis Longus tendons to be pulled respectively (shown in Fig. 6). As a result, the Flexor Digitorum Longus in turn pulls ungual phalanx (flex proc) also called the first digit and the Flexor Hallucis Longus pulls that of the back toe. This action happens simultaneously in all the toes on the claw which leads to the latter close and form a grip. The more the bird relaxes the tighter is the grip [10]. This stays the unmoved until the bird flaps its wings to stand up which causes its knee and ankle joints to rise and Fig. 6 shows a schematic diagram of the bird’s leg with the arrangement of the FDL and FHL tendons. The tendons, together with the knee and ankle joints, form a "pulley system of tendons" which all work together to help the bird with the perching. Therefore it should be noted that perching is a passive process.

![Fig. 6. A schematic diagram of the bird’s leg.](image)

3.2 Structure of the Bird’s Leg

The bird’s leg consists of three parts; the femur, the tibia and fibula and finally the tarsus which is sometimes called the bone of the lower leg. They are connected to the ilium, which is a part of the skeletal structure of the bird as shown in Fig. 7(b). Invisible to the eyes is the femur, which is inside the body of the bird, also shown in the Fig. 7(b).

![Fig. 7(b).](image)
When a bird stands on its legs, the knees are flexed, thus putting the knee joint near the centre of gravity of the bird. This helps to keep balance. The feet are positioned under the centre of gravity of the bird which leads to a stable balance of the body. The tail is used as a counter-balance and also as a means of balancing when the bird walks/hops on the ground or perches. Birds like woodpeckers have stiffened tail feathers, which they use as a prop, helping them in perching and climbing on vertical tree trunks.

3.3 The Relationship between the Bird’s Leg and the Inverted Pendulum

A closer look at the leg of the bird, it can be seen that its shape resembles that of an inverted pendulum, with the body of the bird on the top and the foot on the ground. The bird changes the angle of the ankle joints and knee joints to balance itself while perching, standing or hopping by the pulling and relaxation of the tendons, forming an imaginary system of an inverted pendulum. Inverted pendulums are among the basics for body balance such as the human leg and in this case, of a bird.

The aim of this paper is to determine the relationship of a new landing gear design, which follows closely the structure of the bird’s leg, and the inverted pendulum. Designing a landing gear as stated before requires many considerations. The inverted pendulum is one of them. Geometrically the bird leg is a three order inverted pendulum, also known as the triple inverted pendulum. But a triple inverted pendulum is practically really unstable. Since the femur of the bird is inside the bird’s body, the inverted pendulum can sometimes be considered as double inverted pendulum one, when ignoring the femur. Fig. 8 below shows the bird leg in schematic diagrams as it would be if considered as a double and a triple inverted pendulum, which is the normal type. The claw is represented as just the foot of the bird in the diagrams due to its complex shape.

4 The Inverted Pendulum

An inverted pendulum is a system whereby the centre of mass of the pendulum is located above the point of pivot of the pendulum [11]. There are so far two kinds of inverted pendulum that have been studied extensively: the simple inverted pendulum and the double inverted pendulum. Other orders of the inverted are deemed really unstable and therefore more difficult to study. The inverted pendulum system is one the most difficult systems while being at the same time a standard problem in the field of control systems due to it being really unstable. A proper force balance must be maintained in order for the system to be kept stable, which eventually lead to the need of a proper control theory [12]. The required force balance is achieved, either by a specific torque applied at the point of pivot; horizontally moving the pivot point in the feedback system; changing the speed at which the mass mounted of the pendulum parallel to the axis of pivot rotates, producing a net torque on the pendulum or by oscillation of the pivotal point vertically.

There is a wide range of applications of the inverted pendulum. It serves as an excellent model idea for the automatic landing system of aircrafts, stabilization of the aircraft in turbulent air-flow, stabilization of the cabin in a ship and so on [13]. The process of stabilizing an inverted pendulum is a nonlinear one which is unstable with one input signal and several...
output signals. Fig. 9 shows the simple inverted pendulum and the double inverted pendulum.

Fig. 9 The simple inverted pendulum (a) and the double inverted pendulum (b)

As being a system which is always unstable, the inverted pendulum system is most of the time used as a point of reference to check the performance and how effective a new control method is. In the following sections, the simple inverted pendulum has been introduced first so that the idea of the inverted pendulum can be understood. Then the double inverted pendulum followed by the model of the landing gear device has been introduced respectively.

4.1 The Double Inverted Pendulum

A double inverted pendulum is a combination of the inverted pendulum and the double pendulum. It is more unstable compared to the simple inverted pendulum. This eventually results in the formation complex equations when finding a stabilized position. The double inverted pendulum is said to be a multivariate nonlinear system which has fast reaction as well as being an unstable one.

Stabilization of a double inverted pendulum system is not only a problem which is challenging but also a valuable way in showing the power of the control method. Since the double inverted pendulum system has strong nonlinearity and inherent instability, sometimes the mathematical model of the object near upright position of the pendulum has to be made linear by the inconsistent structure control system [14].

4.2 Modelling

The double inverted pendulum usually consists of a pendulum bob, of mass M, supported by two solid rods, which are of mass \( m_1 \) and \( m_2 \). The solid rods of length \( l_1 \) and \( l_2 \) are attached to a pivot which are allowed free rotation. The movement of the bob, to and fro, causes the system to be unstable. Fig. 10 below shows a schematic diagram of a double inverted pendulum which is restricted to linear motion and with the base connected to a fixed place.

Fig. 10. A double inverted pendulum.

If the mass is tilted to the right, the pendulum moves to the right and vice-versa. Usually the equations of motion of inverted pendulums depend on the constraints that are placed on the movement of the pendulum. They can be derived using the Lagrange’s equations:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = Q
\]

(1)

Where \( L = T - U \) is the Lagrangian; \( T \) and \( U \) are the total kinetic energy and potential energy respectively. \( Q \) is a vector of the generalized forces or moments that is acting in the direction of the generalized coordinates \( \theta \) and is not taken into consideration in forming the equations of kinetic energy and potential energy and is usually considered as \( Q = 0 \) for a stabilized system.

Derivation of the total kinetic energy and potential energy:

Total Kinetic Energy,

\[
T = Kinetic\ Energy + Kinetic\ Energy + Kinetic\ Energy
\]

of rod1 of rod2 of Mass
\[ \mathbf{\dot{\theta}} = \mathbf{\ddot{\theta}} \]

where \( \mathbf{\dot{\theta}} \) is the speed at the time and \( I \) is the moment of inertia of the rods.

**Kinetic energy of rod, \( \theta_t \):**

\[
\frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right)
\]

\[
= \frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right)
\]

\[
= \frac{1}{2} m_{\text{rod}} \left( \frac{d}{dt} (\sin \theta) \right)^2 + \left( \frac{d}{dt} (\cos \theta) \right)^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2
\]

\[
= \frac{1}{2} m_{\text{rod}} \left( \frac{d}{dt} (\sin \theta) \right)^2 + \left( \frac{d}{dt} (\cos \theta) \right)^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2
\]

**Kinetic energy of rod, \( \theta'_t \):**

\[
\frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right)
\]

\[
= \frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right)
\]

\[
= \frac{1}{2} m_{\text{rod}} \left( \frac{d}{dt} (\sin \theta) \right)^2 + \left( \frac{d}{dt} (\cos \theta) \right)^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2
\]

**Kinetic energy of Mass:**

\[
\frac{1}{2} M_{\text{mass}} \left( \dot{\theta}_2^2 + \frac{1}{2} I_{\theta_2} \dot{\theta}_2^2 \right)
\]

\[
= \frac{1}{2} M_{\text{mass}} \left( \dot{\theta}_2^2 + \frac{1}{2} I_{\theta_2} \dot{\theta}_2^2 \right)
\]

**Total kinetic energy, \( T \):**

\[
\sum \frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right) + \frac{1}{2} M_{\text{mass}} \left( \dot{\theta}_2^2 + \frac{1}{2} I_{\theta_2} \dot{\theta}_2^2 \right)
\]

\[
= \sum \frac{1}{2} m_{\text{rod}} \left( \dot{\theta}_t^2 + \frac{1}{2} I_{\theta_t} \dot{\theta}_t^2 \right) + \frac{1}{2} M_{\text{mass}} \left( \dot{\theta}_2^2 + \frac{1}{2} I_{\theta_2} \dot{\theta}_2^2 \right)
\]

**Total Potential Energy,**

\[
U = \text{Potential Energy of rod, } \theta_t + \text{Potential Energy of rod, } \theta'_t + \text{Potential Energy of Mass}
\]

\[
= m_{\text{rod}} g h_{\text{rod}} + m_{\text{rod}} g h_{\text{rod2}} + M g h_{\text{mass}}
\]

**Potential energy of rod, \( \theta_t \):**

\[
= m_{\text{rod}} g \left( \ell_1 \cos \theta_t \right)
\]

**Potential energy of rod, \( \theta'_t \):**

\[
= m_{\text{rod}} g \left( \ell_2 \cos \theta'_t \right)
\]

**Potential energy of Mass:**

\[
= M g \left( L_1 \cos \theta_2 + L_2 \cos \theta_2 \right)
\]

**Total Potential energy, \( U \):**

\[
= m_{\text{rod}} g \left( \ell_1 \cos \theta_t + \ell_2 \cos \theta'_t \right) + M g \left( L_1 \cos \theta_2 + L_2 \cos \theta_2 \right)
\]

**The Lagrange equation is given by:**

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0
\]

**And therefore the equations of motion are:**

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0
\]

Substituting \( L \) in these equations and simplifying leads to the equations that illustrate the motion of the inverted pendulum:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0
\]

Simplifying:
\[
\begin{align*}
(m\ell_1^2 + I_1 + m_2L_2^2 + ML_2^2)\dot{\theta}_1 + m_2L_2\dot{\theta}_2^2 \cos(\theta_1 - \theta_2) \\
+ m_2L_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + ML_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
ML_2L_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1\ell_1 + m_2L_2 + ML_2)g \sin \theta_2 = 0
\end{align*}
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0
\]

\[
\frac{d}{dt} \left[ (m_1\ell_1^2 + I_2 + ML_2^2)\dot{\theta}_2 + m_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) \\
+ m_2L_2\dot{\theta}_2 \sin(\theta_1 - \theta_2) + ML_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
ML_2L_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1\ell_2 + ML_2)g \sin \theta_2 = 0
\]

Simplifying:

\[
\begin{align*}
(m_1\ell_1^2 + I_2 + m_2L_2^2 + ML_2^2)\dot{\theta}_1 + m_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
+ m_2L_2\dot{\theta}_2 \sin(\theta_1 - \theta_2) + ML_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
ML_2L_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1\ell_2 + ML_2)g \sin \theta_2 = 0
\end{align*}
\]

Therefore the final equations are:

\[
\begin{align*}
(m_1\ell_1^2 + I_2 + m_2L_2^2 + ML_2^2)\dot{\theta}_1 + m_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
+ m_2L_2\dot{\theta}_2 \sin(\theta_1 - \theta_2) + ML_2L_2\dot{\theta}_2 \cos(\theta_1 - \theta_2) + \\
ML_2L_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1\ell_2 + ML_2)g \sin \theta_2 = 0
\end{align*}
\]

5 MECHANICAL DESIGN BASED ON THE DOUBLE INVERTED PENDULUM

This results in bending of the knee and ankle joints. Simultaneously, the cable wires passing behind the ankle joint becomes taut. Since it is of definite length, it cannot extend and they are pulled in the process. This causes the first digit (ungual phalanx) to be pulled along with the other part of the toe. This action happens in all the all toes resulting in closing the claw thus forming a grasp. At this time, the thrust of the UAV is to be decreased, and therefore the use of less power.
Now if the UAV wants to go away, it increases it power. This causes less pressure to be applied on the actuators which again is picked up by the sensors and sent to the computer. This time the actuators extend, making the UAV to start rising. The tension on the cable decreases which make the claw to open. The thrust of the UAV is increased more it can fly immediately. While in the air, the pilot of the UAV can activate a gear device highlighting the different parts with different colours. The use of the inverted pendulum to find the stability and balance is important so that the new designed landing gear device can be in the UAV. The greater the order of the inverted pendulum is, the greater the instability which therefore leads to more complex equations for solving. In this paper, most of the assumptions that have been made are based on some papers that have already been published. Stability of the inverted pendulums has many other criteria which must be met in order for pendulum to be stable.

6 Discussions and conclusions

The double inverted pendulum is unstable because it has two rods, which have their own masses. In section 4.2 the equations of motion were derived without the UAV on the double inverted pendulum. When the UAV will be added to the double inverted pendulum, additional masses will be seen in the equations of Lagrange rendering the equations more complex. In the case of the bird’s leg, the equations have been described and formulated as that of a double inverted pendulum when the femur is excluded. If the femur is taken into consideration, this leads the system of the leg to a triple inverted pendulum, which is even more unstable than the simple inverted and double inverted pendulum. This leads to even more complex equations that need solving using a computerized system.

A triple pendulum is a kind of pendulum system which is difficult to control because of it being most of the time unstable and also because of its nonlinear behaviour. In practice, some of the pendulum parameters may not be known accurately, which has a big influence on the system dynamics [15]. For the bird’s leg, considering a triple inverted pendulum can be quite complicated due to the high instability of the triple inverted pendulum. The high instability means more complicated and longer equations are needed to be resolved in order to find a better control.

The work so far in this paper has been to introduce how the inverted pendulum can be used to find the stability of the UAV. In the near future, the control of the inverted pendulum system will be worked on using various ways: fuzzy control systems, control strategies such as PID controller, neural network and gravity compensator.

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REFERENCES