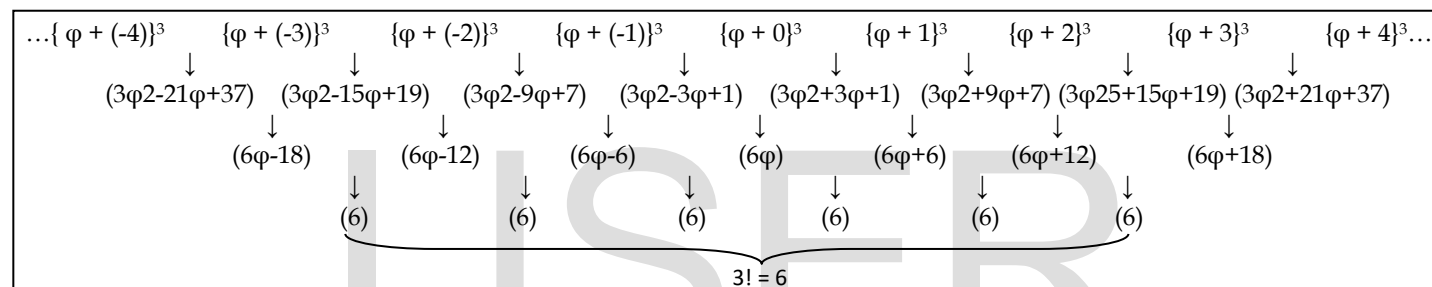


Taking an arbitrary rational number "5/7" as an example in this MJ factorial calculation.

### 2.3 MJ Factorial using Irrational number

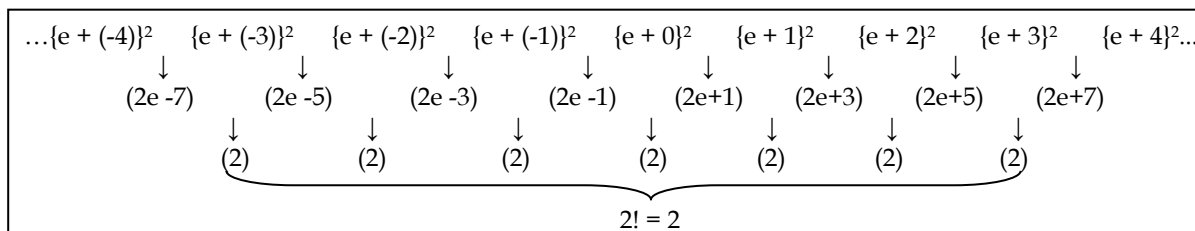
Factorials can also be obtained using Irrational numbers by doing the same procedure as did above with Rational numbers.



"φ" is a greek letter representing the irrational number Golden ratio.

### 3 TRANSCENDENTAL NUMBERS

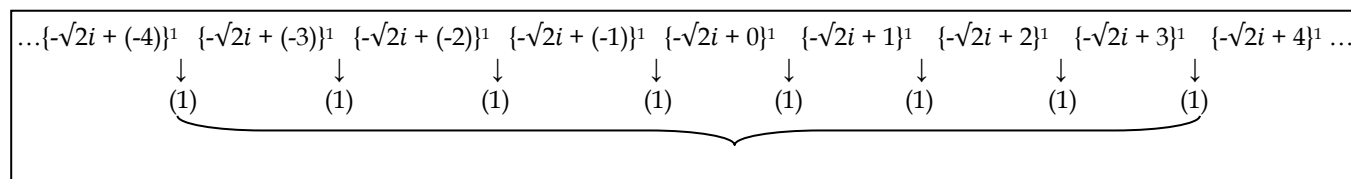
Even transcendental numbers can be used to get Factorials, using MJ Factorials calculation method!



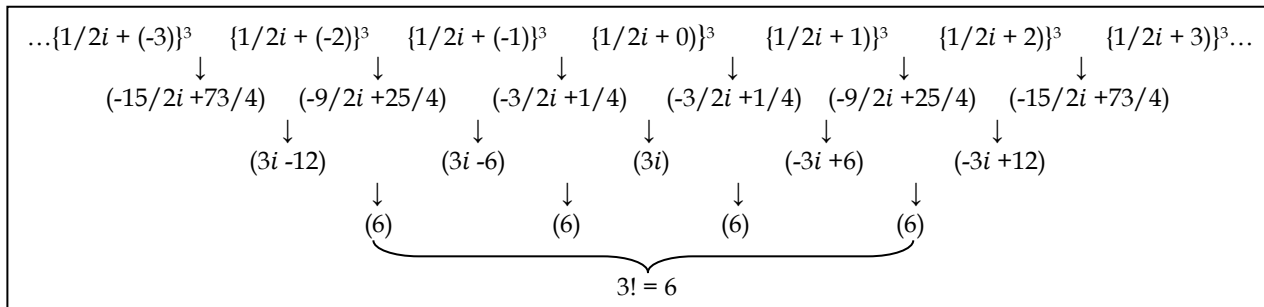
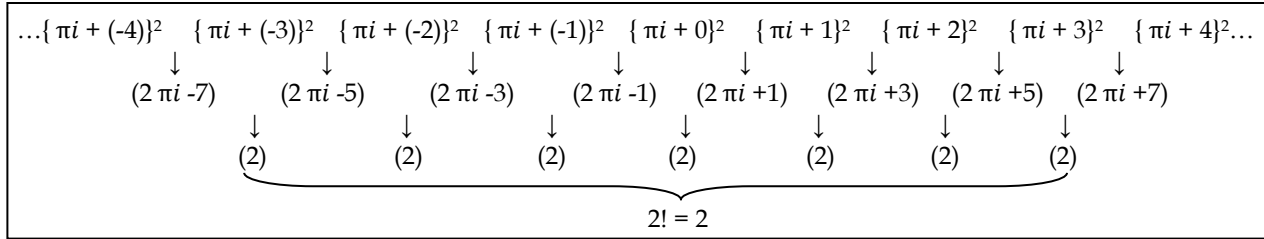
### 4 COMPLEX NUMBERS

Based on the concept of real numbers, a complex number is a number of the form  $a + bi$ , where  $a$  and  $b$  are real numbers and  $i$  is an indeterminate satisfying  $i^2 = -1$ . For getting  $n!$  using complex number, same procedures will be followed as in the

case of real number by raising the sum of a chosen complex number and consecutive integers to the  $n^{\text{th}}$  power and subtract their values  $n$  times repeatedly. For simplicity the value of  $a$  has been taken 0 in the examples shown below.



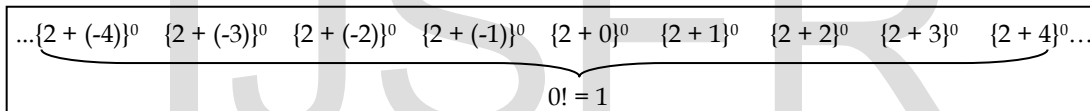
$$1! = 1$$



### 5 0! = 1

And one of the important features of MJ factorials is that it can perfectly explain why  $0! = 1$ <sup>[1]</sup>. According to the definition of MJ form of Factorials, raise the sum of the chosen real or complex number and consecutive integers to the 0 and then subtract

the values zero times which literally means no subtraction at all. For simplicity, take the example of a natural number 2 and add consecutive integers to it and it will give  $0!$ , as shown in the example below.



### 6 MJ FACTORIAL TRIANGLE

Amazingly because of MJ Factorials,  $n!$  can be predicted using 'Staircase of Mount Meru'<sup>[2]</sup>, (which you would probably recognize as Pascal's triangle).

Factorial	Corresponding formulae
0 <sup>th</sup> row	$1a^0$
1 <sup>st</sup> row	$1b_2^1 - 1b_1^1$
2 <sup>nd</sup> row	$1c_3^2 - 2c_2^2 + 1c_1^2$
3 <sup>rd</sup> row	$1d_4^3 - 3d_3^3 + 3d_2^3 - 1d_1^3$
4 <sup>th</sup> row	$1e_5^4 - 4e_4^4 + 6e_3^4 - 4e_2^4 + 1e_1^4$
5 <sup>th</sup> row	$1f_6^5 - 5f_5^5 + 10f_4^5 - 10f_3^5 + 5f_2^5 - 1f_1^5$
6 <sup>h</sup> row	$1g_7^6 - 6g_6^6 + 15g_5^6 - 20g_4^6 + 15g_3^6 - 6g_2^6 + 1g_1^6$
7 <sup>th</sup> row	$1i_8^7 - 7i_7^7 + 21i_6^7 - 35i_5^7 + 35i_4^7 - 21i_3^7 + 7i_2^7 - 1i_1^7$
8 <sup>th</sup> row	$1j_9^8 - 8j_8^8 + 28j_7^8 - 56j_6^8 + 70j_5^8 - 56j_4^8 + 28j_3^8 - 8j_2^8 + 1j_1^8$
9 <sup>th</sup> row	$1k_{10}^9 - 9k_9^9 + 36k_8^9 - 84k_7^9 + 126k_6^9 - 126k_5^9 + 84k_4^9 - 36k_3^9 + 9k_2^9 - 1k_1^9$
10 <sup>th</sup> row	$1l_{11}^{10} - 10l_{10}^{10} + 45l_9^{10} - 120l_8^{10} + 210l_7^{10} - 252l_6^{10} + 210l_5^{10} - 120l_4^{10} + 45l_3^{10} - 10l_2^{10} + 1l_1^{10}$

It is still the same pile of number. I've just reformed it a little bit for making its use in Factorials and to be called as MJ Factorial triangle.

In MJ Factorial triangle variables can be replaced by the sum of any real or complex number and consecutive integers, raise to the  $n$  in  $n^{\text{th}}$  row, but the value of numbers should decrease from left to right. And there is an alternate pattern of - + - + - +

... arithmetic sign in between the different terms of the formulae in MJ Factorial triangle as in the expansion of  $(a - b)^n$ . As in 3<sup>rd</sup> row of "MJ Factorials triangle";  $1c_4^3 - 3c_3^3 + 3c_2^3 - 1c_1^3$ ,  $c$  can be any real or imaginary number but  $c_4 > c_3 > c_2 > c_1$  and they

all should be the sum of real or complex number and consecutive integers raise to the 3rd power. Let's take an example, let  $c_0 = 0^3$ ,  $c_2 = \{1 + (-1)\}^3$  and  $c_1 = \{1 + (-2)\}^3$ , (you can take any desired consecutive integers to the sum of a chosen real or imaginary number, as  $3/2+5$ ,  $\sqrt{n} + 1$ ,  $\sqrt[3]{ki} - 1$  etc). Placing the values in place of variables in the 3rd row of MJ Factorials triangle and solving;

$$= 1 \text{ and now add consecutive integers to it and raise the sum to the } 3^{\text{rd}} \text{ power. As } c_4 = \{1 + 1\}^3, \text{ then } c_3 = \{1 + 1c_4^3 - 3c_3^3 + 3c_2^3 - 1c_1^3\}$$

$$= 1(2)^3 - 3(1)^3 + 3(0)^3 - 1(-1)^3$$

$$= 1 \times 8 - 3 \times 1 + 3 \times 0 - 1 \times (-1)$$

$$= 8 - 3 + 0 - (-1)$$

$$3! = 6$$

### 7 GENERALISING MJ FACTORIAL CALCULATIONS

For the generalized equation of  $n!$  using consecutive subtraction of the power of consecutive numbers, number of terms will be  $2^n$ , in which all the terms get repeated following the

binomial coefficient shown in Pascal's triangle, which is one of the reason to find the hidden factorials in Pascal's triangle.

Number of terms	Generalizations
$2^0$	$0! = k^0$
$2^1$	$1! = \{k^1 - (k + (-1))^1\}$
$2^2$	$2! = [\{k^2 - (k + (-1))^2\} - \{(k + (-1))^2 - (k + (-2))^2\}]$
$2^3$	$3! = [\{k^3 - (k + (-1))^3\} - \{(k + (-1))^3 - (k + (-2))^3\}] - [\{(k + (-1))^3 - (k + (-2))^3\} - \{(k + (-2))^3 - (k + (-3))^3\}]$
$2^n$	$n! = [\{k^n - (k + (-1))^n\} - \{(k + (-1))^n - (k + (-2))^n\}] - [\dots (k + (-n))^n]$

$$10! = 2^8 \times 3^4 \times 5^2 \times 7$$

### 8 PROOF: BOTH REAL AND COMPLEX NUMBER CAN BE USED IN MJ FACTORIAL CALCULATION TO FIND FACTORIALS.

Let's take the example of 2!

$$2! = 1b_3^2 - 2b_2^2 + 1b_1^2$$

{ where  $b_3 > b_2 > b_1$  and all are the sum of consecutive integers and real or complex number which means,  $b_2 = (b_1 + 1)$  and  $b_3 = (b_2 + 1)$ }

Now, placing all the values, we'll get;

$$= \{(b_1 + 2)^2 - 2(b_1 + 1)^2 + b_1^2\}$$

$$= \{b_1^2 + 4b_1 + 4 - 2b_1^2 - 4b_1 - 2 + b_1^2\}$$

$$= 2$$

After cancelling out everything we are left with 2, this shows that whatever be the value of  $b$  either real or complex number we'll always have Factorial in the end.

### 9 Prime (p) Factorization of Factorials

According to the Fundamental theorem of Arithmetic, every integer greater than 1 either is a prime number itself or can be represented as a product of prime numbers<sup>[3]</sup>. Writing a number as a product of prime numbers is called prime factorization. Some of the Factorials' prime factorization (except 0 and 1 factorials) are:

$$2! = 2$$

$$3! = 2 \times 3$$

$$4! = 2^3 \times 3$$

$$5! = 2^3 \times 3 \times 5$$

$$6! = 2^4 \times 3^2 \times 5$$

$$7! = 2^4 \times 3^2 \times 5 \times 7$$

$$8! = 2^7 \times 3^2 \times 5 \times 7$$

$$9! = 2^7 \times 3^4 \times 5 \times 7$$

$$11! = 2^8 \times 3^4 \times 5^2 \times 7 \times 11$$

$$12! = 2^{10} \times 3^5 \times 5^2 \times 7 \times 11$$

$$13! = 2^{10} \times 3^5 \times 5^2 \times 7 \times 11 \times 13$$

$$14! = 2^{11} \times 3^5 \times 5^2 \times 7^2 \times 11 \times 13$$

$$15! = 2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$$

$$16! = 2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$$

$$17! = 2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13 \times 17$$

$$18! = 2^{16} \times 3^8 \times 5^3 \times 7^2 \times 11 \times 13 \times 17$$

$$19! = 2^{16} \times 3^8 \times 5^3 \times 7^2 \times 11 \times 13 \times 17 \times 19$$

$$20! = 2^{18} \times 3^8 \times 5^4 \times 7^2 \times 11 \times 13 \times 17 \times 19$$

$$21! = 2^{18} \times 3^9 \times 5^4 \times 7^3 \times 11 \times 13 \times 17 \times 19$$

$$22! = 2^{19} \times 3^9 \times 5^4 \times 7^3 \times 11^2 \times 13 \times 17 \times 19$$

$$23! = 2^{19} \times 3^9 \times 5^4 \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$$

$$24! = 2^{22} \times 3^{10} \times 5^4 \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$$

$$25! = 2^{22} \times 3^{10} \times 5^6 \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$$

$$26! = 2^{23} \times 3^{10} \times 5^6 \times 7^3 \times 11^2 \times 13^2 \times 17 \times 19 \times 23$$

$$27! = 2^{23} \times 3^{13} \times 5^6 \times 7^3 \times 11^2 \times 13^2 \times 17 \times 19 \times 23$$

$$28! = 2^{25} \times 3^{13} \times 5^6 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23$$

$$29! = 2^{25} \times 3^{13} \times 5^6 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$$

$$30! = 2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$$

$$31! = 2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29 \times 31$$

$$32! = 2^{31} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29 \times 31$$

$$33! = 2^{31} \times 3^{15} \times 5^7 \times 7^4 \times 11^3 \times 13^2 \times 17 \times 19 \times 23 \times 29 \times 31$$

$$34! = 2^{32} \times 3^{15} \times 5^7 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19 \times 23 \times 29 \times 31$$

$$35! = 2^{32} \times 3^{15} \times 5^8 \times 7^5 \times 11^3 \times 13^2 \times 17^2 \times 19 \times 23 \times 29 \times 31$$

$$36! = 2^{34} \times 3^{17} \times 5^8 \times 7^5 \times 11^3 \times 13^2 \times 17^2 \times 19 \times 23 \times 29 \times 31$$

$$37! = 2^{34} \times 3^{17} \times 5^8 \times 7^5 \times 11^3 \times 13^2 \times 17^2 \times 19 \times 23 \times 29 \times 31 \times 37$$

$$38! = 2^{35} \times 3^{17} \times 5^8 \times 7^5 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37$$

$$39! = 2^{35} \times 3^{18} \times 5^8 \times 7^5 \times 11^3 \times 13^3 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37$$

$$40! = 2^{38} \times 3^{18} \times 5^9 \times 7^5 \times 11^3 \times 13^3 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37$$

$k^x$  is being added to  $(\alpha \times k)!$  ( $x$  and  $\alpha$  can be any natural number and  $k$  is always prime number). If  $n = k^x$  then its factorial will be;  $(n - 1)! \times k^x$  and all the factorials after it will be the product

of  $k^{(x+\alpha)}$  for  $(n + \alpha \times k)!$ , where  $\alpha$  depends on how much steps it is far away from  $n$  in terms of  $k$ , until the multiple of  $k^{(x+1)}$ . And if a number is not equal to  $\alpha \times k$  then the exponent of  $k$  in

all these instructions one can predict  $n!$ . Let's take an example of 15!. Write down all the prime numbers  $\geq 15$  (15#, '# is pronounced as Primorial<sup>[4]</sup>);  $2 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13$ . Now think of a number close to  $k^x$  (where  $k = p$ ) for each listed prime number. Let's think of 2, a number close to 15 ( $<15 <$ ) which would be  $2^n$  is both 16 and 8. If you consider 8 ( $2^3$ ) then its factorial will be the product of  $2^{(2^0 + 2^1 + 2^2)} = 2^7$  and 15 is 3 step away from 8 (in respect to 2). So, from 8! to 10! there will be the multiplication  $2^1$  but from 10! to 12!  $2^2$  will join the product because  $2^2$  is the multiplicative factor of 12 ( $3 \times 2^2$ ). Then from 12! to 14! there will be the multiplication of  $2^1$  to the product of 12!. Now, 15! will be the product of  $2^7 \times 2^{(1+2+1)} = 2^{11}$ . Another way of approaching  $2^n$  for 15! is by finding relation with 16 ( $2^4$ ). One of the factor in the prime factorization of 16! will be  $2^{(2^0 + 2^1 + 2^2 + 2^3)} = 2^{15}$ . To get 16! we need to multiply 15! by 24, so to get 15! back we've to divide 16! By 24 ( $2^{(15-4)} = 2^{11}$ ). Doing the same procedure for all the above listed prime numbers we'll get  $2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13 = 1307674368000$  (15!).

## 10 Addition rules for Factorials

The recursive definition of factorials is:

$$\begin{aligned} (n + 1)! &= n! \times (n + 1) \\ \text{(and now distribute the RHS)} \\ (n + 1)! &= n! \times n + n! \end{aligned}$$

So this means, one can get  $(n + 1)!$  using  $n!$ , but what one can do to find  $(n + k)!$  using  $n!$  (where  $k$  is a positive integer). We know  $(5 - 4) = 1$  because they are consecutive, but  $(5 - 2) = ?$  5 and 2 are not consecutive integers but we know that they are distant by  $x$  number of consecutive integers. So, we can solve this problem as

$$\begin{aligned} (5 - 2) &= (5 - 4) + (4 - 3) + (3 - 2) \\ &= 1 \times 3 \\ &= 3 \end{aligned}$$

In case of factorials we know that  $(n + 1)! - n! = n! \times n$ . We can use this fact to determine  $(n + k)! - n!$  using the same logic as I did above with positive integers.

$$\begin{aligned} (5! - 2!) &= (5! - 4!) + (4! - 3!) + (3! - 2!) \\ (4! - 2!) &= 2! \times 3 \times 4 \text{ and } 3! = 2! \times 3, \text{ in respect to } 2) \\ &= (2! \times 3 \times 4^2) + (2! \times 3^2) + (2! \times 2) \\ &= 2! (2 + 3^2 + 3 \times 4^2) \end{aligned}$$

The generalization of the difference of two factorials in terms of  $n$ ;

$$(n + k)! - n! = n! \{n + (n + 1)^2 + (n + 1) \times (n + 2)^2 + (n + 1) \times (n + 2) \times (n + 3)^2 + \dots + (n + 1) \times \dots \times (n + k - 2) \times (n + k - 1)^2\}$$

the prime factorization of it's factorials will remain the same as in the previous factorial, i.e.,  $\alpha \times k$ . If  $n = k^x$  then  $n!$  will have  $k^{(k^0+k^1+k^2+\dots+k^{x-1})}$  as one of it's prime factors. With the help o

## 11 Conclusion

In the early 12<sup>th</sup> century Factorials were used to count Permutations by Indian scholars<sup>[5]</sup>. But now, it has been found buried deep inside the root of many areas of mathematics as in combinations, permutations, algebra via the binomial coefficients, calculus, probability theory, number theory and much more. There is also a connection of hyper cube with factorials. But the computation of factorials is not much efficient. Surely, MJ factorial triangle, some rules for the power of prime numbers appear in the prime factorization of factorials, generalization of MJ factorials calculation method and addition rules for factorials which I've developed in this paper will also help to improve the efficiency of the computation of factorials. MJ Factorial triangle also shows the relation of factorials with Pascal's triangle. The technique that I developed in this manuscript to find factorials using Pascal's triangle is neither purely the binomial expansion nor polynomial expansion, it is something more interesting! The fact that we can use either real or complex numbers to find  $n!$  is showing relation of factorials with the whole set of Complex numbers and may be with the help of these we can even extend factorials. I'm still working on it and there is a lot more hidden in factorials.

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Number of terms =  $k$

With the help of the above equation any factorials of large value can be obtained using smaller factorials by summing up the above equation with the smaller factorial, respect to which the equation is arranged.