Image Compression using Wavelet Transforms of DCT, DST, Hartley and Real-DFT with Variation in Size of Component Transforms

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ABSTRACT- This paper presents simple wavelet based image compression method where wavelet transform is generated from orthogonal component transforms. Wavelets of DCT, DST and Real-DFT and Discrete Hartley Transform (DHT) are generated. Each generated wavelet transform is applied separately on R, G, and B plane of 256x256x3 colour image. Performance of these wavelet transforms is evaluated using two parameters. One is compression ratio and other is rate- distortion graph. Size of component transforms in their respective wavelet transform is varied and results are compared. From results it is observed that DCT and Real-DFT wavelet gives better compression using component transform size of m=8 and n=32. DST wavelets show considerable blocking effect. Blocking effect is more intense if local component transform size (i.e. 'n') is larger. These wavelet transforms are compared in terms of bit rate as a function of distortion. It shows that DCT wavelet gives best performance among all four. Real-DFT ranks second followed by Hartley wavelet and then Discrete Sine wavelet transform.

Index Terms- Wavelet Transform, DCT, DST, Real-DFT, Hartley Wavelet Transform, Compression Ratio, Bit Rate.

1. INTRODUCTION

Images are essential elements of communication. With increasing use of internet and multimedia use of images to convey the information has been increased. More use of images indicates more time, storage space and bandwidth required to process, store and transmit the information. Solution for this is to represent the information in compact form. Hence image compression is used to reduce the bandwidth required for image transmission. In order to compress the image, redundancies must be exploited [1].

Many techniques of image compression have been experimented and used. Use of transforms in image compression is not new. An effective transform will concentrate useful information into a few of low frequency transform coefficients. Discrete cosine Transform (DCT) is most widely used in image and video compression. DCT maps a block of pixel colour values in spatial domain to values in frequency domain [2]. For larger size images DCT coefficients are stored with insufficient accuracy resulting in deteriorated image quality. Hence image is divided into pxp size block and DCT is applied on each block. But it introduces blocking artifacts in images [3]. Wavelet transforms are now being adopted for various applications like biometric identification [4,5] image steganography [6] and many more. It is good tool to replace the Fourier Transform [7,8]. Since there is no need to block the images wavelet transform coding avoids blocking artifacts at higher compression ratio. Hence it is preferred over simple orthogonal transform. Compression ratio and bit rate [9] are two important parameters to measure effectiveness of lossy image compression technique. In lossy data compression decompressed data need not have to be same as original data. Shannon developed a rate- distortion theory for lossy data compression. It says that if D is tolerable amount of distortion then R(D) is the best possible compression rate or if R is minimum bit rate used to transfer data then D is maximum possible error at that bit rate. In rate distortion theory, rate is normally understood as number of bits per data sample to be stored or transmitted.

2. RELATED WORK

Many transform based and wavelet transform based techniques have been studied with variations. Warped DCT for image compression has been proposed by Mitra and Nam Cho in [10]. It computes the DCT samples by warping the frequency axis by means of an all pass transform. Depending on the frequency distribution of the input, the input signal is warped by all pass filters. So it has frequency distribution that is more suitable for DCT. A lossless image compression technique using combination of approximate matching and run length coding is proposed in [11] by Sumir Kumar,
It uses intrinsic property of the images that they have similar pattern in localized area of image. Multi wavelet transform based on zero tree coefficient shuffling has been proposed in [12]. Image compression based on luminance and chrominance using binary wavelet transform and raster line technique is proposed by Thipanna and Reddy in [13]. A non-linear transform called peak transform is proposed in [14]. It minimizes the high frequency components in the image to a greater extent thus making the image to get compressed to a greater extent. Performance of column, row and full transform for image compression is evaluated in [15]. Column transform can be used for image compression instead of full transform at the cost of slight increase in RMSE. But it saves number of computations required. A simple method of orthogonal wavelet transform generation is proposed in [16]. Apart from Harr wavelet which was studied till now, wavelets of Walsh, Cosine, Hartley and Kekre transform have been proposed in [166]. Based on wavelet generation technique proposed in [166], wavelets of Walsh, Cosine, Sine, Slant and Kekre are generated in [17] and performance of column, row and full wavelet transform is compared. Column wavelet transform can be used in place of full wavelet transform as its performance is approximately same as full wavelet transform. It helps to save the computations. Also image compression using new transform Real-DFT that combines features of sine and cosine with only real values is proposed in [18].

3. PROPOSED METHOD

3.1 Generation of Wavelet Transform

Wavelet transform is generated using two orthogonal transform matrices as its component transforms. Kronecker product of two component transforms of different sizes give wavelet transform of required size. Let 'A' is of size MxM and 'B' is of size NxN then wavelet transform 'TAB' is of size MNxMN. It is given by following matrix where Im is identity matrix of size MxM which is size of local component transform.

\[
T_{AB} = \begin{bmatrix}
A_m \otimes B_n(1) \\
I_m \otimes B_n(2) \\
I_m \otimes B_n(3) \\
\vdots \\
I_m \otimes B_n(n)
\end{bmatrix}
\]

3.2 Algorithm

1. Choose colour image of size 256x256x3
2. Separate Red, Green and Blue component
3. To generate wavelet transform of image size, select orthogonal component transform DCT/DST/Real-DFT/DHT of size 8x8 and 32x32 i.e. M=8 and N=32
4. Generate wavelet transform of size 256x256 i.e. MN x MN.
5. Apply wavelet transform on each plane of image. Full wavelet transform of each plane is obtained as \(F = [T][f][T]^{T}\) where T is wavelet transform matrix, \(T^{T}\) is transpose of wavelet transform matrix and \(f\) is individual plane of an image
6. Sort the elements in transformed plane in descending order of their energy and eliminate lowest 256x8 elements. i.e. make them zero.
7. Apply inverse transform to reconstruct the image
8. Calculate RMSE between original image and reconstructed image at various compression ratios.
9. Calculate rate distortion function at different bit rates
10. Change the size of component transform to 16x16 i.e. M=16 and N=16 and repeat the steps 4 to 9 for each transform
11. Select component transform of size 32x32- 8x8 and 64x64-4x4 and execute steps 4 to 9 again.
12. Compare the results after variation in component transforms and their sizes.

4. EXPERIMENTS AND RESULTS

Twelve different colour images are selected for experimental purpose. Each image is of size 256x256x3. Experiments are performed using Matlab 7.0 on AMD dual core processor with 4 GB RAM. Images chosen are shown in figure 1.
Fig. 1. Set of twelve test images of different classes used for experimental purpose namely (from left to right and top to bottom) Mandrill, Peppers, Lord Ganesha, Flower, Cartoon, dolphin, Birds, Waterlily, Bud, Bear, Leaves and Lenna.

Fig. 2 shows plot of average RMSE against compression ratio with different sizes of DCT component matrix selected. From graph it can be observed that for component transform of size 16, DCT wavelet gives lower RMSE values.

DCT wavelet gives lowest distortion at smallest bit rate 0.25. Size of respective DCT matrix is m=8 and n=32. DST wavelet with component size M=32 and N=8 gives maximum error of all.

Graph in Fig. 4 shows average RMSE vs. Compression Ratio for wavelet of Real-DFT. Up to compression ratio 10.67, component transform of m=n=16 size gives better results. Onwards, for higher compression ratios wavelet generated from m=8 and n=32 size component gives lower RMSE values. Plot of Average RMSE vs. compression ratio for Discrete Hartley Wavelet Transform is presented in Fig. 5. At lower compression ratios, size of Hartley wavelet as 8x8 and 32x32 gives good performance. At higher compression ratios, 16x16 size gives lower value of RMSE. Fig. 6 presents graph of distortion versus bit rate. At lower bit rate i.e. when less number of bits are used to represent a pixel, maximum error has been observed. Distortion is measured in terms of RMSE.
Performance of Real-DFT wavelet and Hartley wavelet is approximately same and ranks second as observed from the graph. With increase in bit rate, Distortion decreases as seen in the graph.

Fig. 7 shows reconstructed image ‘Flower’ at lowest possible bit rate 0.25. Lowest distortion (RMSE) is obtained using DCT wavelet transform whose component orthogonal transforms are of size $m=8$ and $n=32$. Blocking effect in reconstructed images is observed when DST wavelet transform is used. It prominently depends on the size of local component transform, i.e. ‘N’. Hence in figure it can be clearly seen for N=32 than other values of N like 16, 8 and 4.

![Graph showing average RMSE vs. Compression Ratio for Hartley wavelet using different sizes of DHT](image)

![Graph showing error vs. Bit Rate (BPP) for wavelets of DCT, DST and Real-DFT with different sizes of respective orthogonal component transforms](image)

<table>
<thead>
<tr>
<th>Component Transform Sizes</th>
<th>Bit Rate=0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>M=8, N=32</td>
<td>M=16, N=16</td>
</tr>
<tr>
<td>DCT Wavelet</td>
<td>DST Wavelet</td>
</tr>
<tr>
<td>RMSE</td>
<td>RMSE</td>
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<tr>
<td>3.579438</td>
<td>3.791956</td>
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<tr>
<td>13.42467</td>
<td>16.36421</td>
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</tbody>
</table>
Hartley wavelet and Real-DFT wavelet also show blocking effect but it is less intense than in DST wavelet. With decrease in size of local component transform matrix, blocking effect becomes less prominent. In DCT this blocking effect is negligible.

5. CONCLUSION

In this paper wavelet transform is generated from orthogonal component transforms of different sizes and it is applied on colour images. DCT, DST, Hartley and Real-DFT are used as component transforms to generate their respective wavelet transforms. RMSE is calculated in DCT wavelet generated from different sizes of Discrete Cosine Transform. It has been observed that, for all wavelets, component size of m=8 and n=32 gives minimum RMSE values. Blocking effect is observed in all wavelets except DCT and it depends upon the size of local component transform. Performance of all four wavelet transforms is evaluated according to bit rate as a function of distortion. Distortion is represented in terms of RMSE. Bit rate is bits per pixel to be stored or transmitted. Four different sizes of respective component transform are tried for each wavelet transform. After comparing all combinations it has been observed that DCT wavelet of size 256x256 which is generated from components from m=8 and n=32 size component transform gives best results i.e. lowest distortion at smallest bit rate of 0.25.

REFERENCES


