Heat and Mass Transfer in MHD Boundary Layer Flow past an Inclined Plate with Viscous Dissipation in Porous Medium

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Abstract— In the present paper, the problem of a steady two dimensional hydromagnetic convective flow of a viscous, incompressible electrically conducting fluid past an inclined plate in porous medium has been studied. The convective flow starts under the simultaneous action of the buoyancy forces caused by the variations in density due to temperature and species concentration differences. The effects of viscous dissipation have been taken into account and the partial differential equations governing the boundary layer flow are converted into a system of ordinary differential equations by using suitable similarity transformations. These equations are solved numerically and effects of various parameters on the flow fields are investigated and presented graphically.

Key worlds: MHD, heat and mass transfer, viscous dissipation, porous medium, inclined plate,

1 Introduction

Due to immense practical importance as well as to meet the requirements of the contemporary technological needs, the convective flow problems, which involve the combination and interaction of various kinds of phenomena, have attracted the attention of many workers in the area. Convective boundary layer flow problems in the cases of horizontal and vertical flat plates have been investigated quite extensively. The boundary layer flows adjacent to inclined plates or wedges have received less attention. The study of Sparrow et al. [1] is related to the convection flow about an inclined surface in which the combined forced and free boundary layer problem has been discussed using the similarity method. An investigation on the longitudinal vortices in the boundary layer flow arising due to natural convection has been carried out by Sparrow and Husar [2]. The flow over a wedge with variable temperature has been studied by Gebhart [3].

The convective heat and mass transfer flows in porous medium find a number of applications in many branches of science and technology, such as, in petrology to study the flow of natural gases and oil through oil reservoirs, geothermal energy extraction processes, filteration and purification processes in chemical engineering etc. The convective flow problem about an inclined surface through porous media has been studied by Cheng [4], where the convection problem was of mixed kind, i.e., free and forced type. It is well understood that the free convection heat transfer phenomena occurs as a result of temperature differences in the vicinity of the surfaces. Gebhart and Pera [5] studied another kind of laminar flows which arises due to the mutual interplay of the forces of gravity and density differences caused by the simultaneous diffusion of thermal energy and species concentration and this investigation on the nature of vertical natural convection flows, has been applied in the study of flows past inclined surfaces. Ganesan and Palani [6] have dealt with the natural convection past an inclined plate wherein they have studied the effects of magnetic field under the conditions of variable surface heat and mass flux. A number of researchers [7, 8, 9] have considered, mainly due to its wide industrial and engineering applications, the convective flow problems about inclined surfaces under different kinds of physical situations. Mansour et al.[10] have analysed the free convection flow with viscous dissipation past an inclined surface in a Newtonian fluid-saturated porous medium taking into account the Rosseland approximation to account for the radiative heat flux. Badruddin et al.[11] have considered the effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus. Pal and Mondal [12] and Abel et al.[13] have investigated the effect of viscous dissipation in the case of flows over stretching sheets.

We find from the preceding investigations that viscous dissipation in porous media is an interesting phenomenon in boundary layer flows and at the same time the effect of viscous dissipation has been modelled in many ways by different workers. Nield [14] and Amgad Salama[15] have discussed quite extensively the Brinkman equation and the concept and modelling of viscous dissipation in porous media . The aim of the present paper is to consider the effects of viscous dissipation on the MHD boundary layer flow adjacent to a an inclined plate in porous medium.

Mathematical formulation:

Let us consider a steady two dimensional hydromagnetic flow of a viscous, incompressible electrically conducting fluid along an inclined plate with an acute angle α_1 to the vertical. x direction is taken along the leading edge of the inclined plate and y is normal to it ,i.e., the plate starts at x

=0 and extends parallel to the x axis and is of semi infinite length. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. The uniform plate temperature and concentration are maintained at T_w and C_w. The plate temperature T_w is higher than the temperature T_∞ of the fluid far away from the plate and concentration C_w is greater than the concentration C_∞. A steady flow parallel to the plate with free stream velocity U_∞ is assumed. Let u and v be the velocity components along the x and y axes respectively in the boundary layer region. The medium of the flow is porous which is assumed to be homogeneous. Then, under the Boussinesq and the usual boundary layer approximations, the governing equations for the Darcy type flow, following Schlichting [16] and Nield and Bejan[17], are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\ (T-T_\infty)\ Cos\alpha_1+g\beta^*\ (C-C_\infty)\ Cos\alpha_1-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{k}(u-U_\infty) \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\nu}{kc_p}u^2 \tag{3}$$

Boundary conditions

$$u=0,\ v=0,\ T=T_w,\ C=C_w \text{ at } y=0$$
$$u=U_\infty, T=T_\infty,\ C=C_\infty \text{ at } y\to\infty \qquad (5)$$

We introduce the following non-dimensional variables:

$$\eta=y\left(\frac{U_\infty}{\nu x}\right)^{\frac{1}{2}}$$

$$\psi=\sqrt{\nu U_\infty x}\,f(\eta)\text{, such that } u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

Now, we have $u=U_\infty f'(\eta)$, $v=\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\,(\eta f'-f)$

And these values of the velocity components satisfy the equation (1). Also,
And these values of velocity components satisfies equation (1) . Also,

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

$$\varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

Now using above non-dimensional variables equations (2) to (4) become

$$f'''+\frac{1}{2}ff''+Gr\,\theta\cos\alpha_1+Gc\,\phi\cos\alpha_1-(\delta+M)f'+\delta=0 \tag{6}$$

$$\theta''+\frac{1}{2}P_r f\theta'+P_rE_c f''^2+NP_r f'^2=0 \tag{7}$$

$$\phi''+\frac{1}{2}Scf\phi'=0 \tag{8}$$

Where

$$Gr=\frac{g\beta\ (T_w-T_\infty)\ x}{U^2_\infty}$$

$$Gc=\frac{g\beta^*\ (C_w-C_\infty)\ x}{U^2_\infty}$$

$$Sc=\frac{\nu}{D}$$

$$P_r=\frac{\nu}{\alpha}$$

$$E_c=\frac{U_\infty^{\ 2}}{c_p(T_w-T)}$$

$$N=\frac{x\nu U_\infty}{kc_p(T_w-T_\infty)}$$

$$M=\frac{\sigma B_0^2 x}{\rho U_\infty}$$

$$\delta=\frac{\nu x}{kU_\infty}$$

The transformed boundary conditions are given by

$$\left.\begin{array}{lllll} f=0 & f'=0 & \theta=1 & \varphi=1 & \text{at } \eta=0 \\ f'=1 & \theta=0 & \varphi=0 & & at\ \eta\to\infty \end{array}\right\} \tag{9}$$

Result and discussion:

The equations (6)-(8) together with the boundary conditions (9) are solved numerically using BVP 4C method. The results depicting the nature of effects of various parameters on the profiles of velocity f', temperature θ and the species concentration ϕ are displayed with the help of graphical illustrations.

In figure 1, the effects of Prandtl Number Pr on the velocity, temperature and concentration distribution are shown. It is observed that Pr has very little effect on the velocity profile.

But, it has very significant effect on the temperature profile- as we increase the value of Pr, we find a narrowing pattern in the temperature boundary layer. This result is in conformity with the known and observed facts that in liquid metals (Pr <1) the heat diffuses faster as compared to the lubricant oils (Pr >1). The Prandtl no. does not appear to have any direct effect on the concentration distribution.

Figure 2 demonstrates the effects of Eckert number Ec on the flow field. We find the effects of Ec similar to that we have in the case of Pr in the sense that Ec has very pronounced effect on the temperature field. As we increase the value of Ec, we find an increasing effect on the temperature profile,but, it has negligible effect on the velocity and species concentration profiles.

The effects of N, the dissipation parameter due to porous medium, are shown in figures 3. The effects are almost the same as those of Eckert number.

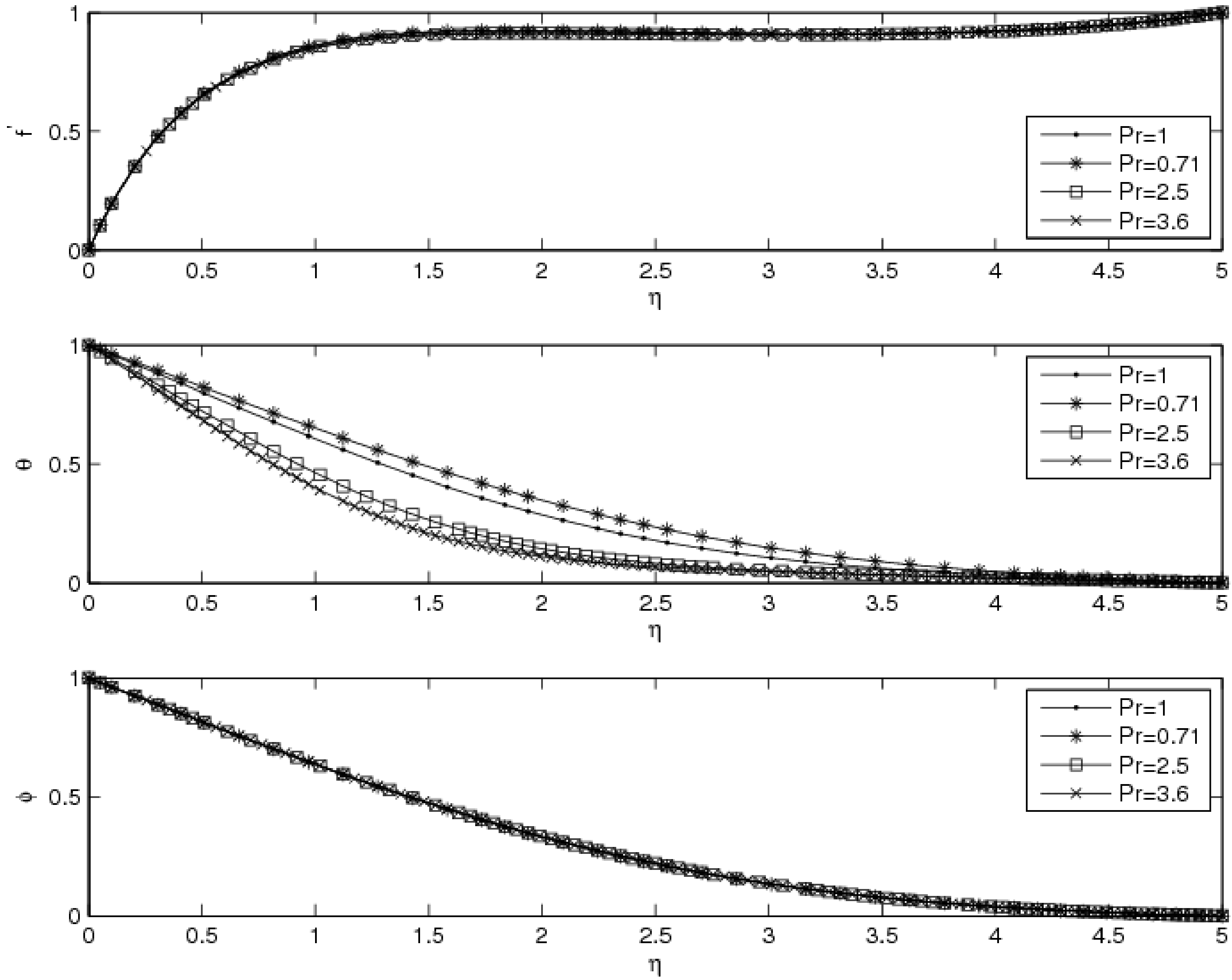
Thus we see that the two parameters, Ec and N, both showing the effects of dissipation of viscosity, have similar effects.

The effects of Schmidt number Sc are shown in the figures 4. As is obvious from the governing equation 3, we find that this parameter has very significant influence on the species concentration profile and this effect is quite similar to that we got in the case of Pr on the temperature field. In fact, if we set Ec = N = 0 in equation 2 and take Pr =Sc, equations 2 and 3 become similar.

In figures 5 and 6, we find the effects of Grashof no. Gr and Gc, parameters due to the temperature and concentration respectively, on the flow field. Both the parameters have very little effects, almost indistinguishable, on the temperature and concentration profiles, but they quite significantly affect the velocity field. An increase in Gr increases the velocity profile.

The effects of magnetic field parameter M are shown in figures 7. We observe that the parameter M has quite noticeable effects on the two , θ and ϕ , profiles. An increase in M has the effect of thickening the temperature and concentration profiles. On the other hand, the parameter M has an inhibiting effect on the velocity field, i.e., any increase in M makes the velocity boundary layer thinner and thinner.

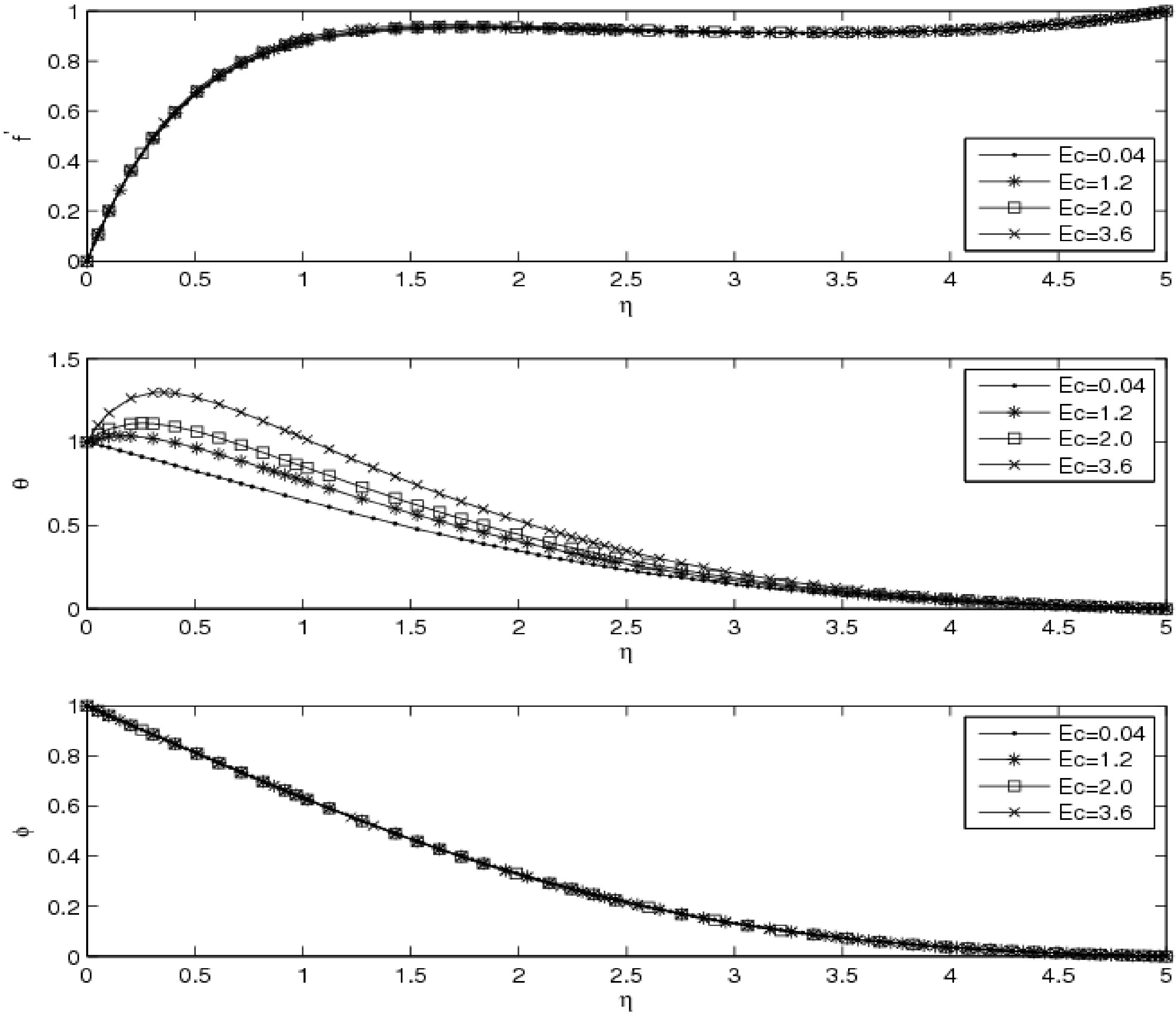
Conclusions: From the present study of convective MHD boundary layer flow on inclined plates with combined buoyancy forces arising due to thermal and mass diffusion, the main observations are- both Prandtl and Eckert numbers affect the temperature field quite significantly. An increase in the Eckert number Ec and viscous dissipation parameter N has a corresponding increasing effect on the temperature profile. . An increase in the magnetic field parameter M has the effect of thickening the temperature and concentration profiles , while it has an inhibiting effect on the velocity field, i.e., any increase in M makes the velocity boundary layer thinner and thinner.



Gr = 3.2; Gc = 3.5; M = 0.5; δ = 4; Ec =0.02; N =0.04; Sc=0.6; ω = 0.1

Figure 1

ISSN 2229-5518



Gr = 3.2; Gc = 3.5; M = 0.5; δ = 4; Pr =0.71; N =0.04; Sc=0.6; ω = 0.1

Figure 2

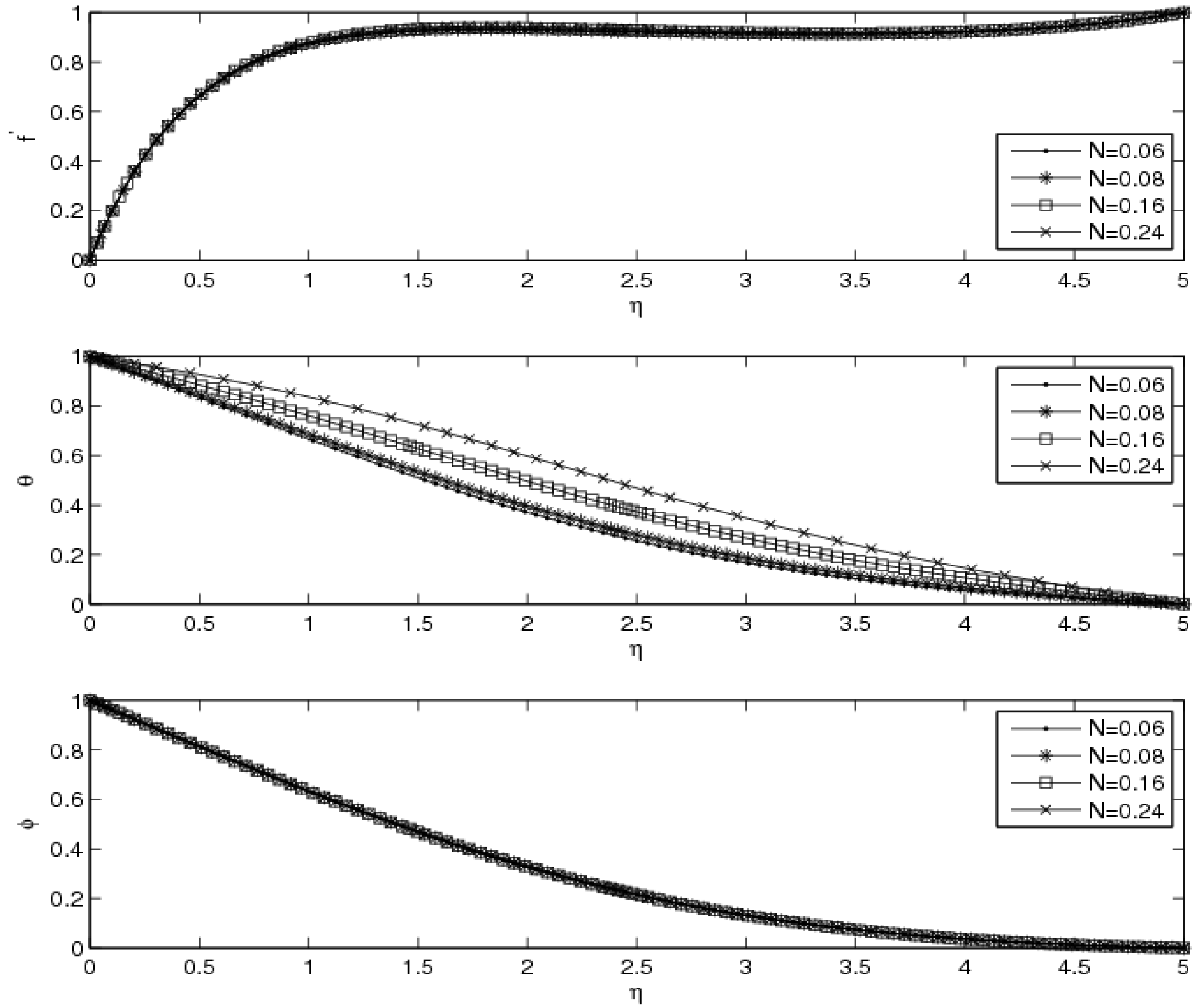


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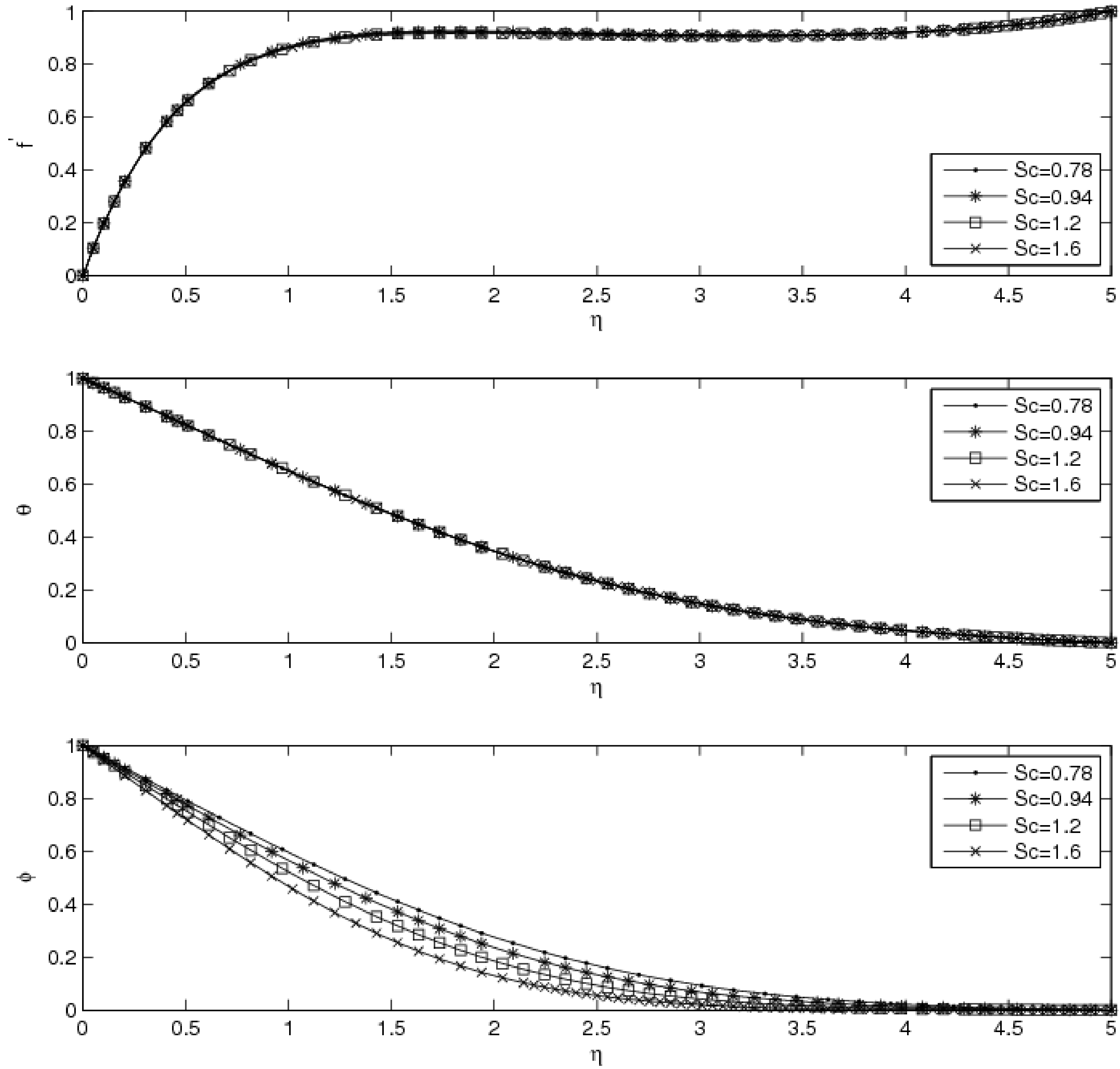
Figure 3



$Gr = 3.2; Gc = 3.5; M = 0.5; \delta = 4; Pr = 0.71; Ec = 0.02; N = 0.04; \omega = 0.1$

Figure 4

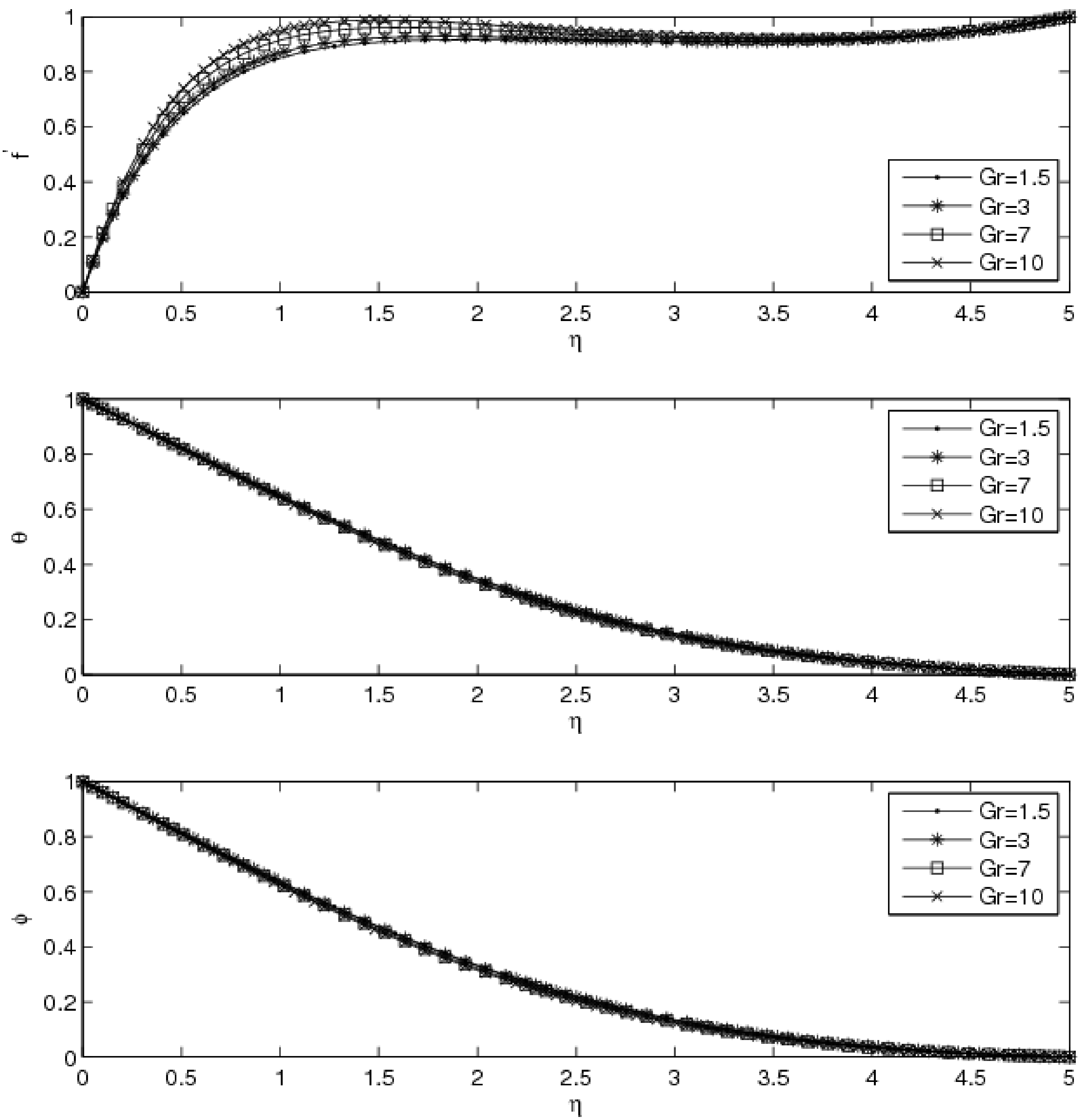


Figure 5

Gc = 3.5; M = 0.5; δ = 4; Pr =0.71; Ec =0.02; N = 0.04; Sc=0.6; ω = 0.1

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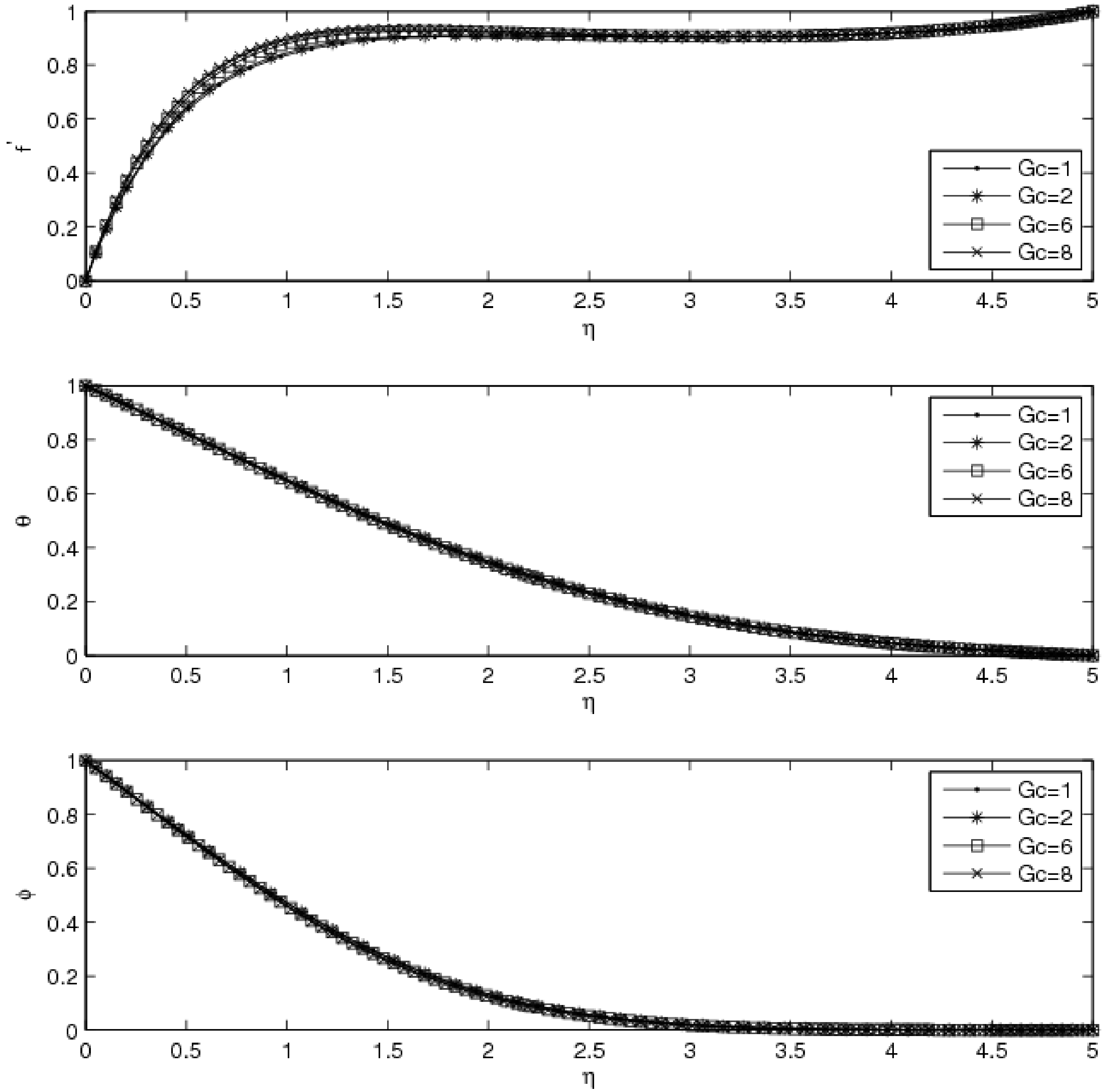
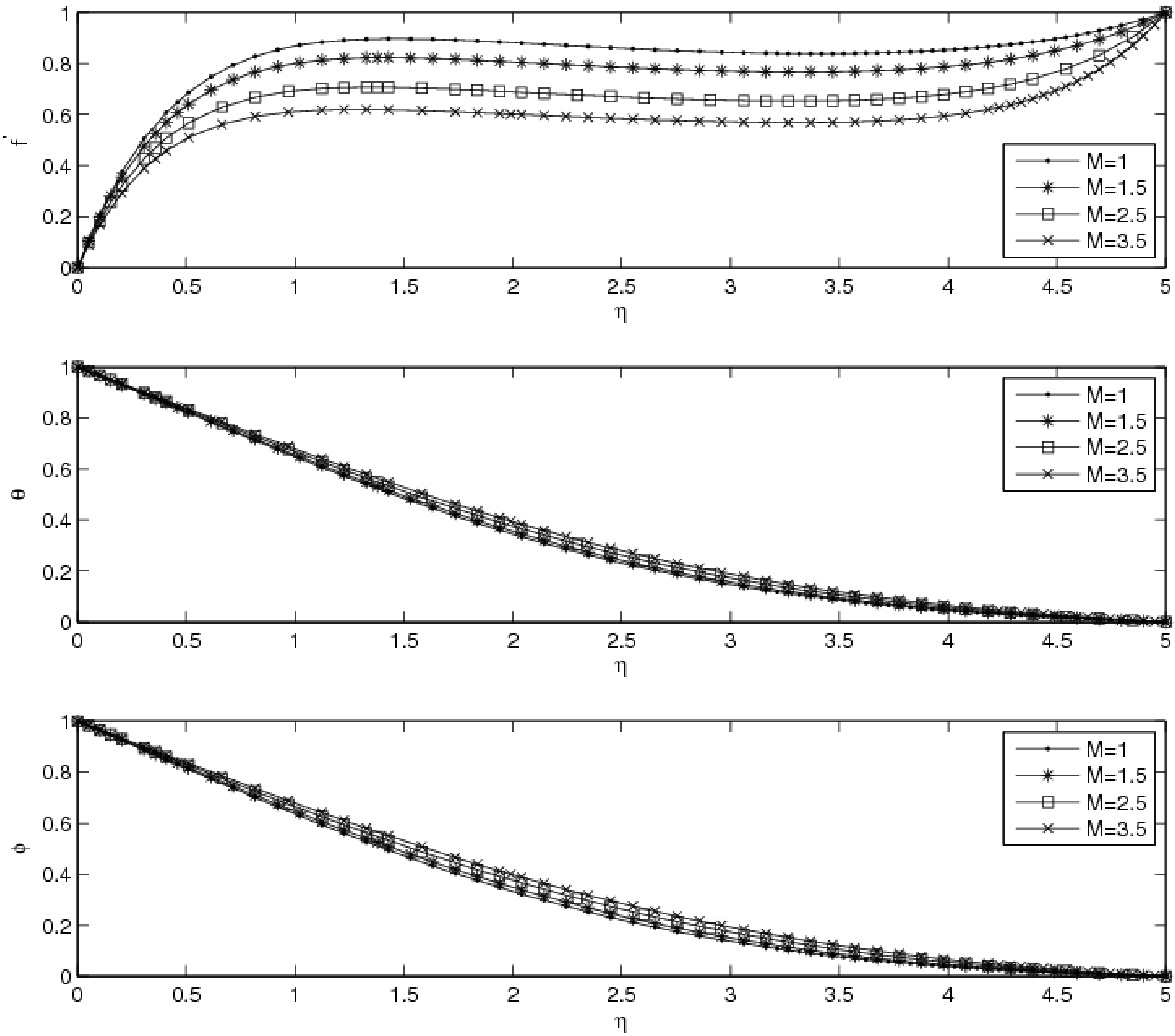


Gr = 3.2; M = 0.5; δ = 4; Pr =0.71; Ec =0.02; N = 0.04; Sc=1.6; ω = 0.1

Figure 6



Gr = 3.0; Gc = 3.5; δ = 4; Pr =0.71; Ec =0.02; N = 0.04; Sc=0.6; ω = 0.1

Figure 7

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