Modeling and Simulation of a Differential Drive Mobile Robot
Anirudh Topiwala

Abstract—Differential drive robots have wide application in the fields of defense, reconnaissance, in house hold applications like house cleansing robot and many others. This paper intends to present a simple and a reliable method to circumvent the physical intricacies of the actual world, by developing a realistic simulation system. A prototype model was first made to understand the logic of obstacle detection using ultrasonic sensors and Arduino Uno as the microcontroller, but optimizing the robot physically by experimentation was very difficult. Therefore to solve this problem an identical simulation is carried out in matlab for easy optimization. The Kinematic equations used to control the robot is presented. The output data of the kinematic model is used to actuate the 3D model in virtual reality toolbox of matlab. The rotation of the robot about the central axis is also observed. The obstacles are detected by the ultrasonic sensors mounted on the robot which enables the robot to navigate in an unknown environment.

Index Terms— Arduino, differential drive robot, kinematics, matlab, mobile robot, object avoidance, simulation, simulink.

1 INTRODUCTION
Robotics today has become a part in our day to day life. We can now develop highly maneuverable, functional and intelligent robots for the high-end military applications to the more mundane applications of the household. Locomotion system for mobile robots can be broadly classified under legged and wheeled systems. Legged robots are very useful under circumstances of extremely difficult terrain or when there are high terrain discontinuities. The main advantage of this articulated walking machines is that they can adapt their posture to the uneven terrain making locomotion very easy. However, articulated robots are difficult to control, have size restrictions and are not as fast as compared to wheeled robots. [1]

Locomotion systems bases on wheels are relatively simple as the number of articulations in a walking robot is far less than that in a wheeled robot. Therefore, wheeled robot are far faster, reliable simple and easy to control if it is required to operate it on a rugged ground, roads, or any other relatively level surface.[2]

Understanding how wheeled mobile robots (WMR) move with reference to the command inputs is essential for navigation tasks like path planning, obstacle avoidance and guidance systems.

2. Wheeled Mobile Robots
The wheeled mobile robots are classifies under five classes corresponding to the pair of indices (m, s), where m is the mobility degree and s is the steer ability. [3]

- Class (3, 0) robots also called omnidirectional robots have no steering wheels(s=0) that is they are only equipped with Swedish or castor wheels. They have complete mobility in a plane which means that they can move in any direction without changing their orientation. (m=3)
- Class (2, 0) robots also have no steering wheel(s=0) but at least one of the wheels is fixed with a common axle, restricting the mobility of the plane to two-dimensional motion in the plane. (m=2)
- Class (2, 1) robots have at least one steering wheel (s=1) and no fixed wheel, which means that all the wheels can rotate independently. If the number of steering wheels is more than two then they must be coordinated and the mobility is restricted to a two dimensional plane. (m=2)
- Class (1, 1) robots have one or more fixed wheels on a common axle and also one or more steering wheels such that the centers of the steering wheels should not coincide with the common axle of the fixed wheel and the orientations must be coordinated.(s=1). The Mobility of the robot depends upon the orientation of the steering wheel and is restricted to one dimensional plane. (m=1)
- Class (1, 2) robots should at least have two steering wheels(s=2) and no fixed wheel. The Mobility is dependent on the orientation of the two steering wheels and is restricted to one dimensional plane. (m=1)

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In this paper I have developed a class (2, 0) type of differential drive robot.

### 3. The Prototype Model

#### 3.1 The Mechanical aspects

The prototype model made here is of class (2, 0) [3] which means that it has no steering wheel and the mobility is restricted to two dimensional plane. The chassis is made up of a rectangular hard wood which has motor mountings mounted on it. The two driving wheels used here are of plastic material and has a rubber layer on them to increase the tractive force. To balance the robot and preventing it from not falling over two support castor wheels are added at the front and the back side. If the material of the chassis is changed to aluminum than it can have practical applications in industries as it can carry heavy weights with high mobility and speed. The capability of the robot to rotate about its own axis adds in to the advantages of using it in industries.

#### 3.2 The Electronic aspects

An Arduino Uno board controls the robot. [4] The bot is equipped with two high torque 800 rpm Robokits motors which are bidirectional and operate at 12 volts. The motors are mounted such that their axis of rotation is in one line. The two ultrasonic sensors used here are of HC-SR04 model that operate at 5 volts provided by the Arduino board. The Arduino measures the distance by calculating the time required for the ultrasonic waves emitted by the sensor to reflect back from the obstacle. Using this data the Arduino determines whether to take a left or right turn and change the direction of motor accordingly with the help of motor controller. The motor controller used here is RKI-1004 (Robokits India). The motor controller helps us connect a higher power source to the motors, here it is 12 volts. Thus once the robot is able to detect obstacles and move accordingly it just needs an input signal to start after which it can act autonomously. A Bluetooth module connected to the Arduino provides this input signal. The Bluetooth module used here is HC-06. In case of any failures, this Bluetooth module can also provide manual actuations to the robot. Communication to the module can be established in many ways, here we have developed an app in MIT appinventor which uses a mobile phone’s Bluetooth to control the robot. [5]

#### 3.3 Need for simulation

While making the prototype robot we faced a lot of difficulties. Some of them include the availability of certain components. Also the sensors and the Arduino board used are very expensive. Another important problem is the time required to experiment different data values which can be easily done using simulations. Optimizing the model experimentally can be a painstaking task which can be easily and efficiently done using different simulators. Graphs can be plotted between input and output values to generate a comprehensive view of the collected data.

### 4. Differential Drive Robot

The mobile robot developed for the simulation is a class (2, 0) type differential drive robot which is very similar to the prototype model developed. There are many design alternatives but the two wheel drive robot is by far the most popular design. The CAD software used to develop the model for simulation is Solid Works 2014. The Differential drive robot consists of four main parts which are the chassis, the wheels, the castor wheel mounting and the castor wheel.

The chassis is cuboidal shaped and hollow at the center to accommodate for the circuitry of the robot and also to reduce weight of the robot. The chassis is symmetric about the central planes which helps the robot to rotate about the central axis at its own position.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>angular velocity of the robot</td>
</tr>
<tr>
<td>$R$</td>
<td>distance between ICC and the midpoint between the wheels</td>
</tr>
<tr>
<td>$l$</td>
<td>distance between the wheels</td>
</tr>
<tr>
<td>$V_r$</td>
<td>linear velocity of the right wheel</td>
</tr>
<tr>
<td>$V_l$</td>
<td>linear velocity of the left wheel</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angle of the robot with X axis</td>
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The different parts of the robot are connected to the chassis by giving relations between the parts, which is called giving mates. The rotary parts are connected to the chassis by mainly giving two mates the coincident mate and the cocentric mate. The castor wheel is connected to the far end of the chassis and is only used to give support to the robot. The two independent driving wheels are connected to the front end of the chassis.

The robot also has eight ultrasonic sensor mountings on the front side for the sensors. The mechanical integrity of the robot is maintained by having a front bumper which protects the robot from impacts or collisions.

Different parts of the model-robot have different material considerations. Aluminium alloy is the preferred material of construction, because the parts are light in weight and are not very expensive. The wheels have an aluminium rim and the tires are made out of plastic which a rubber covering for traction.

5. Kinematic Equations for differential drive robot

5.1 Differential Drive Kinematics

For Navigation of the robot autonomously it has to know its position at all times, that is the translational and the rotational matrices. When we change the velocity of the wheels to have different motions the robot must rotate about a point that lies along the common axis of both the driving wheels, this point is known as the Instantaneous Center of Curvature (ICC) (Fig.2(b))[6]. The angular velocity developed due to this rotation is denoted by $\omega$ and the following equations can be written:

\[
\omega (R + 1/2) = V_r
\]
\[
\omega (R - 1/2) = V_l
\]

Where $l$ is the track of the robot or the distance between the two wheels, $V_r$ and $V_l$ are the right and the left wheel linear velocities respectively, $R$ is the distance between the ICC (Instantaneous Center of Curvature)[6][7] and the midpoint between the wheels. At any instant of time we can solve for $R$ and $\omega$ as:

\[
R = \frac{l(V_l + V_r)}{2(V_l - V_r)}
\]
\[
\omega = \frac{(V_l - V_r)}{l}
\]

The trajectory of the robot solely depends upon the velocity of the wheels,

1. If $V_l = V_r$, than the robot will have a linear motion
2. If $V_l = -V_r$, than the robot will rotate about its own central axis
3. If the velocities are not equal than the robot will have a mixed motion of translation and rotational.[8]

5.2 Forward and Inverse Kinematics of Differential Drive Robot

To understand this, assume that the robot is at some location $(x, y)$ making an angle of $\theta$ with the X axis. After some time the position of the robot will change and the new positions would be $(x', y')$ and the new angle is $\theta'$. [8]

\[
ICC = [x - R \sin(\theta), y + R \cos(\theta)]
\]

and at time $t + \delta t$ the new positions would be:

\[
\begin{pmatrix}
x' \\
y' \\
\theta'
\end{pmatrix} = \begin{pmatrix}
\cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
\sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x - ICC_x \\
y - ICC_y \\
\theta
\end{pmatrix} + \begin{pmatrix}
ICC_x \\
ICC_y \\
\omega \delta t
\end{pmatrix} \delta t
\]
Using this equation we can find the position of the robot at any instant.

To sum up the above equations we can describe the position of the robot moving in a particular direction $\theta$ at a given velocity $V$ (where $V$ is the average of the left wheel and the right wheel velocity) by: 

$$P_x = x_t = \int V \sin \theta \, d\theta$$  

$$P_y = y_t = \int V \cos \theta \, d\theta$$

$$\theta = \int \omega \, dt$$

6. Simulation of Differential Drive Robot

In this section we will learn about the various simulations carried out on the robot. The simulations are carried out in Simulink which is an extension of Matlab R2015A.

6.1 Basic Simulink Model

Once we are done with the modeling of the robot it is imported into Matlab using the Simulink Link add-in in the CAD software. The imported XML file is of second generation type. The block diagram for the cad model is than generated using the built in function $H = \text{smimport}(\text{"File Name.xml")}$. This generates the block diagram wherein the relations between the chassis and the different components can be easily seen. The mates in the cad software for the robot are represented by revolute blocks and can be altered so that we can change parameters like internal mechanics, actuation, and sensing. We can give constant or varied velocity inputs to the robot through this revolute block.

6.2 Robot governed by kinematic equations.

When we simulate the basic model of the robot, we see that Matlab had produced a similar model to that of the CAD software used.

Fig. 3. Basic block diagram of the differential drive robot

Fig. 4. It shows the model of the robot produced by Simulink.

Fig. 5. It shows the connection of the kinematic subsystem block and the constant value blocks to the basic model.

To give velocity inputs in this model we will have to connect a constant value block or a variable input block based upon the requirement, to the revolute joint. When we will simulate this we will see that the wheel rotates but the robot does not move with respect to the environment or the world, this is because the robot is not aware of its current position therefore it is also not able to find its position for the next instance. To solve this we will have to add a kinematic subsystem which is a block consisting of the three governing equations (equation 7, equation 8, equation 9). This block will take input reading of velocity from the revolute joints of both the wheels and calculate the position of the robot, this will then enable the robot to move with respect to the world.
Fig. 6. It is the kinematic subsystem block. This block takes in the input velocities \( w_l \) and \( w_r \) and calculates the final positions of the robot according to the governing equations described above.

6.3 Control Logic for the simulation

Once the robot can move in the environment the next step is to be aware of the obstacles present in the environment. There are many ways to sense the obstacles present in the vicinity of the robot but the sensors used here are ultrasonic sensors. These sensors work on the principle of reflection of ultrasonic waves. The distance of the obstacle can be calculated by measuring the time taken by the ultrasonic wave to reflect back from the obstacle. [10]

In this simulation we have used a virtual sensor subsystem to detect and find the range of the obstacles from the robot. In this subsystem first we use two transform sensors to passively sense the 3D time varying transformation and send it to the user defined matlab function through the ‘SSC to SSL’ subsystem as seen in Fig. 7, which converts the physical signal into simulink signal.

The first transform sensor senses the rotation of the robot while the second sensor senses the ‘x’ and the ‘y’ translations.

The Matlab Function box consists of a user defined code to calculate the position of the sensors with respect to the environment which is possible because of the know position of the robot. The obstacles present in the environment are in the form of ones and zeroes in the simulation and therefore the sensor ranges can be calculated by interpolating values from the virtual sensor lookup block. [10] Once we get the different sensor values we can use stateflow which is the control logic of the robot, to decide whether to take a left or a right turn depending on the value of the allowable sensor ranges. In this project depending upon the sensor readings we have a total of six conditions, which are wide turn, tight turn and turn in place for both left and right side. Thus, the robot will be able to navigate in the unknown environment autonomously.

Fig.7. Various blocks inside the virtual sensor subsystem
6.4 Virtual Reality Output
Exploiting the virtual reality feature of matlab, I have tried to simulate the robot in a predefined existing environment. As the VRML model from SolidWorks need to be processed before being used, it is passed through a V-Realm Builder 2.0. This is done so as to rearrange the nodes and parent-child relationship, which then can be used by Simulink. Once the model is imported, the actuatuons can be passed to the robot through a VR Sink block. [10]
The problem with simulations is that, they do not have actual sensors. Therefore, for the robot to detect obstacles, the objects in the environment are in the form of ones and zeros. Using this data the robot with the help of its own location can successfully detect the obstacles and avoid them using the control algorithm, which is the sataeflow.

7. Conclusion
This paper intends to present a simple and a reliable method to avoid the intricacies of physical modelling, by carrying out a simulation of the robot, in matlab. In particular, the paper starts with the making of a prototype model which can detect obstacles and navigate accordingly, using ultrasonic sensors and Arduino uno as the microcontroller. Because of the difficulties faced during physical modeling of the robot, an alternative solution is used, to carry out an identical simulation of the robot in simulink, which can be further optimized easily. The kinematic equations associated with a differential drive robot of class (2, 0) is then discussed and implemented in the simulation using the simulink library. Finally, the control aspect of the robot is discussed, and 3D outputs are obtained through the virtual reality toolbox of matlab.

8. Acknowledgments
I would like to thank Institue of Technology, Nirma University for accepting and approving my research project.
I would like to thank my faculty incharge Prof. Nipun Raval for guiding me throughout the project and proving me with astute insights.

9. References