Mathematical Model for Future Population Scenario In India And China – An Econometric Approach

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Abstract—Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences. A mathematical model including dynamical systems, statistical models and differential equations involves variety abstract structures. Population growth is one of the main issues in India and China which are located in Asia. These two countries are over populated and the growth in resources has not been keeping pace with the growth in population. So the increasing trend in population is great threat to the nations. The use of the logistic growth model is widely established in many fields of modeling and forecasting . In this paper, we will determine the carrying capacity and the vital coefficients governing the population growth of India and China. Further this study gives an insight on how to determine the carrying capacity and the vital coefficients, governing population growth, by using the least square method. Future Population growth rates and Global ranks of India and China are predicted.

Index Terms— Logistic growth model, Carrying capacity, Vital coefficients, Annual growth rate.

1. Introduction

Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences. A mathematical model including dynamical systems, statistical models and differential equations involves variety abstract structures. The "population growth rate" is the rate at which the number of individuals in a population increases in a given time period, expressed as a fraction of the initial population. Specifically, population growth rate refers to the change in population over a unit time period, often expressed as a percentage of the number of individuals in the population at the beginning of that period. Population growth is one of the main issues in India and China which are located in Asia. These two countries are over populated and the growth in resources has not been keeping pace with the growth in population. So the increasing trend in population is great threat to the nations. The most populous country on the earth, China accounts for more than 19 % of the world population of the surface area with 9,596,960 sq.km, the second most populated country on the world, India accounts for more than 17.5% of the world population of the surface area with 3,287,590 sq.km. The population of India, at 1210.2 million on 2012, is almost equal to the combined population of U.S.A, Indonesia, Brazil, Pakistan, Bangladesh and Japan put together the population of these six countries totals 1214.3 million.

The use of the logistic growth model is widely established in many fields of modeling and forecasting [1]. The ordinary differential equations with which students are most familiar are the equations for exponential and logistic population growth. Historically, Thomas Malthus initiated the mathematical treatment of population dynamics. First order differential equations govern the growth of various species. At first glance it would seem impossible to model the growth of a species by a differential equation since the population of any species always changes by integer amounts. Hence the population of any species can never be a differentiable function of time. However if a given population is very large and it is suddenly increased by one, then the change is very small compared to the given population [2]. Thus we make the assumption that large populations change continuously and even differentiable with time. The projections of future population are normally based on present population. In this paper, we will determine the carrying capacity and the vital coefficients governing the population growth of India and China. Further this study gives an insight on how to determine the carrying capacity and the vital coefficients, governing population growth, by using the least square method. This paper is organized as follows: In Section 2 model development and some definitions are described. India and China population growth rates are predicted using carrying capacity values, vital coefficient values and half of the carrying capacity values in Section 3 and Section 4 respectively. Analysis on predicted population growth rates and Global ranks of India and China and Conclusion of the study is presented in Section5.

2. Development of the model

- Let $P(t)$ denote the population of a given species at time t
- let α denote then difference between its birth rate and death rate.

If this population is isolated, then $\frac{d}{dt}P(t)$, the rate of change of the population, equals $\alpha\, P(t)$ where α is a constant that does not change with either time or population. The differential equation governing population growth in this case is

$$\frac{d}{dt}P(t) = \alpha\, P(t) \qquad (1)$$

where, t represents the time period and α- referred to as the Malthusian factor, is the multiple that determines the growth rate. This mathematical model, of population growth, was proposed by an Englishman, Thomas R. Malthus [3], in 1798. Equation (1) is a non-homogeneous linear first order differential equation known as Malthusian law of population growth. $P(t)$ takes on only integral values and it is a discontinuous function of t. However, it may be approximated by a continuous and differentiable function as soon as the number of individuals is large enough [7].The solution of equation (1) is

$$P(t) = P_0\, e^{\alpha t} \qquad (2)$$

Hence any species satisfying the Malthusian law of population growth grows exponentially with time. This model is often referred to as *The Exponential Law* and is widely regarded in the field of population ecology as the first principle of population Dynamics. At best, it can be described as an approximate physical law as it is generally acknowledged that nothing can grow at a constant rate indefinitely. As population increases in size, the environment's ability to support the population decreases. As the population increases per capita food availability decreases, waste products may accumulate and birth rates tend to decline while death rates tend to increase. Thus it seems reasonable to consider a mathematical model which explicitly incorporates the idea of carrying capacity (limiting value). A Belgian Mathematician Verhulst [5], showed that the population growth not only depends on the population size but also on how far this size is from its upper limit i.e. its carrying capacity (maximum supportable population). He modified Malthus's Model to make the population size proportional to both the previous population and a new term

$$\frac{\alpha - \beta P(t)}{\alpha} \qquad (3)$$

where α and β are called the vital coefficients of the population that reflects how far the population is from its maximum limit. However, as the population value grows and gets closer to $\frac{\alpha}{\beta}$, this new term will become very small and tend to zero, providing the right feedback to limit the population growth. Thus the second term models the competition for available resources, which tends to limit the population growth. So the modified equation using this new term is:

$$\frac{d}{dt}P(t) = \frac{\alpha P(t)(\alpha - \beta P(t))}{\alpha} \qquad (4)$$

This is a nonlinear differential equation unlike equation (1) in the sense that one cannot simply multiply the previous population by a factor. In this case the population $P(t)$ on the right of equation (4) is being multiplied by itself. This equation is known as the logistic law of population growth. Putting

$P = P_0$ for $t = 0$, where P_0 represents the population at some specified time, $t = 0$, equation (4) becomes

$$\frac{d}{dt}P = \alpha P - \beta P^2 \qquad (5)$$

Separating the variables in equation (5) and integrating, we obtain

$$\int \frac{1}{\alpha}\left(\frac{1}{P} + \frac{\beta}{\alpha - \beta P}\right) dP = t + c, \text{ so that}$$

$$\frac{1}{\alpha}\left(\log P - \log(\alpha - \beta P)\right) = t + c \qquad (6)$$

Using $t = 0$ and $P = P_0$ we see that

$c = \frac{1}{\alpha}(\log P - \log(\alpha - \beta P))$. Equation (6) becomes

$\frac{1}{\alpha}(\log P - \log(\alpha - \beta P)) = t + \frac{1}{\alpha}(\log P_0 - \log(\alpha - \beta P_0))$

Solving for P yields

$$P = \frac{\frac{\alpha}{\beta}}{1 + \left(\frac{\frac{\alpha}{\beta}}{P_0} - 1\right)e^{-\alpha t}} \qquad (7)$$

If we take the limit of equation (7) as $t \to \infty$, we get (since $\alpha > 0$)

$$P_{max} = \lim_{t \to \infty} P = \frac{\alpha}{\beta} \qquad (8)$$

Next, we determine the values of α, β and P_{max} by using the least square method. Differentiating equation (7), twice with respect to t gives

$$\frac{d^2P}{dt^2} = \frac{C\,\alpha^3 e^{\alpha t}(C - e^{\alpha t})}{\beta(C + e^{\alpha t})^3} \qquad (9)$$

where $C = \frac{\frac{\alpha}{\beta}}{P_0} - 1$.

At the point of inflection this second derivative of P must be equal to zero. This will be so, when

$$C = e^{\alpha t} \qquad (10)$$

Solving for t in equation (10) gives

$$t = \frac{InC}{\alpha} \qquad (11)$$

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This is the time when the point of inflection occurs, that is, when the population is a half of the value of its carrying capacity. Let the time when the point of inflexion occurs be $t = t_k$. Then $C = e^{\alpha t}$ becomes $C = e^{\alpha t_k}$. Using this new value of C and replacing $\frac{\alpha}{\beta}$ by K equation (7) becomes

$$P = \frac{K}{1+e^{-\alpha(t-t_k)}} \qquad (12)$$

Let the coordinates of the actual population values be (t, p) and the coordinates of the predicted population values with the same abscissa on the fitted curve be (t, P). Then the error in this case is given by $(P - p)$. Since some of the actual population data points lie below the curve of predicted values while others lie above it, we square $(P - p)$ to ensure that the error is positive. Thus, the total squared error, e, in fitting the curve is given by

$$\mathbf{e} = \sum_{i=1}^{n} (P_i - p_i)^2 \qquad (13)$$

Equation (13) contains three parameters K, α and t_k. To eliminate K we let

$$P = Kh \qquad (14)$$

where

$$h = \frac{1}{1+e^{-(t-t_k)}} \qquad (15)$$

Using the value of P in equation (14) and algebraic properties of inner product to equation (13), we have

$$\mathbf{e} = \sum_{i=1}^{n} (P_i - p_i)^2$$

$= (P_1 - p_1)^2 + \ldots\ldots + (P_n - p_n)^2$

$= (Kh_1 - p_1)^2 + \ldots\ldots + (Kh_n - p_n)^2$

$= |\,(Kh_1 - p_1, \ldots\ldots, Kh_n - p_n)\,|^2$

$= |\,(Kh_1, \ldots\ldots, Kh_n) - (p_1, \ldots\ldots, p_n)\,|^2$

$= |\,KH - W\,|^2$

$= < KH - W,\ KH - W >$

$= K^2 < H, H > - 2K < H, W > + < W, W >$

where $H = (h_1, \ldots\ldots, h_n)$ and $W = (p_1, \ldots\ldots, p_n)$.

Thus, $e = K^2 < H,H > -2K < H,W > + < W,W >$ *(16)*

Taking partial derivative of e with respect to K and equating it to zero, we obtain $2K < H, H > -2K < H, W > = 0$

This give

$$K = \frac{< H,W >}{< H,H >} \qquad (17)$$

Substituting this value of K in the equation (16), we get

$$e = < W, W > - \frac{< H,W >^2}{< H,H >} \qquad (18)$$

This equation is defined as an error function.

3. Population growth rate of India [11]

Annual percentage growth rate can be calculated from the formula [10]

$$APG = \left(\frac{V_{present} - V_{past}}{N} \right) \times 100$$

where,

APG = Annual percentage growth

$V_{Present}$ = Present or Future Value
V_{Past} = Past or Present Value

The annual percentage growth rate is simply the percent growth divided by N, the number of years.

Actual values of Population of India are collected from International Data Base and using the equations in section 2, actual population and predicted populations are given in Table 1:

Table 1

No	Years	Actual Population	Predicted population	Annual growth rate
1	2008	1140566211	1140566211	1.46
2	2009	1156897766	1151439316	1.43
3	2010	1173108018	1162365099	1.40
4	2011	1189172906	1173342850	1.36
5	2012	1205073612	1184437184	1.33
6	2013	1220800359	1195451329	1.30
7	2014	1236344631	1206580552	1.27

The graph of actual and predicted population values against time is given in Fig1 and Fig 2 respectively.

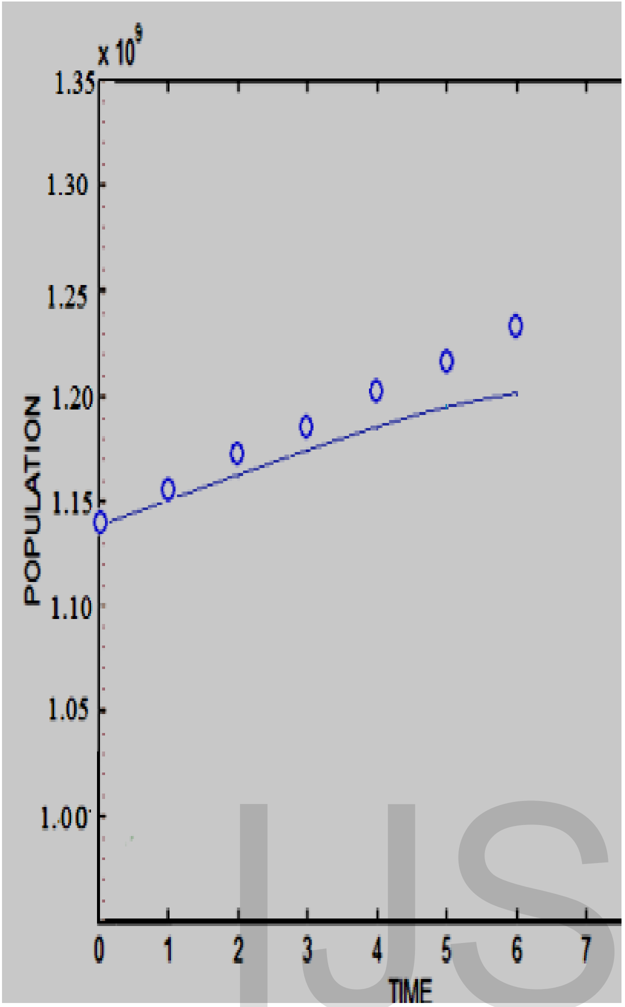

Figure 1

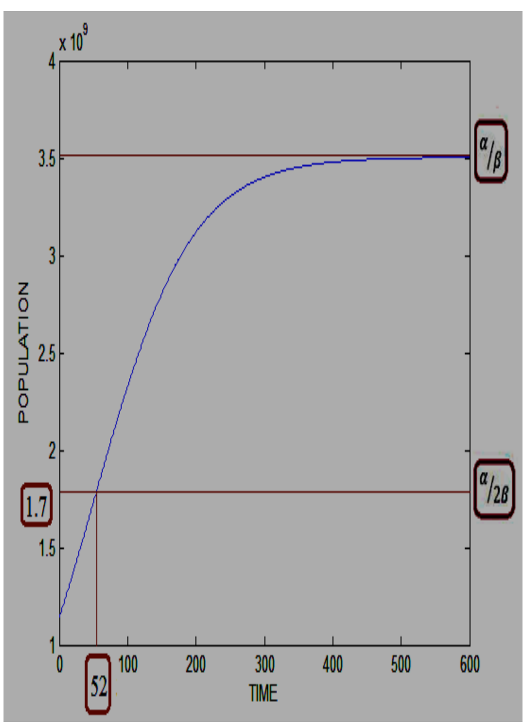

Figure 2

From fig 2, we understand that the population of India will reach the carrying capacity after 600 years.

4. Population growth rate of China [10]

Actual values of Population of China are collected from International Data Base and using the equations in section 2, actual population and predicted populations are given in Table 2.

Table 2

NO	Years	Actual population	Annual growth rate	Predicted population
1	2008	1323480266	0.53	1323480266
2	2009	1330233426	0.51	1327228815
3	2010	1336680939	0.48	1330979756
4	2011	1342946472	0.47	1334733044
5	2012	1349160868	0.46	1338488635
6	2013	1355286257	0.45	1342246483
7	2014	1361344333	0.45	1346006543

The graph of actual and predicted population values against time is given in Fig3 and Fig 4 respectively.

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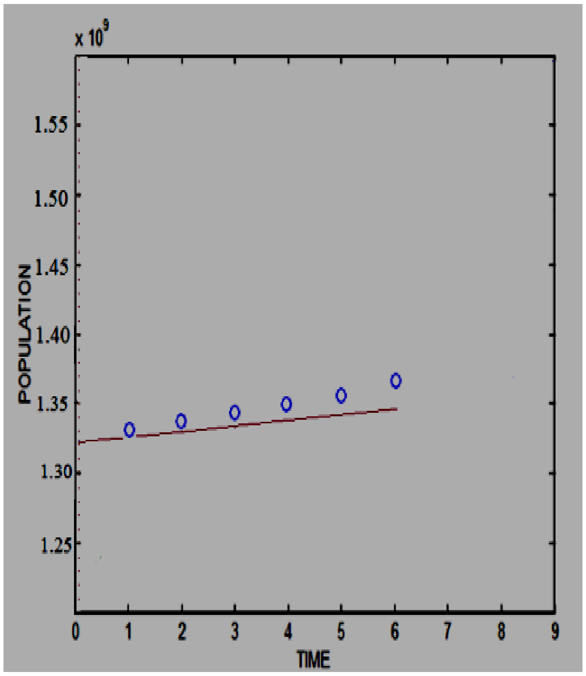

Figure 3

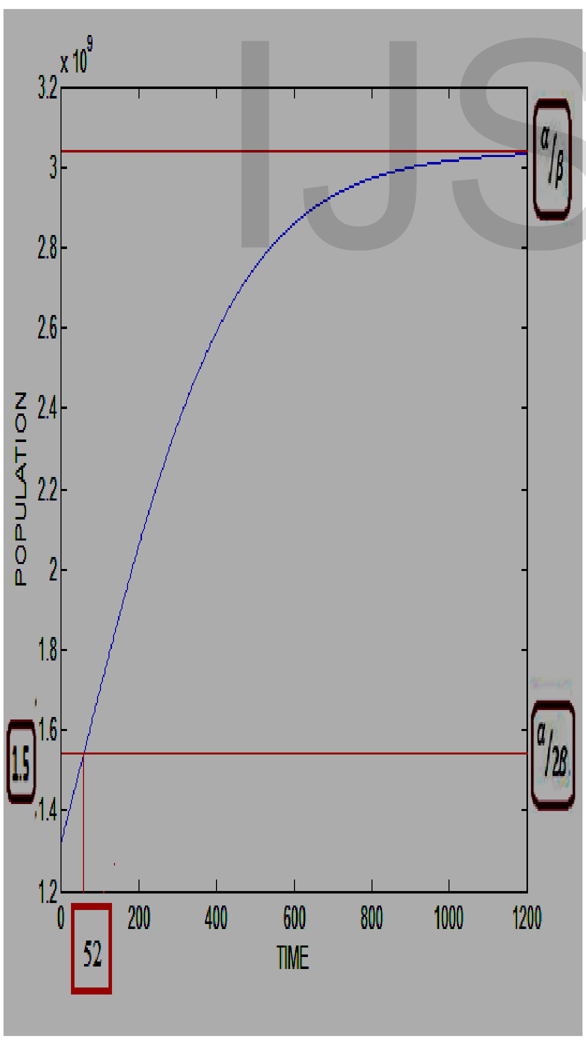

Figure *4*

From fig 4, we understand that the population of China will reach the carrying capacity after 1200 years.

5. Conclusion

5.1 ANALYSIS OF POPULATION GROWTH RATE OF INDIA AND CHINA FROM 2017 -2040

The value of the population growth rate of India is 1.34% and China is 0.49% approximately per annum. In January 2016 India's population is 1304162999 and China's population is 1377583156 and it is collected from country meters. From this data the Carrying capacity of India and China are calculated and presented in Table 3.

Table 3

NO	Years	Carrying Capacity of India (million)	Carrying Capacity of China (million)
1	2017	2.6259	2.7619
2	2018	2.6438	2.7687
3	2019	2.6618	2.7756
4	2020	2.6801	2.7824
5	2021	2.6987	2.7893
6	2022	2.7175	2.7963
7	2023	2.7366	2.8032
8	2024	2.7559	2.8102
9	2025	2.7755	2.8173
10	2026	2.7953	2.8243
11	2027	2.8154	2.8315
12	2028	2.8358	2.8386
13	2029	2.8565	2.8458
14	2030	2.8774	2.8530
15	2031	2.8987	2.8602
16	2032	2.9202	2.8675
17	2033	2.9420	2.8748
18	2034	2.9641	2.8822
19	2035	2.9865	2.8896
20	2036	3.0092	2.8970
21	2037	3.0322	2.9045
22	2038	3.0555	2.9120
23	2039	3.0791	2.9195
24	2040	3.1030	2.9271

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The population and global rank of India and China is calculated and presented in Table 4.

Table 4

No	Year	Predicted population of India (millions)	Predicted population of China (millions)	Global rank of India	Global rank of China
1	2017	1313	1381	2	1
2	2018	1321.9	1384.4	2	1
3	2019	1330.9	1387.8	2	1
4	2020	1340.1	1391.2	2	1
5	2021	1349.3	1394.7	2	1
6	2022	1358.8	1398.1	2	1
7	2023	1368.3	1401.6	2	1
8	2024	1378.0	1405.1	2	1
9	2025	1387.7	1408.6	2	1
10	2026	1397.7	1412.2	2	1
11	2027	1407.7	1415.7	2	1
12	2028	1417.9	1419.3	2	1
13	2029	1428.2	1422.9	1	2
14	2030	1438.7	1426.5	1	2
15	2031	1449.3	1430.1	1	2
16	2032	1460.1	1433.8	1	2
17	2033	1471	1437.4	1	2
18	2034	1482	1441.1	1	2
19	2035	1493.2	1444.8	1	2
20	2036	1504.6	1448.5	1	2
21	2037	1516.1	1452.2	1	2
22	2038	1527.7	1456	1	2
23	2039	1539.5	1459.8	1	2
24	2040	1551.5	1463.5	1	2

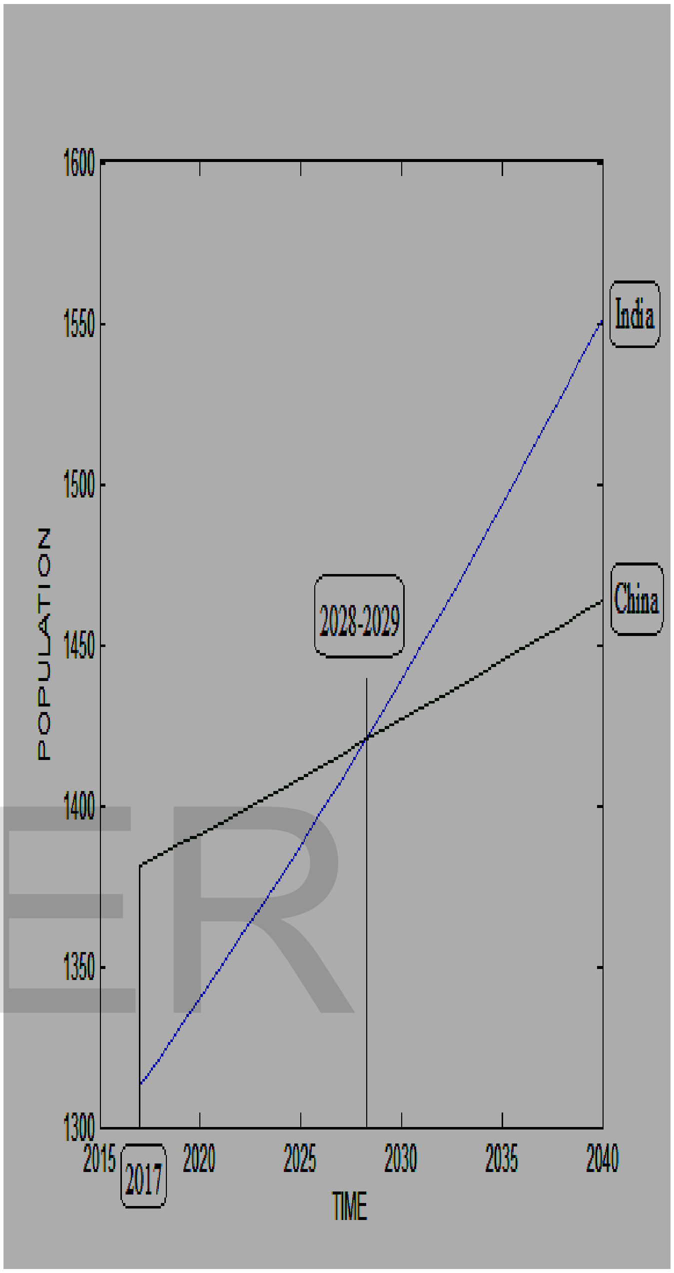

Figure 5: Graph of future population of India and China

5.2 RESULT

From the graph given in fig 5 we can understand that the population of India in 2028-2029 will reach its global rank as **1**. If the Population continues to grow without bound, then India will continue to remain in its global rank as 1 and it is an alarming situation. So the government and people of India must take necessary measures to control increasing Annual growth rate of India.

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