Generation of Ultrashort pulses in a Highly Nonlinear Negative Index Material

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Abstract: In this paper, we consider the electromagnetic pulse propagation in a highly nonlinear negative index material wherein the pulse propagation is governed by cubic-quintic nonlinear effects. By adopting coupled amplitude phase technique, we delineate the generation of ultrashort pulse in terms of bright soliton. In addition, we also calculate the peak power and pulse width required for generating this ultrashort pulse.

Keywords— Bright soliton, metamaterials, negative index materials, ultrashort pulses, modified nonlinear Schrodinger equation.

1 INTRODUCTION

Metamaterials (MMs) are artificially structured materials which exhibit electromagnetic properties, such as negative magnetic permeability, negative refraction, reversed Cerenkov radiation, sub-diffraction imaging, invisibility cloaking, that are not available in natural materials [1]. These unique properties led to several applications in science and technology [2]. For the first time, Veselago theoretically proposed that a material could also support the propagation of electromagnetic wave even if the material exhibits negative dielectric permittivity, \(\varepsilon\), and negative magnetic permeability, \(\mu\). This kind of special material possesses the negative refractive index and hence it is named as negative index material (NIM) [3]. One decade ago, these NIMs had first been practically realized in the microwave region using arrays of wire and split-ring resonators (SRRs) on the printed circuit board [4]. Various linear properties have been explored very well in the microwave regime. Recently, nonlinear effects have also been observed in NIMs when these materials are embedded with the weakly nonlinear dielectric [5]. First theoretical model was initiated by Scalora et al. who arrived at the generalized nonlinear Schrodinger equation which supports a wide class of solitary waves in NIMs [6]. Lazarides et al. derived the coupled nonlinear Schrodinger equations and theoretically demonstrated the formation of bright and dark solitons through Manakov model [7]. Wen et al. proposed a theoretical for generating ultrashort pulses with few optical cycles based on the Drude dispersive model [8].

In this paper, we study the generation ultrashort pulse in terms of bright soliton in nonlinear NIMs by using the modified nonlinear Schrodinger equation.

2 THEORETICAL MODEL

The governing model for propagation of ultrashort pulses in nonlinear NIMs with Kerr and non-Kerr polarization (P\(_{\text{NL}}\)) is given as [9]

\[
\frac{\partial E}{\partial \xi} + i \delta_1 \frac{\partial^2 E}{\partial \tau^2} - \delta_2 \frac{\partial^3 E}{\partial \tau^3} - i \sigma_0 \frac{\partial |E|^2 E}{\partial \tau} + \sigma_0 s_1 \frac{\partial (|E|^2 E)}{\partial \tau} - i \eta_0 |E|^4 E + \eta_0 s_3 \frac{\partial (|E|^4 E)}{\partial \tau} = 0, \\
\]

Here \(\delta_1\) is the group velocity dispersion, \(\delta_2\) is the third order dispersion, \(\sigma_0\) is the Kerr nonlinearity, \(s_1\) is the self-steepening coefficient, \(\eta_0\) is the quintic nonlinearity, \(s_3\) self-steepening parameter due to quintic nonlinearity. Based on equation (1), the modulation instability has been studied in nonlinear fiber Bragg gratings [10].

In order to generate the soliton type ultrashort pulses in NIMs, we consider the solution of the form of equation (1) as,

\[
E(\xi, \tau) = U(\chi) \exp[i(k\chi - \Omega \tau)],
\]

where \(U(\chi)\), with \(\chi = \tau - \lambda \xi\), is a real quantity, \(\lambda\) is a real parameter, \(k\) and \(\Omega\) being the wavenumber and circular frequency, respectively. Substituting equation (2) in equation (1), we obtain,

\[
\begin{align*}
[-i \lambda U_x + ikU] + i \delta_1 [-\Omega^2 U - 2i \Omega U_x + U_{xx}] \\
- \delta_2 [i \Omega^2 U - 3 \Omega^2 U_x - 3i \Omega U_{xx} + U_{xxx}] - i \sigma_0 U^3 \\
+ \sigma_0 s_1 [-i \Omega U^3 + 3 \Omega^2 U_x] - i \eta_0 U^5 \\
+ \eta_0 s_3 [-i \Omega U^5 + 5 \Omega^4 U_x] = 0,
\end{align*}
\]

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Separating the real and imaginary parts, we have

\[ \partial_z U_{xx} + (\Lambda - 2\Omega_1 - 3\Omega_2^2\delta_2)U_{x} - 3\sigma_0 s_1 U^2 U_x - 5\eta_0 s_3 U^4 U_x = 0, \]  
(3)

\[ (\delta_1 + 3\delta_2)U_{xx} + (k - \delta_1\Omega_2^2 - \delta_2\Omega_3^2)U - \sigma_0 (1 + s_3)\Omega_3^3 U - \eta_0 (1 + s_3)\Omega_5^5 = 0. \]  
(4)

Where \( U_x \) is the first order derivative with respect to \( \chi \) and so on.

On integrating equation (3), we get

\[ U_{xx} = -\frac{(\Lambda - 2\Omega_1 - 3\Omega_2^2\delta_2)}{\delta_2} U + \frac{\sigma_0 s_1}{\delta_2} U^3 + \frac{\eta_0 s_3}{\delta_2} U^5 + c, \]  
(5)

c is the integration constant, for simplicity \( c=0 \).

Rewriting equation (4) as,

\[ U_{xx} = -\frac{(K - \delta_1\Omega_2^2 - \delta_2\Omega_3^2)}{(\delta_1 + 3\delta_2\Omega)} U + \frac{\sigma_0 (1 + s_3)\Omega_3^3}{(\delta_1 + 3\delta_2\Omega)} U^3 + \frac{\eta_0 (1 + s_3)\Omega_5^5}{(\delta_1 + 3\delta_2\Omega)} U^5. \]  
(6)

Equation (5) and equation (6) are equivalent only under the following conditions:

\[ \alpha = -\frac{(\Lambda - 2\delta_1\Omega - 3\delta_2\Omega^2)}{\delta_2} = -\frac{(K - \delta_1\Omega^2 - \delta_2\Omega^3)}{\delta_1 + 3\delta_2\Omega}, \]

\[ \beta = -\frac{\sigma_0 (1 + s_3)\Omega_3^3}{\delta_1 + 3\delta_2\Omega} = -\frac{\sigma_0 s_1}{\delta_2}, \]

\[ \gamma = -\frac{\eta_0 (1 + s_3)\Omega_5^5}{\delta_1 + 3\delta_2\Omega} = -\frac{\eta_0 s_3}{\delta_2}. \]

From the above relations, we find \( k \) and \( \Omega \) as

\[ k = \frac{(\Lambda - 2\delta_1\Omega - 3\delta_2\Omega^2)(\delta_1 + 3\delta_2\Omega) + \delta_1\Omega^2 + \delta_2\Omega^3}{\delta_2}, \]  
(7)

Equation (5) can be written as,

\[ U_{xx} - \alpha U + \beta U^3 + \gamma U^5 = 0. \]

Integrating above equation, we get first order differential equation,

\[ \left( \frac{dU}{d\chi} \right)^2 = 4\alpha U^2 - 2\beta U - \frac{2}{3}\gamma + c_1, \]  
(9)

and \( c_1 \) is an integration constant.

Equation (9) is a first-order ordinary differential equation, we end up with the bright soliton pulses when the integration constant, \( c_1=0 \) and the same is given by,

\[ U(\chi) = \frac{2\alpha}{\sqrt{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + 4\alpha\gamma \cosh[2\sqrt{\alpha}\chi]}}}. \]  
(10)

Now, by plugging this amplitude value in equation (2), we arrive at the generation of bright soliton type ultrashort pulse and the field envelope for the same is given by,

\[ E(\xi, \tau) = \frac{2\alpha}{\sqrt{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + 4\alpha\gamma \cosh[2\sqrt{\alpha}(\tau - \Lambda\xi)]}} \times \exp[i(K\xi - \Omega\tau)]. \]  
(11)

![Fig. 1 Evolution of bright soliton type ultrashort pulse generated in a nonlinear NIMs under the influence of higher order nonlinear effect.](http://www.ijser.org)
Figure (1) portrays the three dimensional view of ultrashort pulse in terms of bright soliton pulse profile in the nonlinear NIMs. From the bright soliton pulse profile, we also calculate the important and interesting physical parameters such as soliton power and pulse width and they are found to be:

\[
P_0 = \frac{2(2\delta_1 \Omega + 3\delta_2 \Omega^2 - \Lambda)}{\beta + \left(\frac{\beta}{2}\right)^2 + \frac{4}{3}(2\delta_1 \Omega + 3\delta_2 \Omega^2 - \Lambda)\gamma}
\]

(12)

\[
T_0 = \frac{1}{\sqrt{2\delta_1 \Omega + 3\delta_2 \Omega^2 - \Lambda}}.
\]

(13)

with the known values for \(\delta_1, \delta_2\) and \(s_1\), we can calculate \(\Omega\) using equation (8). Hence, for a given \(T_0\), we can easily calculate the power required for generating the bright soliton type ultrashort pulse.

Figure (2) depicts the comparison of bright soliton pulse profiles for various peak powers when \(\Lambda = 0.2\) and \(\xi = 10\). It is obvious that the pulse width gets reduced as and when the power is increased.

3 Conclusion

We have successfully generated the bright soliton type ultrashort pulses in nonlinear negative index materials by solving the higher order nonlinear Schrodinger equation using coupled-amplitude phase method under the influence of cubic and quintic nonlinear effects. Besides, we have also calculated the minimum power required for realizing the ultrashort pulses. We do believe that the results presented in this paper would highly be useful for realizing optical switches and terahertz optical modulators.

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