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Mathematical Model to Design Rack And Pinion Ackerman Steering Geomtery

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Abstract— In order to turn the vehicle, steering mechanism is required. Nowadays most of the fourwheelers are having steering mechanism based on ackerman principle. In order to design steering mechanism based on ackerman principle, one method is to use rack and pinion with tierods. In the present work, new mathematical model is developed in order to design steering geometry mentioned above considering different geometry parameters. This mathematical model includes three equations. By solving this three equations we can get different steering geometry parameters by fixing some variables according to restriction and considering optimum steering geometry with respect to steering effort and %ackerman. This model can be used for ackerman as well as reverse ackerman steering geometry and further it can be used for two wheel steering as well as four wheel steering by applying this model on front and rear steering design.

Index Terms— Ackerman Principle, Steering Geometry, Mathematical Model, Inner Wheel Angle, Outer Wheel Angle, % Ackerman

1 INTRODUCTION

1.1 Objective

Ackerman steering geometry is most widely used today in commercial vehicle. One way to design ackerman steering geometry is to use Rack and pinion with tierods.
Objective of this paper is to develop a new mathematical model to design such ackerman steering geometry instead of try and error method.

1.2 Ackerman principle

During turning if I-centers of all wheels meet at a point, then the vehicle will take turn about that point which results in pure rolling of the vehicle. The condition is called the Ackerman condition and this principle is known as ackerman principle.

1.3 Ackerman condition for two wheel steering

Ackerman condition for two wheel steering is expressed as:

$$cot\delta o - cot\delta i = \frac{B}{L} \quad (1)$$

Where,

- δ_o = outer wheel angle
- δ_i = inner wheel angle
- W = Track width of the vehicle
- B = distance between left and right kingpin centerline
- L = wheel base of the vehicle

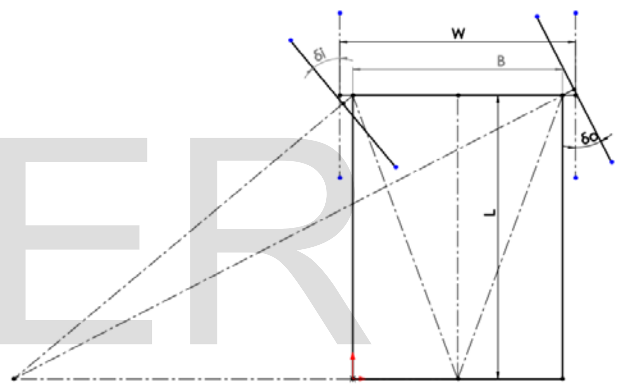

Fig. 1. Ackerman condition for two wheel steering

Here, ackerman condition is satisfied when i-centres of front wheels meet at a point on the rear axis of the vehicle which is turning point of the vehicle.

2 MATHEMATICAL MODEL

2.1 Rack and pinion geometry

Rack and pinon steering geometry is one of the way to design steering geometry which is based on ackerman principle. Here is a list of various steering geometry parameters in case of rack and pinion geometry.

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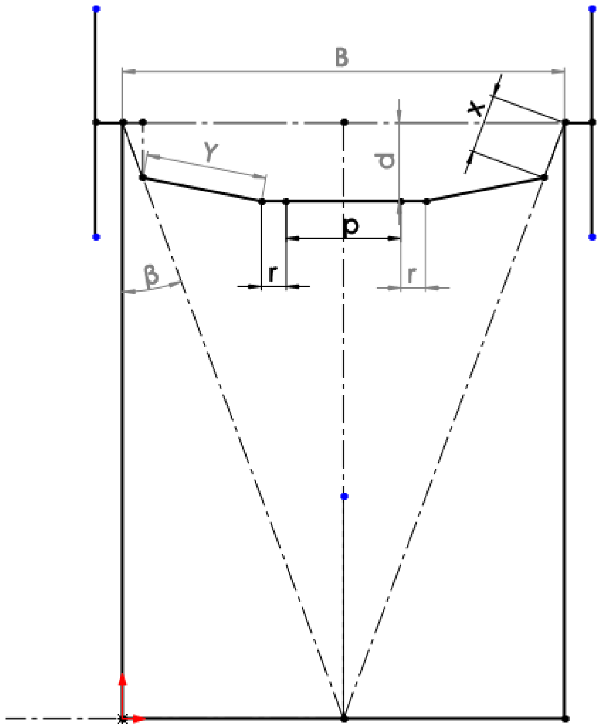

Fig. 2. Various steering geometry parameters

Where,

- x= steering arm length
- y= tie-rod length (in top view)
- p= rack casing length
- p+2r= rack ball joint center to center length
- q= travel of rack
- d= distance between front axis and rack center axis
- β= Ackerman angle

2.2 Equation: For toe zero condition

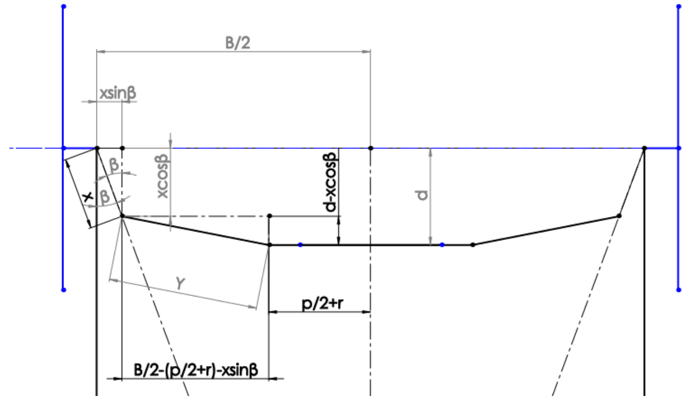

Fig. 3. Toe zero condition

From above fig 3, it is clear that

$$y^2 = [\frac{B-(p+2r)}{2} - xsin\beta]^2 + [d\text{-}xcos\beta]^2 \qquad (2)$$

2.3 Equation: From inner wheel geometry

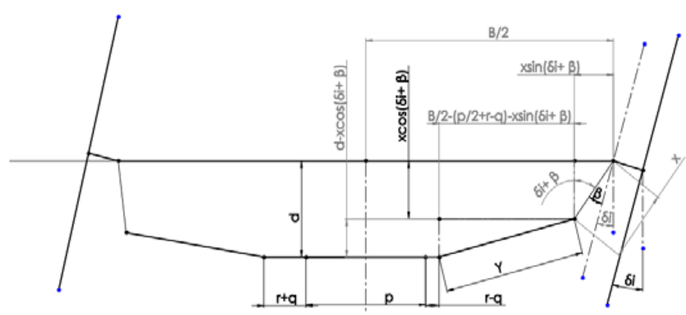

Fig. 4. Inner wheel geometry

From fig 4, it is clear that

$$y^2 = [\frac{B}{2} - (\frac{p}{2} + r - q) - xsin(\delta i + \beta)]^2 + [d - xcos(\delta i + \beta]^2 \qquad (3)$$

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2.4 Equation: From outer wheel geometry

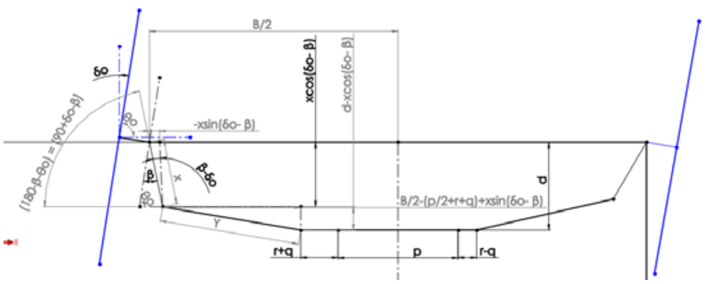


Fig. 5. Outer wheel geometry

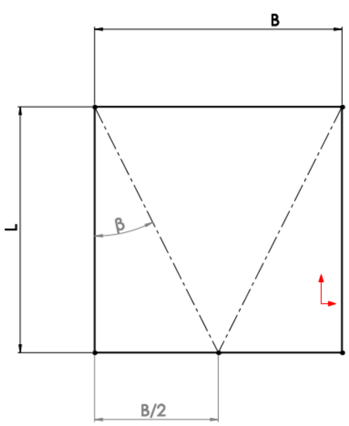
From fig, it is clear that,

$$y^2 = [\frac{B}{2} - \left(\frac{p}{2} + r + q\right) + xsin(\delta o\text{-}\beta)]^2 + [d - xcos(\delta o - \beta]^2 \quad (4)$$

3 Methodology

3.1 Design of steering geometry

- For given rack and pinion, value of **p** and **r** is known.
- Value of **B** is fixed by track width of the vehicle and distance between wheel center and kingpin center.
- Value of **β** is fixed by value of **B** and wheelbase **L**.



From above fig,

$$\beta = \tan^{-1}(2L/B)$$

- In (3) Angle $\boldsymbol{\delta_i}$ is the value of certain inner wheel angle at which we want to satisfy the ackerman principle in order to calculate outer wheel angle $\boldsymbol{\delta_o}$ for (4). From (1),

$$\delta o = cot^{-1}(cot\delta i + \frac{B}{L}) \quad (5)$$

- Now, we have 4 unknowns : **x, y, d, q** and 3 equations so any one variablewe can fix either according to restriction if any or as per our comfort. After fixinng any one variable, we can calculate value of other three variable by solving these three equations.
- Thus we have values of all **x, y, d, q-** steering geometry parameters and for this we will get perfect ackerman condition when inner wheel is at angle $\boldsymbol{\delta_i}$ and therefore outer wheel is at angle $\boldsymbol{\delta_o}$.

3.2 Rack travel q for any particular inner wheel angle δi

From (3),

$$q = x * sin(\delta i + \beta) - A + [y^2 - (d - x * cos(\delta i + \beta))^2]^{0.5}$$

$$\text{Where, } A = B/2 - (p/2 + r)$$

3.3 Calculation of actual outer wheel angle δ_o for any particular inner wheel angle δ_i

From (4),

$$y^2 = [\frac{B}{2} - (\frac{p}{2} + r + q) - xsin(\delta o\text{-}\beta)]\text{^}2 + [d - xcos(\delta o - \beta]\text{^}2$$

$$\therefore c = (A - q)sin\gamma - dcos\gamma$$

$$\text{Where, } c = \frac{[y^2 - d^2 - x^2 - (A-q)^2]}{2x},$$

$$\gamma = \delta_o\text{-}\beta$$

$$\therefore c = (A - q)sin\gamma - d(1 - \sin^2\gamma)^{0.5}$$

$$\therefore[(A - q)^2 + d^2] * \sin^2\gamma - 2c(A - q)sin\gamma) + c^2 - d^2 = 0$$

$$\therefore sin\gamma = c * (A - q) + [c^2 * (A - q)^2 - \{(A - q)^2 + d^2\} * (c^2 - d^2)]^{0.5} = K$$

$$\therefore \gamma = \sin^{-1}(\boldsymbol{K})$$

But,

$$\gamma = \delta o - \beta$$

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$$\therefore \delta o = \sin^{-1}(K) + \beta \qquad (6)$$

Thus actual value of outer wheel angle δo can be calculated for each and every inner wheel angle. To see deviation of designed steering geometry from the perfect ackerman geometry (Geometry in which at every point ackerman principle is satisfied), plot graph for two curves:

1. Outer wheel angle δo as per designed steering geometry v/s inner wheel angle δi
2. Outer wheel angle δo as per perfect ackerman steering geometry v/s inner wheel angle δi
3.

Now design steering geometries for different values (iterations) of inner wheel angle δi at which ackerman condition is satisfied and select such geometry for which get optimum deviation from perfect ackerman geometry as well as steering effort are achieved.

4 Experimental Stetup

Data:

1. Wheelbase L = 1.524 m
2. Track width W = 1.27 m
3. B = 1.137 m
4. β = 20.457 deg
5. For Tata Nano rack p = 0.273 m and r = 0.0635 m
6. x = 0.0753 m is fixed in this example due to restriction in length of steering arm because of knuckle design.

Now, here suppose we want to achieve ackerman condition when inner wheel angle $\boldsymbol{\delta_i}$ **= 40 deg** and therefore as per ackerman principle $\boldsymbol{\delta_o}$ **= 27.296 deg** for given data.

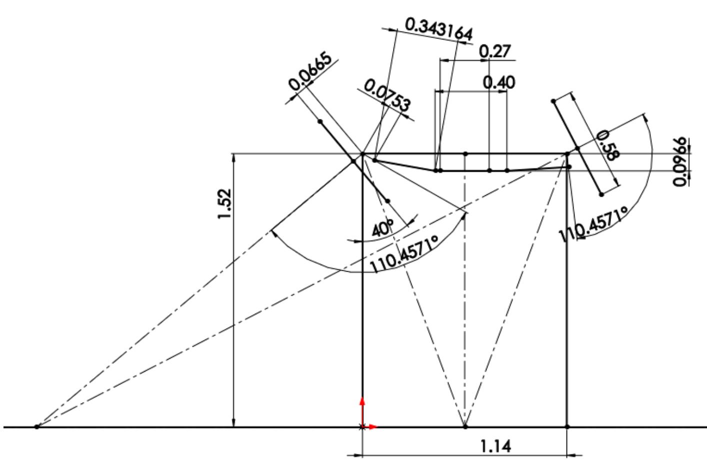
From (2), (3) & (4),

$$y^2 = [0.3685 - 0.3495x]^2 + [d - 0.9369x]^2,$$

$$y^2 = [(0.3685 + q) - 0.87x]^2 + [d - 0.4931x]^2,$$

$$y^2 = [0.3685 - q + 0.1191x]^2 + [d - 0.9929x]^2$$

By solving these three equations, we will get values **Y, d** and **q** **(travel of rack when inner wheel angle is 40 deg).**

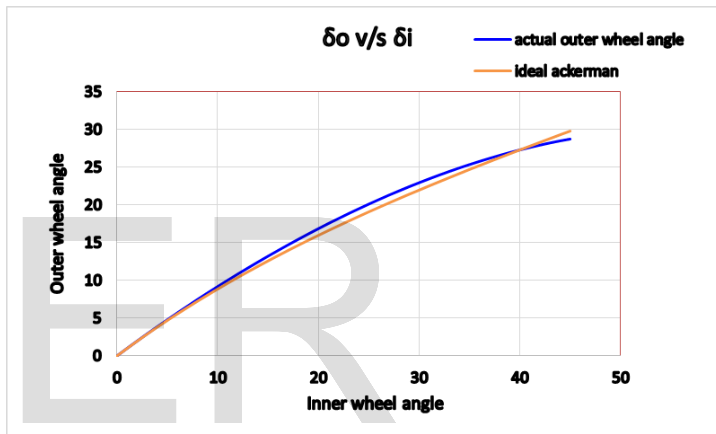


Y = 0.3431 m
d = 0.0966 m
Travel of rack q = 0.0349 m (when inner wheel angle δ_i is 40 deg)

Fig. 6. Designed steering geometry as per given data

Now, for different inner wheel angle get value of outer wheel angle as per ideal ackerman as well as actual steering geometry by using (5) and (6) respectively and then plot the graph for outer wheel angle (for ideal ackerman and for actual geometry) v/s inner wheel angle.

Here graph is as shown below.



5 Conclusion

By applying and solving three equations of mathematical model for any vehicle, rack and pinion ackerman steering geometry for any vehicle can be designed. Steering geometry can be optimized by using mathematical model for ackerman condition for different inner wheel angles and select geometry for which percentange ackerman as well steering effort is optimum. This mathematical model can be applied to rear wheel steering also. To design four wheel steering in which rack and pinio geometry is at front as well as rear side, this mathematical model should be applied on front and rear side separately.

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REFERENCES

[1] Reza N. Jazar , Vehicle Dynamics: Theory and Application, Springer, Volume 3 , Page No: 379-451, March 2008

[2] http://www.rctek.com/technical/handling/ackerman_steering_principle.html

[3] http://www.auto-ware.com/setup/ack_rac.htm