HEAT TRANSFER OVER A FLAT PLATE IN LAMINAR FLOW USING COMPUTATIONAL FLUID DYNAMICS

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ABSTRACT

The aim of this paper is the theoretical estimations for boundary layer thickness and heat transfer coefficient which is analysed using CFD with certain parametric functions and partial differential equations as well. Along with it the feasibility of the convective heat transfer coefficient for the calculations examined during their rendering certainty with CFD methods. The equations have been taken in validating and iterating the values achieved during the grid sensitivity analysis of the determination of convective heat transfer coefficients. For other coefficients the validation is done through analytical solution. While taking the accountability of the approach the local nusselt numbers as well as prandtl numbers are obtained which can be used in the analysis of flow.

The results tell us that for the laminar forced convection simulations the convective heat transfer coefficients differed from analytical values by 7%

KEYWORDS: boundary layer thickness, convective heat transfer coefficient, free and forced convection, laminar flow, local nusselt number.

INTRODUCTION

Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Application of Convective heat transfer studies are very important, which lies in the fact involving in processes having high temperatures such as gas turbines, nuclear plants, thermal energy storage, etc.

Convection is usually the dominant form of heat transfer in liquids and gases. The convection heat transfer mode comprises
one mechanism. In addition to energy transfer due to specific molecular motion (diffusion), energy is transferred by bulk, or macroscopic, motion of the fluid.

Two types of convective heat transfer may be distinguished:-

**Free or natural convection:** when fluid motion is caused by buoyancy forces that result from the density variations due to variations of thermal temperature in the fluid.

**Forced convection:** when a fluid is forced to flow over the surface by an external source such as fans, by stirring, and pumps, creating an artificially induced convection current.

The classical problem i.e., fluid flow along a horizontal, stationary surface located in a uniform free stream was solved for the first time in 1908 by Blasius [1]; it is still a subject of current research [2] and, moreover, further study regarding this subject can be seen in most papers [3]. Moreover, Bataller presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow) and about a moving plate in a quiescent ambient fluid (Sakiadis flow). Aziz [4] investigated a similarity solution for laminar thermal boundary layer over a flat-plate with a convective surface boundary condition. Numerous studies such as Refs [5–6] considered different variations in temperature and heat flux at the plate. Thompson [7] discusses solution of partial differential equations involved in areas such as Fluid Mechanics, Elasticity and Electromagnetic Field by using FEM. Jackson simulated the periodic behavior of a two-dimensional laminar flow past various shaped bodies, including flat plates aligned over a range of angles of attack with respect to the incoming free-stream. Knisely performed Strouhal number measurements of several rectangular cylinders with side ratios ranging from 0.04 to 1.0 and with the angles of attack ranging from 0° to 90°. Lam investigated the flow past an inclined flat plate at \( \gamma = 15^\circ \), using phased-averaged LDA measurements. The flow field around flat plates, characterized by sharp leading and trailing edges, was also investigated by Breuer and Jovicic. Breuer et al. simulated the flow over an 18° inclined plate, showing how the trailing edge vortices are able to dominate the wake features. Convective heat transfer or, simply, convection is the study of heat transport processes by the flow of fluids. Problems related to convective heat transfer rest on basic thermodynamics and fluid mechanics principles, which essentially involved with
partial differential equations. This paper demonstrates how CFD can be used for the determination of the heat transfer process in the boundary layer for laminar air flow. The commercial CFD code Fluent 6.1.22 was used for all simulations. The coefficients are validated using analytical solutions. A grid sensitivity analysis is performed for the CFD solutions, and it is used to determine the grid independent solution for convective heat transfer coefficients. A typical mesh is shown below in Figure 1.

![fig1. mesh used in laminar CFD simulation](image)

**MESHING:**

The partial differential equations that govern fluid flow and heat transfer are not usually amenable to analytical solutions, except for very simple cases. Therefore in order to analyze fluid flow, flow domains are split into smaller subdomains (made up of geometric primitives like hexahedra and tetrahedra in 3D and quadrilaterals and triangles in 2D). The governing equations are then discretized and solved inside each of these subdomains. Typically one of three methods is used to solve the approximate version of the system of equations: finite volumes, finite elements, or finite differences. The subdomains are often called elements or cells, and the collection of all elements or cells is called a mesh or grid. The origin of the term mesh (or grid) goes back to early days of CFD when most analyses were 2D in nature. For 2D analyses, a domain split into elements resembles a wire mesh, hence the name. The process of obtaining an appropriate mesh (or grid) is termed mesh generation (or grid generation), and has long been considered a bottleneck in the analysis process due to the lack of a fully automatic mesh generation procedure.

**PROBLEM FORMULATION:**

![fig2. flow over a flat plate](image)
The heat transfer coefficient may be obtained from analytically derived values of the Nusselt number. The values differ slightly based upon the heating conditions. It is found that the Nusselt number can be expressed as:

\[ Nu = \frac{hL}{k} \quad (1) \]

Where \( L \) is the length of plate (m), \( k \) is the thermal conductivity of air (w/m\(^2\) k), \( h \) is the heat transfer coefficient (w/m\(^2\) k) and \( Nu \) is the Nusselt number. Heat transfer coefficient for a flat plate can be determined by solving the conservation of mass, momentum, and energy equations (either approximately or numerically). The appropriate parameters may then be input to yield the following analytical values for \( h \):

\[ h = \frac{Nu k}{L} \quad (2) \]

Convective heat transfer between a moving fluid and a surface can be defined by the following relationship:

\[ Q = h(T_S - T_f) \quad (3) \]

Where \( Q \) is the heat flux (W/m\(^2\)), \( h \) is the convective heat transfer coefficient (W/m\(^2\)K), \( T_s \) is the surface temperature (K), and \( T_f \) is the fluid reference temperature (K). The figure shows the schematic diagram laminar flow over a flat plate with constant wall temperature.

**fig3. Schematic representation of laminar flow over flat plate with constant wall temperature.**

**DISCRETIZATION**

In mathematics discretization concerns the process of transferring continuous models and equations into discrete counterparts. This process is usually carried out as a first step toward making them suitable for numerical evaluation and implementation on digital computers. In order to be processed on a digital computer another process named quantization is essential.

The governing equations are further discretized and used for computer modelling.

Continuity equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4) \]

x-momentum equation

\[ -\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \beta g(T - T_w) + v\nabla^2 u \quad (5) \]

y-momentum equation
\[-\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \]  \hspace{1cm} (6)

Energy equation

\[-u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T \]  \hspace{1cm} (7)

Types of discretization methods:
1. finite element method
2. finite volume method
3. boundary element method

**FINITE ELEMENT METHOD**

Analogous to the idea that connecting many tiny straight lines can approximate a larger circle, FEM encompasses all the methods for connecting many simple element equations over many small subdomains, named finite elements, to approximate a more complex equation over a larger domain.

**BOUNDARY ELEMENT METHOD**: In the boundary element method, the boundary occupied by the fluid is divided into a surface mesh.

**Laminar CFD simulation results**

The convective heat transfer coefficients for the constant wall temperature indicates the convective heat transfer coefficients differ from analytical values by 7%.

![Graph of convective heat transfer coefficient for constant wall temperature](image)

**Fig2: convective heat transfer coefficient for constant wall temperature**

The graph is plotted against convective heat transfer coefficient and position x.

If a flat plate is put in a flow with zero incidences, the flow at the plate surface is slowed down because of the friction. This region of slowed down flow becomes even larger, since more and more fluid particles are caught up by the retardation. The thickness of the boundary layer is therefore a monotonically increasing function of x.

The transition from boundary layer flow to outer flow, at least in the case of laminar
flow, takes place continuously. That means that a precise boundary cannot be given. It is often used in practice that the boundary is at the point where the velocity reaches a certain percentage of the outer velocity. Thickness of boundary-layer for a flat plate at a location x is given by:

\[
\delta = \frac{5.48x}{\sqrt{Re}}
\]  

(8)

\[\text{fig4. temperature profile}\]

**CONCLUSION:**

In this study heat transfer coefficient in laminar flow is analyzed over a flat plate by using computational fluid dynamics and results showed a 7% error with the analytical solutions, indicating performance of the CFD code, at least for the cases studied. A grid sensitivity analysis was performed on the mesh for laminar air flows. The convective heat transfer coefficient (h) was calculated over a flat plate.

**REFERENCES:**