Improved Space-Time Coding Scheme over Nakagami Fading Channels

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Abstract—In this paper, the performance of space-time coding scheme is investigated over Nakagami fading channels. In the first part, the performance of super-orthogonal space-time trellis code is presented while the performance of its concatenated version is presented in the second part. The concatenation coding scheme with iterative decoding involves convolutional code as the outer code and super-orthogonal space-time trellis code (SOSTTC) as the inner code. The pairwise error probability (PEP) for the coding schemes were derived and their performances evaluated by computer simulation. Simulation results shows that the diversity order of the coding schemes guaranteed by quasi static fading channel increases by m times in the presence of Nakagami fading with inverse fading parameter m and the coding gain varies with the general Nakagami fading channel.

Index Terms— Channel, coding gain, diversity, Nakagami, pairwise error probability, space-time, super-orthogonal space-time trellis code

1 INTRODUCTION

The information capacity gain of a wireless system can be increased by employing multiple transmit antenna and/or receive antenna in a communication system [1-5]. Space-time coding (STC), which combine in its design, channel coding, modulation, transmit diversity and/or receive diversity, has been introduced as a power and bandwidth efficient method of communication over fading channels [6].

A class of new STCs known as super-orthogonal space-time trellis codes (SOSTTC), was introduced in [7-8]. These codes are concatenations of a super set of orthogonal space-time block codes with space-time trellis codes. SOSTTC combines the set partitioning principle in [9] and a super set of orthogonal space-time block code (STBC) in a systematic way, to provide full diversity and improved coding gain over the earlier space time trellis code (STTC) schemes [10]. SOSTTC are full rates and full diversity STCs that provides improved coding gain. The transmission matrices of SOSTTC for two transmit antenna is given as [4]

\[
C(x_1, x_2, \theta) = \begin{pmatrix}
    x_1 e^{j\theta} & x_2 \\
    -x_2^* e^{j\theta} & x_1^*
\end{pmatrix},
\]

(1)

where for M-PSK signal constellations, the signals x1 and x2 which are selected by input bits can be represented by \(e^{j2\pi l/M}\), where \(l = 0, 1, ..., M-1\) and \(\theta\) which is the rotation angle can take on the values \(\theta = 2\pi l'/M\), where \(l' = 0, 1, ..., M-1\).

The first row corresponds to the symbols transmitted in time slot 1 and the second row corresponds to the symbol in time slot 2. The first column corresponds to the symbols transmitted by antenna 1, while the second column to the symbol by antenna 2.

SOSTTCs are designed based on the rank and determinant criteria and its trellis structure has a large number of parallel transitions. In [11], a new SOSTTC was designed for fast fading Rayleigh channel and in [12] the pair-wise error probability (PEP) was obtained for SOSTTC. The generator matrix notation for SOSTTC was recently introduced in [13] to allow for systematic and exhaustive computer search for optimal codes with higher number of states. Rules that govern the components of the generator matrix were given in [14].

The invention of turbo code with its astonishing performance has attracted the interest of researchers into the subject of concatenated coding scheme in recent times. Turbo codes which are built from parallel concatenation of convolutional codes with iterative decoding perform close to the Shannon limit in AWGN channels [15]. Serially concatenated convolutional codes were investigated in [16] with turbo principles while in [17] hybrid concatenated convolutional codes were proposed with a Soft-Input Soft-Output (SISO) maximum a posteriori decoding module.

To improve the coding gain of STC, various concatenated topologies have been proposed in literature with reported improve performance over conventional ST codes [18-25]. In [23], serial concatenated space-time trellis code (STTC) was proposed while in [17], double concatenated scheme was proposed which consist of a serial concatenation of a parallel concatenated convolutional code with STTC.

The Nakagami fading model is a more versatile fading model that is based on Nakagami distribution also called the m distribution [27, 28]. The Nakagami distribution include the Rayleigh distribution and one sided Gaussian distribution as two special cases, and can model fading channel that are more or less severe than that of the Rayleigh distribution. Since Rayleigh fading cannot account for large-scale effect of shadowing, the Nakagami-distributed fading may be

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encountered in practical situations especially in mobile wireless communications [27].

In [27], the performance of STTC in Nakagami fading channel was investigated and an upper bound was obtained for the pairwise error probability. It was shown that the diversity order of STTC guaranteed by quasi static fading channel increases by m times in the presence of Nakagami fading with inverse fading parameter m and that the coding gain varies with the general Nakagami fading channel. In [29], the performance of STTC designed based on the Euclidean distance criteria (EDC) was also investigated and it was shown that STTC designed for Rayleigh fading channel are suitable for Nakagami fading channel. In [30], the performance of STBC over Nakagami-m fading channel and a closed form expression for the exact symbol error rate for orthogonal space time block code over independent identically distributed Nakagami-m fading channel was obtained. Also in [31], the PEP for space-time codes in Rician-Nakagami channels was obtained while in [32], the performance of MIMO systems through Nakagami Fading channels with arbitrary fading parameter was conducted.

In this paper, the performance of space-time coding scheme is presented over Nakagami fading channel. In the first part, the performance of designed 16, 32 and 64-states SOSTTC is investigated over Nakagami fading channel. In order to improve the performance of the coding scheme, two concatenated coding scheme with constituent code of SOSTTC and convolutional code is presented in the second part. The first concatenated scheme consists of a serial concatenation of a convolutional code with a SOSTTC (CC-SOSTTC) while the second involves parallel concatenation of two serially concatenated convolutional and SOSTTC codes (HC-SOSTTC). The two schemes are from [33], but are now investigated over Nakagami-m fading channel. Simulations results are presented for the case of two transmit and one receive antenna in quasi-static Nakagami-m fading channels.

The rest of the paper is organized as follows. Section II describes the system model consisting the channel model, the encoder and the decoder structure. In section III, the PEP of the coding schemes is presented. The performance of the concatenated scheme is evaluated by computer simulations in section IV, while conclusions are presented in section V.

2 SYSTEM MODEL

2.1 Channel Model

We consider a quasi-static Nakagami fading channel with the fading amplitude of \( \alpha \). The PDF of \( \alpha \) is given by

\[
(2) \quad p(\alpha) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^m \alpha^{2m-1} e^{-\frac{\alpha^2 m}{\Omega}},
\]

where \( \Gamma(x) \) denotes the Gamma function of \( x \), and

\[
(3) \quad \Omega = E[\alpha^2],
\]

The notation \( E[x] \) denotes the expected value of \( x \) and \( m \) is the inverse fading parameter. When \( m = 1 \), we have Rayleigh fading and when \( m = \infty \) we have non fading channel.

Consider a coding scheme with \( n_T \) transmit antenna and \( n_R \) receive antenna. The signal at each receive antenna is a mixture of the faded signal and noise. In this paper, a quasi-static fading channel is assumed in which the fading is constant over a frame of length L and vary from frame to frame. Assuming that the transmitted symbol from the i-th antenna at time \( t \) is \( x_i^t \), and the receive symbol at time \( t \) of the receive antenna \( j \) is \( r_j^t \), then

\[
(5) \quad r_j^t = \sum_{i=1}^{n_T} h_{i,j} x_i^t \sqrt{E_s} + n_j^t,
\]

where \( h_{i,j} \) is the complex path gain from transmit antenna \( i \) to receive antenna \( j \), whose envelopes obey the Nakagami \( m \)-distribution with identical \( m \) and \( \Omega \) and \( \sqrt{E_s} \) is the energy per symbol. The \( n_j^t \) symbol is the noise term modeled as independent samples of a zero mean complex Gaussian random process with variance \( N_0/2 \) per dimension.

2.2 System Model

SOSTTC: Fig. 1 shows a SOSTTC coding scheme with \( n_T \) transmit antenna and \( n_R \) receive antenna. The signal at each receive antenna is a mixture of the faded signal and noise. The information signal is SOSTTC encoded before transmission by the \( n_T \) transmit antenna. At the receiver, matched filtering is performed and the received signal is decoded using Vertibi decoding algorithm.

![SOSTTC system model](http://www.ijser.org)
CC-SOSTTC Encoding: The encoder block diagram of the CC-SOSTTC is shown in Fig. 2.0. In the system, a block of \( N \) independent data bits is encoded by the convolutional outer encoder and the output block of the coded bits is interleaved by using a random bit interleaver (\( \pi \)). The interleaved sequences are then passed to the SOSTTC encoder to generate a stream of QPSK symbols which is transmitted from the antennas.

\[ \text{Source bit} \xrightarrow{\text{CC Encoder}} \xrightarrow{\pi} \text{SOSTTC Encoder} \]

Fig. 2.0: Encoder block diagram of the CC-SOSTTC system [34]

CC-SOSTTC Decoding: The simplified block diagram of the CC-SOSTTC decoder is shown in Fig. 3.0. The subscript of the \( c \) or \( u \) specifies the decoder where \( st \) is used for the SOSTTC encoder and \( cc \) is used for the convolutional encoder. The coded intrinsic LLR for the SOSTTC SISO module is computed as

\[
\lambda(c_{st}, I) = -\frac{1}{2\sigma^2} \sum_{j=1}^{n_{c}} \sum_{i=1}^{n_{c}} \rho_{i,j} s_{0}^{j} \left[ 1 - \frac{1}{2\sigma^2} \sum_{j=1}^{n_{c}} \sum_{i=1}^{n_{c}} \rho_{i,j} s_{0}^{j} \right]^{2} \]

(6)

where \( s_{0}^{j} \) is the reference symbol and \( \sigma^2 \) is the variance of the AWGN.

The SOSTTC SISO takes \( \lambda(c_{st}, I) \) and the \textit{a priori} information from the CC-SISO which is initially set to zero to compute the extrinsic information \( \hat{\lambda}(c_{st}, O) \). This extrinsic information is de-interleaved (\( \pi^{-1} \)) and fed to the CC-SISO to become its \textit{a priori} information \( \lambda(c_{cc}, I) \). The \textit{a priori} information is then used to compute the extrinsic LLR \( \hat{\lambda}(c_{cc}, O) \) for the convolutional code SISO (CC-SISO). The extrinsic LLR is then interleaved to become the \textit{a priori} information \( \lambda(u_{st}, I) \) for the SOSTTC SISO for the next iteration. During the first iteration, \( \lambda(u_{st}, I) \) is set to zero as no \textit{a priori} information is available at the SOSTTC-SISO. The transmitted source symbols are assumed to be equally likely and therefore the input LLR \( \lambda(c_{cc}, I) \) to the CC-SISO is permanently set to zero. The process is iterated several times and on the final iteration, a decision is taken on the extrinsic information \( \hat{\lambda}(c_{cc}, O) \) to obtain the estimate of the original transmitted bit stream.

\[ \text{SOSTTC SISO} \xrightarrow{\pi} \text{CC-SISO} \]

Fig. 3.0: The block diagram of the CC-SOSTTC decoder [34]

HC-SOSTTC Encoding: In Fig. 4.0, the transmitting block diagram of the HC-SOSTTC system is shown. The HC-SOSTTC topology consists of a parallel concatenation of two serially concatenated schemes. Each of the serial concatenated schemes consists of an outer convolutional code concatenated via an interleaver with an inner SOSTTC encoder. In the system, a block of \( N \) independent bits is encoded by the convolutional outer encoder (CC1) of the upper serial part of the scheme. The output of the upper convolutional encoder is then passed through a random bit interleaver (\( \pi_{1} \)). The permuted bits from the interleaver are then fed to the upper SOSTTC encoder to generate a stream of complex data that are transmitted from each of the transmit antennas using the SOSTTC transmission matrix.

In the lower serial part of the encoding, the lower convolutional encoder (CC2) receives the permuted version of the block of \( N \) independent bits and generates blocks of coded bits which are passed through another interleaver (\( \pi_{2} \)) to the lower SOSTTC encoder. The complex data from the output of the lower SOSTTC encoder are transmitted from the transmit antennas. It should be noted that the same convolutional and SOSTTC codes are used in the upper and lower encoding of the systems. Each of the encoders is terminated using appropriate tail bits. All the four transmit antennas are well separated by at least half of the wavelength of the signal.

\[ \text{bits} \rightarrow \text{CC1} \rightarrow \pi_{1} \rightarrow \text{SOSTTC Encoder1} \]

Fig. 4.0: Encoder block diagram of the HC-SOSTTC system [34]

Decoding of HC-SOSTTC: The HC-SOSTTC decoder consists of two serial arms and one parallel sector as shown in Fig. 5.0. The decoder is specified by the subscript of the \( c \) or \( u \), where for the upper SOSTTC encoder \( st1 \) is used, and \( st2 \) is used for the lower SOSTTC encoder, 1 is used for the upper convolutional encoder CC1 while 2 is used for the lower convolutional encoder CC2. The coded intrinsic LLR for the SOSTTC SISO module is computed as in (5).
The SOSTTC1 SISO takes the intrinsic LLR \( \lambda(c_{u1}, I) \) and the a priori information from the CC1 SISO which is initially set to zero and computes the extrinsic LLR \( \hat{\lambda}(c_{u1}, O) \). This extrinsic LLR from the SOSTTC1 SISO is passed through the interleaver \( \pi^{-1} \) to obtain \( \lambda(c_1, I) \).

The LLR’s output of the CC1 SISO module which are \( \lambda(c_1, O) \) and \( \lambda(u_1, O) \) are calculated. The LLR \( \lambda(c_1, I) \) is subtracted from \( \lambda(c_1, O) \) to obtain the LLR \( \hat{\lambda}(c_1, O) \) which is then sent via interleaver \( \pi_1 \) to obtain the intrinsic information \( \lambda(c_{u1}, I) \) for the SOSTTC1-SISO for the next iteration.

For the lower parallel arm, the SOSTTC2 SISO takes the intrinsic LLR \( \lambda(c_{u2}, I) \) and the a priori information from the CC2 SISO, which is also initially set to zero, and computes the extrinsic LLR \( \hat{\lambda}(u_{2}, O) \). This extrinsic LLR from the SOSTTC2 SISO is passed through the interleaver \( \pi^{-1} \) to obtain \( \lambda(c_2, I) \). The LLRs \( \lambda(c_2, O) \) and \( \lambda(u_2, O) \) from the output of the CC2 SISO module are then calculated. The LLR \( \lambda(c_2, I) \) is subtracted from \( \lambda(c_2, O) \) to obtain the LLR \( \hat{\lambda}(c_2, O) \) which is then passed through the interleaver \( \pi_2 \) to obtain the intrinsic information \( \lambda(c_{u2}, I) \) for the SOSTTC1-SISO.

For the parallel interconnection component of the iterative decoding process, the LLR \( \hat{\lambda}(u_{2}, O) \) obtained by subtracting the LLR \( \lambda(u_{2}, I) \) from the LLR \( \lambda(u_{2}, O) \) is sent via the de-interleaver \( \pi^{-1} \) to obtain the LLR \( \hat{\lambda}(u_{1}, I) \) which is the uncoded a priori information from the CC2 SISO into the CC1 SISO. Also the LLR \( \hat{\lambda}(u_{1}, O) \) obtained by subtracting LLR \( \lambda(u_{1}, I) \) from the LLR \( \lambda(u_{1}, O) \) is sent via the interleaver \( \pi \) to obtain the LLR \( \hat{\lambda}(u_{2}, I) \) which is the uncoded a priori information from the CC1 SISO into the CC2 SISO.

The process is iterated several times and the bit with the maximum APP is chosen by the decision device in the last iteration using the summed values of the output uncoded LLRs of both the CC1 and CC2 SISO decoders.

![Decoding block diagram of the HC-SOSTTC system](http://example.com/figure5.png)

3 **Pairwise Error Probability**

Let the transmitted codeword and the erroneously decoded codeword be denoted by \( \hat{C} \) and \( C \) respectively. If we denote the symbol-wise Hamming distance between \( C \) and \( \hat{C} \) by \( d(C, \hat{C}) \) and assume maximum likelihood (ML) decoding, the conditional PEP that the receiver will select \( \hat{C} \) over \( C \) conditioned on the channel gains assuming perfect channel state information (CSI) at the receiver is given by [6]

\[
P(C \rightarrow \hat{C} \mid H) = Q\left(\frac{E_c d^2}{2 N_0} \sum_{i=1}^{n_r} \sum_{j=1}^{n_k} |h_{i,j}|^2\right),
\]

where \( d^2 = \sum_{l=1}^{d(C, \hat{C})} C(l) - \hat{C}(l) \) is the squared Euclidean distance of the outer code.

By using \( Q(x) = \exp(-\frac{x^2}{2}) \), we have

\[
P(C \rightarrow \hat{C} \mid H) = \exp\left(-\frac{E_c d^2}{4 N_0} \sum_{i=1}^{n_r} \sum_{j=1}^{n_k} |h_{i,j}|^2\right).
\]

The amplitude of \( h_{i,j} \) are identically independent m-distributed and the pdf of \( |h_{i,j}|^2 \) is given by
\[ P(|h_{i,j}|^2) = \frac{1}{\Gamma m(\Omega)} \left( \frac{m}{\Omega} \right)^m |h_{i,j}|^{2(m-1)} e^{-|h_{i,j}|^2/\Omega}, \quad (9) \]

where \( \Omega = E[|h_{i,j}|^2] \) and

\[ m = \frac{\Omega^2}{E[|h_{i,j}|^2 - \Omega^2]}, \quad m \geq 1/2. \]

If (8) is average with respect to the distribution of \(|h_{i,j}|^2\), the PEP can be approximated at high SNR as

\[ P(C \rightarrow \hat{C}) \approx \text{fm} \left( \frac{E_s d^2}{4 N_0} \right)^{-mn_f n_g} \]

\[ = \text{fm} \left( \frac{d^2(C, \hat{C})}{4 N_0} \right)^{-mn_f n_g}, \quad (10) \]

where

\[ f(m) = (m/\Omega)^{m-n_f n_g} \Gamma(m)^{n_f n_g - 1}. \]

In Rayleigh fading channel, \( m = 1 \) and \( \Omega = 1 \) and (10) is simplified as

\[ P(C \rightarrow \hat{C}) = \left( \frac{E_s d^2}{4 N_0} \right)^{-mn_f n_g} \quad (11) \]

Equation (10) shows that the diversity order of coding scheme guaranteed by slow fading channel increase by \( m \) times in the presence of general Nakagami fading with inverse fading parameter, and the coding gain is multiplied by a factor of \( f(m)^{-1/mn_f n_g} \).

4 RESULTS AND DISCUSSIONS

In this section, we evaluate the performance of the space-time coding scheme and its concatenated version by computer simulation under a narrow band frequency flat Nakagami fading channels. Narrow band transmission is assumed. Therefore, the results illustrate the performance in time division multiple access (TDMA) type systems, like the global system for mobile communication (GSM), IS 136, or enhance data rates on GSM Evolution (EDGE). For all the simulations, 130 symbols per frame are transmitted from each transmit antenna and CSI is assumed at the receiver. It is assumed that the fading channel is quasi-static i.e. the fading channel coefficient is constant over one frame but varies from one frame to another.

Figs. 6, 7 and 8 present FER performance curves for 16-, 32- and 64-state QPSK SOSTTC for one receive antenna over Nakagami fading channels with fading parameters \( m = 0.5, 1, \) and 2, respectively. In Fig. 6, the simulation result for 16-state STTC from [6] for \( m = 2 \) using 1 receive antenna is also shown.

From the FER performance curve, it is observed that the diversity order of the system in a Rayleigh fading channel is a multiple of the fading parameter \( m \) in a Nakagami fading channel. Also, the coding gain achieved by the system is also seen to increase with a multiple of \( m \). The 16-state SOSTTC is observed to maintain its superior coding gain advantage over the STTC counterpart over Nakagami-m fading with no diversity order advantage.
For the concatenated part, two transmit antennas and a single receive antenna are used for each part of the connection. For the outer code, the RSC rate-1/2, 4-state convolutional codes are employed for the CC-SOSTTC and the HC-SOSTTC. The 16 states SOSTTC presented in [23] are used as the inner code.

Fig. 9 shows the FER performance of the CC-SOSTTC over Nakagami fading channels with $m = 0.5$, 1, and 2, respectively, where the constant $m$ denote the fading parameter with $m = 1$ and $m = \infty$ corresponding to the Rayleigh and non-fading channel, respectively. As can be observed from the FER plot, the diversity order of the concatenated grows linearly with the fading parameter $m$, which agrees with the observation from the pairwise error probability analysis. The coding gain is also observed to increase with an increase in $m$ value. The CC-SOSTTC topology achieves a diversity order of 1 for $m = 0.5$, diversity order of 2 for $m = 1$ and diversity order of 4 for $m = 2$.

Fig. 10 shows the FER performance of the HC-SOSTTC over Nakagami fading channels with $m = 0.5$, 1, and 2, respectively. As can also be observed from the figure, the diversity order of the concatenated increases linearly with the fading parameter $m$, which is consistence with the pairwise error probability analysis. The topology achieves a diversity order of 2 for $m = 0.5$, diversity order of 4 for $m = 1$ and diversity order of 8 for $m = 2$. The coding gain is also observed to increase with an increase in $m$ value.

## 5 CONCLUSION

In this paper, the performance of space-time time coding scheme is evaluated over Nakagami-$m$ fading channel. The first consisted the performance of 16, 32, and 64-state SOSTTC while the second part consisted its concatenated version. The CC-SOSTTC system consisted of a serial concatenated convolutional code and an inner SOSTTC, while the HC-SOSTTC system consisted of parallel concatenation of two serially concatenated convolutional and SOSTTC codes. The encoding and the iterative decoding of the two topologies were discussed. Simulation results are presented for the case of quasi-static Nakagami fading channel involving 130
symbols per frame length from each transmit antenna. The PEP for the coding schemes was presented and it was shown that the diversity order of the coding scheme is a multiple of the fading parameter m. Results shows that the diversity order of the schemes guaranteed by quasi static fading channel increases by m times in the presence of Nakagami fading with inverse fading parameter m and the coding gain varies with the general Nakagami fading channel.

REFERENCE


