Fuzzy Set Theory in Medical Diagnosis

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ABSTRACT:

A fuzzy set is a class of objects in with a continuation of grades of membership. Such a set is characterized by a membership function (characteristic function) which assigns to each object. A grade of membership function is ranging between zero and one. Fuzziness is an inherent characteristic in all decision-making problems especially in medical science. Fuzzy set theory has a number of properties that make it suitable for validating the inaccurate information in which medical diagnosis and treatment is usually based. In this paper first of all, it defines inaccurate medical entities as fuzzy sets. Then it provides a linguistic approach with an excellent approximation to texts. At last, fuzzy logic offers reasoning methods capable of drawing approximate inferences. These facts suggest that fuzzy set theory might be a suitable basis for the development of a computerized diagnosis system.

KEYWORDS: Fuzziness, Medical Diagnosis, Linguistic Approach

I. **INTRODUCTION**: It is widely accepted that the information available to the physician about his patient and about medical relationships in general is inherently uncertain. Nevertheless, the physician is still quite capable of drawing conclusions from this information. This paper describes an attempt to provide a formal model of this process using fuzzy set theory, and to II. implement the model in the form of a CADIAG-2(computerized diagnostic system). we now know that real-world knowledge is characterized by incompleteness, inaccuracy & inconsistency. Fuzzy set theory, which was developed by Zadeh [1], makes it possible to define inexact medical entities as fuzzy sets. It offers a linguistic approach that represents an excellent approximation to medical texts [2], [3]. In addition, fuzzy logic provides reasoning methods capable of making approximate inferences [4], [5]. These facts suggest that fuzzy set theory might be a suitable basis for the development of a computerized diagnosis system [6].

Current developments and applications of some medical expert systems on the basis of fuzzy set theory and fuzzy logic show that this is indeed the case [22]-[7]. CADIAG-2(computer-assisted diagnosis), an expert system especially designed for internal medicine, which is presently being clinically tested, will be described in more detail in order to provide an example and report some results.

II. **REAL WORLDKNOWLEDGE**: Precision exists only through abstraction. Abstraction may be defined as the ability of human beings to recognize and select the appropriate properties of real-world phenomena and objects. This leads to the construction of conceptual models defining abstract classes of these phenomena and objects. Abstraction forms the basis of human thought & human knowledge is its result.

A. **Incompleteness**: Abstraction, is not a static concept. The process of abstraction is continuous and is continually producing new results. The set of properties of real-world phenomena and objects under attention is constantly being enlarged and changed. Knowledge is therefore always and necessarily unfinished.

B. **Inaccuracy**: Unlimited precision is impossible in the real world. Anything said to be "precise" can only be considered as "precise to a certain extent." Heisenberg's Uncertainty Principle [23] states the limits to accurate measurement very clearly. Of course, the principle applies only to the world of micro-phenomena and micro-objects, but its philosophical implications go further. It shows that nature possibly is fundamentally indeterministic. And it seems meaningless to ask whether nature inherently lacks determinism or whether uncertainty stems only from experimentation.

C. **Inconsistency**: Abstraction does not always lead to the same results, which in turn are not always interpreted in the same way. Knowledge may differ according to nation, culture, religion, social status, education, etc., and information from different sources may therefore be inconsistent.

III. **MEDICAL INFORMATION**: In medicine, it is not necessary to deal with two phenomena ie; micro-phenomena and micro-objects to run into the problems of incompleteness, uncertainty, and inconsistency. The lack of information, and its rough and sometimes contradictory nature, is much more a fact of life in medicine than in, say, the physical sciences. These problems have to be taken into account in every medical decision, where they may have important, even vital consequences for the object of medical attention to the patient.

A. **Information About the Patient**: Data about the patient can be divided into a number of different categories, which are all characterized by an inherent lack of certainty.

1. **Medical History of the Patient:** The medical history of the patient is given by the patient himself. It is highly subjective and may includesimulated, exaggerated, or understated symptoms. Ignorance of previous diseases in himself or his family, failure to mention previous operations, and general poor recollection often raise doubts about a patient's medical history in the mind of the doctor. On the other hand, however, the information that finally leads to the correct diagnosis is very often found here.

2. **Physical Examination**: The physician subjects the patient to a physical examination from which he obtains more or less objective data. But of course, physicians can make mistakes, overlook important indications, or fail to carry out a complete examination. Furthermore, they may misinterpret other indications because the boundary between normal and pathological status is not always clearly defined.

3. **Results of Laboratory Tests**: The results of laboratory tests are considered to be objective data. However, measurement errors, organizational problems or improper behavior on the part of the patients prior to examinations can lead to imprecise and sometimes even totally incorrect data. Again, the boundaries between normal and pathological results are generally not strict: there are always borderline values that cannot be said to be either normal or pathological.

4. **Results of Histological, X-ray, Ultrasonic, and Other Clinical Investigations:** These results again depend on correct interpretation by medical or other staff. Such findings are often crucial, because they frequently indicate invasive therapy. In many cases, consideration of uncertainty is part of the evaluation procedure; for example, in cell counts, cell determination, picture analysis, etc.

B. **Information on Medical Relationships:** Medical knowledge consists of medical descriptions and assertions that are incomplete and uncertain. It has been built up step by step, and is based partly on theoretical studies (in areas such as anatomy and physiology) and partly on almost purely empirical observations (made in the course of surgery, for example). Medical knowledge may be said to comprise knowledge about causal relationships based in theory, statistical information, pure definitions, and personal judgment.

C. **Medical inference:** This is the process by which the physician uses his medical knowledge to infer a diagnosis from the symptoms displayed by the patient, his lab test results, and his medical history. It is a complex and partly uninvestigated process in which the physician is obviously able to work with uncertain and imprecise sets of data.

IV. MEDICAL EXPERT SYSTEM CADIAG-2:

CADIAG-2 is intended to be an active assistant to the physician in diagnostic situations. In this way the experience, creativeness, and intuition of the physician is supplemented by the knowledge-based computational power of the computer. The general structure of CADIAG-2 is shown in Fig. 1.

A. **Representation of Medical Knowledge**:

CADIAG-2 considers four classes of medical entities:

- symptoms, signs, test results, and findings (Si)
- diseases, and diagnoses (Dj)
- intermediate combinations (ICk)
- symptom combinations (SCI).

Symptoms Si take values t, μSi in [0,1] u{v}. The value μSi indicates the degree to which the patient exhibits symptom Si. In the language of fuzzy set theory, μSi expresses the grade of membership to which the patient's symptom manifestation Si belongs to the patient.

Diseases or diagnoses also take values in [0,1] u {v}. Fuzzy values 0.00 <, μDj < 1.00 represent possible diagnoses, while the values. μDj. = 1.00 and μDj = 0.00 correspond to confirmed and excluded diagnoses, respectively. Diagnoses that have not yet been considered take the value μDj = V. Formally, a relationship RPD ⊂ π X is established, defined by μRPD (Pq, Dj) = LD for patient Pq, where Dj ∈ (= {D1………., Dn}).

Intermediate combinations (fuzzy logical combinations of symptoms and diseases) were introduced to model the pathophysiological states of patients; symptom combinations are combinations of symptoms, diseases, and intermediate combinations.

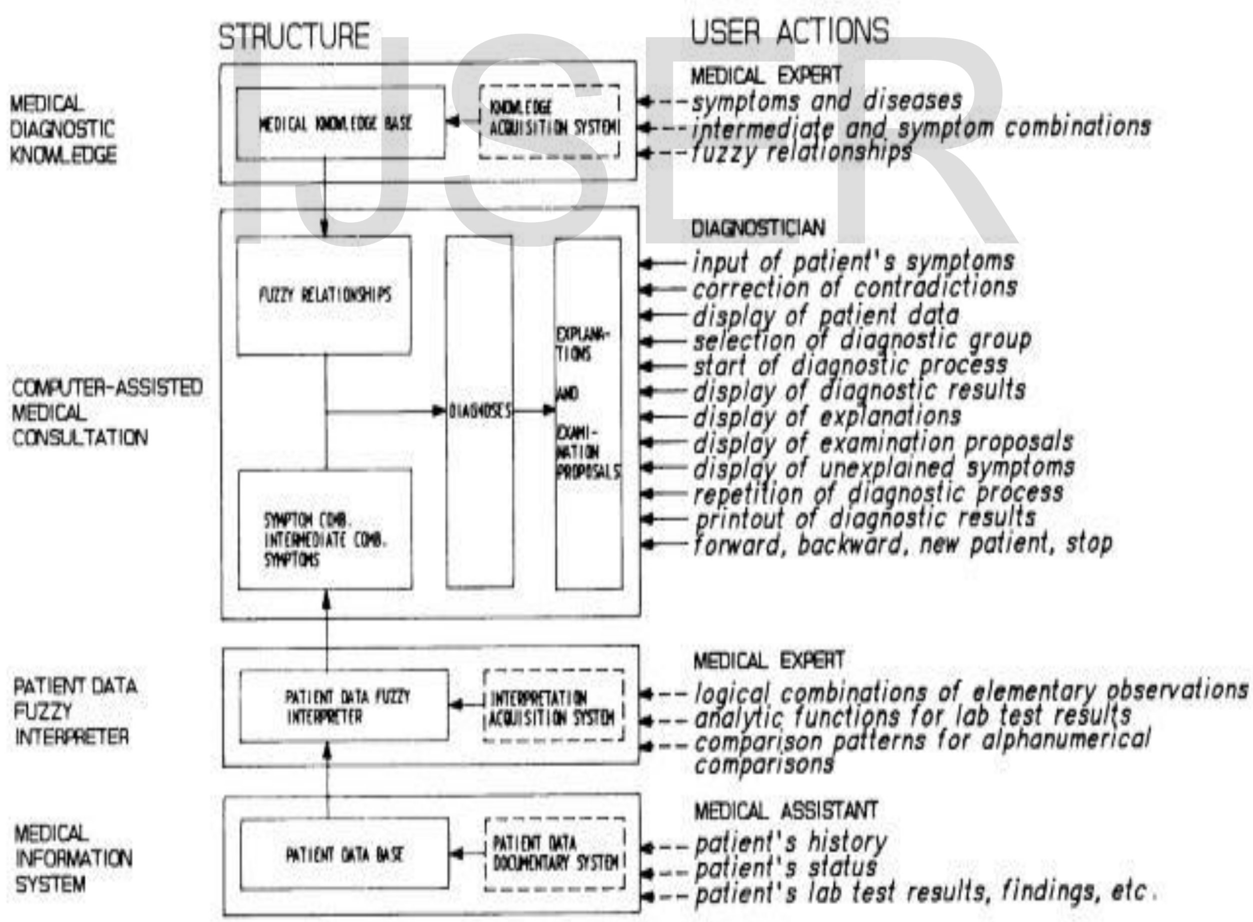

Fig. 1. Structure of CADIAG-2 with connection to a medical information system (dashed lines mark components effective before starting the individual consultation).

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Both entities take their values μICK, and μSCI, (respectively) in [0, 1] U {v}, where v implies that the actual value has not yet been determined. The relationship Rpsc ⊂ π X K is defined by μRpsc (Pq, SCI) = μSCI, for patient Pq, where SCI ∈K (K = {SC1…SCi}) formally describes the symptom combinations observed in the patient.

The fuzzy logical connectives are defined as follows.

Conjunction:

$$x_1 \wedge x_2 = \begin{cases} \min(x_1, x_2), & \text{if } x_1 \in [0,1] \text{ and } x_2 \in [0,1] \\ \nu, & \text{if } x_1 = \nu \text{ and/or } x_2 = \nu. \end{cases}$$

Disjunction:

$$x_1 \vee x_2 = \begin{cases} \max(x_1, x_2), & \text{if } x_1 \in [0,1] \text{ and } x_2 \in [0,1] \\ x_1, & \text{if } x_1 \in [0,1] \text{ and } x_2 = \nu \\ x_2, & \text{if } x_1 = \nu \text{ and } x_2 \in [0,1] \\ \nu, & \text{if } x_1 = \nu \text{ and } x_2 = \nu. \end{cases}$$

Negation:

$$\bar{x}_1 = \begin{cases} 1 - x_1, & \text{if } x_1 \in [0,1] \\ \nu, & \text{if } x_1 = \nu. \end{cases}$$

The following relationships between medical entities are considered in medical expert system CADIAG-2:

- symptom-disease relationships (Si, Dj)
- symptom combination-disease relationships (Sci, Dj)
- symptom-symptom relationships (Si, Sj)
- disease-disease relationships (Di, Dj).

These relationships are characterized by two parameters:

- frequency of occurrence (o)
- strength of confirmation (c).

For a relationship between medical entities X and Y (where X and Y might be symptoms, diseases, or symptom combinations), the frequency of occurrence describes the frequency with which X occurs when Y is present. Similarly, the strength of validation reflects the degree to which the presence of X implies the presence of Y.

The relationships between medical entities are given in the form of relationship rules with associated relationship tuples. The general formulation of these rules is;

IF (antecedent) THEN (consequent) WITH (o, c).

The relationship tuples (o, c) contain either numerical values μo and, μc, or linguistic fuzzy values λo and λc, or both [3].

The definitions of the linguistic values X and XA., the intervals that they cover, and their representative numerical values are given in Table I. Some examples of relationship rules are given below.

Example 1:

IF (ultrasonic of pancreas is pathological)

THEN (pancreatic cancer)

WITH (0.75 = often, 0.25 = weak).

Example 2:

IF (tophi)

THEN (gout)

WITH (0.25 = seldom, 1.00 = always).

Example 3:

IF (lower back pain Λ limitation of motion of the lumbar spine Λ diminished chest expansion Λ male patient Λ age between 20 and 40 years)

THEN (ankylosing spondylitis)

WITH (v, 0.90 = very strong).

Frequency of Occurrence			Strength of Confirmation		
Value λ_o	Interval	Representative Value μ_o	Value λ_c	Interval	Representative Value μ_c
Always	[1.00, 1.00]	1.00	Always	[1.00, 1.00]	1.00
Almost always	[0.98, 0.99]	0.99	Almost always	[0.98, 0.99]	0.99
Very often	[0.83, 0.97]	0.90	Very strong	[0.83, 0.97]	0.90
Often	[0.68, 0.82]	0.75	Strong	[0.68, 0.82]	0.75
Medium	[0.33, 0.67]	0.50	Medium	[0.33, 0.67]	0.50
Seldom	[0.18, 0.32]	0.25	Weak	[0.18, 0.32]	0.25
Very seldom	[0.03, 0.17]	0.10	Very weak	[0.03, 0.17]	0.10
Almost never	[0.01, 0.02]	0.01	Almost never	[0.01, 0.02]	0.01
Never	[0.00, 0.00]	0.00	Never	[0.00, 0.00]	0.00
Unknown		ν	Unknown		ν

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TABLE I: LINGUISTIC FUZZY VALUES, NUMERICAL INTERVALS, AND REPRESENTATIVE NUMERICAL VALUES DESCRIBING FREQUENCY OF OCCURRENCE AND STRENGTH OF CONFIRMATION

The values μo and, μc, are interpreted as the values of the fuzzy relationships between antecedents and consequents. Thus

S_iD_j (occurrence relationship)	$R^o_{SD} \subset \Sigma \times \Delta$
S_iD_j (confirmation relationship)	$R^c_{SD} \subset \Sigma \times \Delta$
SC_iD_j (occurrence relationship)	$R^o_{SCD} \subset K \times \Delta$
SC_iD_j (confirmation relationship)	$R^c_{SCD} \subset K \times \Delta$
S_iS_j (occurrence relationship)	$R^o_{SS} \subset \Sigma \times \Sigma$
S_iS_j (confirmation relationship)	$R^c_{SS} \subset \Sigma \times \Sigma$
D_iD_j (occurrence relationship)	$R^o_{DD} \subset \Delta \times \Delta$
D_iD_j (confirmation relationship)	$R^c_{DD} \subset \Delta \times \Delta$.

B. Fuzzy Logical Inference:

The compositional inference rule proposed by Zadeh [4] and introduced into medical diagnosis by Sanchez [28], [29] is adopted as an inference mechanism. It accepts fuzzy descriptions of the patient's symptoms and infers fuzzy descriptions of the patient's diseases by means of the fuzzy relationships described in the previous section.

Three such inference rules (compositions) are used to deduce the diseases Dj suffered by patient Pq from the observed symptoms Si:

1) Composition for S_iD_j (hypotheses and confirmation):

$$R^1_{PD} = R_{PS} \circ R^c_{SD} \qquad (1)$$

defined by

$$\mu_{R^1_{PD}}(P_q, D_j) = \max_{S_i} \min\left[\mu_{R_{PS}}(P_q, S_i); \mu_{R^c_{SD}}(S_i, D_j)\right].$$

2) Composition for S_iD_j (exclusion (by present symptoms)):

$$R^2_{PD} = R_{PS} \circ (1 - R^c_{SD}) \qquad (2)$$

defined by

$$\mu_{R^2_{PD}}(P_q, D_j) = \max_{S_i} \min\left[\mu_{R_{PS}}(P_q, S_i); 1 - \mu_{R^c_{SD}}(S_i, D_j)\right].$$

3) Composition for S_iD_j (exclusion (by absent symptoms)):

$$R^3_{PD} = (1 - R_{PS}) \circ R^o_{SD} \qquad (3)$$

defined by

$$\mu_{R^3_{PD}}(P_q, D_j) = \max_{S_i} \min\left[1 - \mu_{R_{PS}}(P_q, S_i); \mu_{R^o_{SD}}(S_i, D_j)\right].$$

The following diagnostic results are obtained.
A diagnosis is confirmed if

$$\mu_{R^1_{PD}}(P_q, D_j) = 1.00. \qquad (4)$$

A diagnosis is possible if

$$\epsilon \leqslant \mu_{R^1_{PD}}(P_q, D_j) \leqslant 0.99. \qquad (5)$$

The boundary value ϵ is a heuristic value which precludes diagnoses with very low evidence (e.g., $\epsilon = 0.10$). A diagnosis is excluded if

$$\mu_{R^2_{PD}}(P_q, D_j) = 1.00 \qquad (6)$$

or

$$\mu_{R^3_{PD}}(P_q, D_j) = 1.00. \qquad (7)$$

Symptom combination-disease inferences (called compositions4, 5, and 6) are carried out and interpreted in an analogous way. Symptom-symptom inferences (called compositions 7, 8, and 9)are computed in order to complete the patient's symptom patterns. Disease-disease inferences (called compositions 10, 11, and12) are performed in order to confirm general disease categories from the presence of differential diagnoses, to exclude entire areas of differential diagnoses if a particular general disease category is definitely absent, and to exclude mutually exclusive diseases if one of these diseases is confirmed.

C. Acquisition of Medical Knowledge:

The knowledge acquisition system is capable of acquiring information on medical entities and the relationships between them. In CADIAG-2, relationships are stored as numerical values in the range [0, 1]. Medical information can be acquired in two ways:

1) Through numerical or linguistic evaluation by medical experts

2) By statistical evaluation of a database containing medical data on patients with confirmed diagnoses.

CADIAG-2 relationships have the important property that they can be interpreted statistically. The values of the frequency of occurrence μo and the strength of confirmation μc, are defined as follows:

$$\mu_o = \frac{F(S_i \cap D_j)}{F(D_j)} = F(S_i / D_j) \quad (8)$$

$$\mu_c = \frac{F(S_i \cap D_j)}{F(S_i)} = F(D_j / S_i) \quad (9)$$

where

$F(S_i \cap D_j)$ absolute frequency of occurrence of S_i and D_j
$F(D_j)$ absolute frequency of occurrence of D_j
$F(S_i)$ absolute frequency of occurrence of S_i
$F(S_i / D_j)$ conditional frequency of S_i given D_j
$F(D_j / S_i)$ conditional frequency of D_j given S_i.

D. The Diagnostic Process:

1. **Symptoms:** The symptoms of the patient can be entered intoCADIAG-2 in three ways :
   a. By natural language input of symptoms Si;
   b. By natural language input of keywords that trigger whole groups of symptoms S,;
   c. Byaccessingadatabase containing the patient's data and transferring information via a fuzzy interpreter.

Input of keywords such as "present complaints," "previous complaints," "blood count," or "ultrasonic" causes whole sections of the symptom thesaurus to be displayed. Subsequently, fuzzy values can be linked with these symptoms by the physician.

The existence of a database that already contains the patient's symptoms suggests the automatic transfer of information from the database to CADIAG-2. During this transfer, the data is passed through a fuzzy interpreter, which contains instructions about the assignment of fuzzy values to observations, lab test results, and even simple alphanumeric texts.

After the patient's symptoms have been collected, symptom-symptom inferences are performed. The symptom list contains all necessary items of data, including fuzzy value, origin(measured; inferred), predefined symptom class (routine; specially requested; invasive or expensive), numerical value, units, and date of observation. The list of symptoms is then checked for contradictions.

2. **Symptom Combinations**: Intermediate combinations of symptoms are evaluated in the next step. Having passed the consistency check, fuzzy values for all symptom combinations are computed. The resulting lists are now as complete as possible and do not contain any contradictions.

3. **Confirmed diagnoses**: The fuzzy values μDj = 1.00, i.e., confirmed diagnoses Dj, for patient Pq, are identified using the following equation:

$$\mu_{D_j} = 1.00, \quad \text{if} \begin{cases} \mu_{R^1_{PD}}(P_q, D_j) = 1.00 \\ \text{or} \\ \mu_{R^4_{PD}}(P_q, D_j) = 1.00. \end{cases} \quad (10)$$

4. **Excluded Diagnoses**: The fuzzy values μDj = 0.00, i.e., excluded diagnoses Dj for patient Pq, are identified using

$$\mu_{D_j} = 0.00, \quad \text{if} \begin{cases} \mu_{R^2_{PD}}(P_q, D_j) = 1.00 \\ \text{or} \\ \mu_{R^3_{PD}}(P_q, D_j) = 1.00 \\ \text{or} \\ \mu_{R^5_{PD}}(P_q, D_j) = 1.00 \\ \text{or} \\ \mu_{R^6_{PD}}(P_q, D_j) = 1.00. \end{cases} \quad (11)$$

Disease-disease relationships now allow the inference of further diagnoses (confirmed or excluded):

$$\mu_{D_j} = \begin{cases} 1.00, & \text{if } \mu_{R^{10}_{PD}}(P_q, D_i) = 1.00 \\ 0.00, & \text{if } \mu_{R^{11}_{PD}}(P_q, D_i) = 1.00 \\ 0.00, & \text{if } \mu_{R^{12}_{PD}}(P_q, D_i) = 1.00. \end{cases} \quad (12)$$

5. **Possible Diagnoses**: Method a): Fuzzy values μDj such that E< μDj < 0.99 indicate possible diagnoses. These are determined as follows:

$$\mu_{D_j} = \max\left[\mu_{R^1_{PD}}(P_q, D_j); \mu_{R^4_{PD}}(P_q, D_j); \mu_{R^{10}_{PD}}(P_q, D_i)\right],$$
$$\text{if} \begin{cases} \epsilon \leqslant \mu_{R^1_{PD}}(P_q, D_j) \leqslant 0.99 \\ \text{and/or} \\ \epsilon \leqslant \mu_{R^4_{PD}}(P_q, D_j) \leqslant 0.99 \\ \text{and/or} \\ \epsilon \leqslant \mu_{R^{10}_{PD}}(P_q, D_i) \leqslant 0.99. \end{cases} \quad (13)$$

Method b): Because the values, μDj calculated by (13) are independent of the number of rules that can be used to support Dj, a heuristic function is introduced which considers the number of criteria present or partly present, which suggest but do not confirm disease Dj. The function then calculates the corresponding number of points PNDj. The values of PNDj are helpful in judging between the various possible diagnoses, although the ultimate aim should be to obtain a confirmed diagnosis. The number of points PND is calculated as follows:

$$PN_{D_j} = 100 \sum_{i=1}^{m^*} \left\{ \alpha \min\left[\mu_{R_{PS}}(P_q, S_i); \mu_{R_{SD}^{o}}(S_i, D_j) \right] + \beta \min\left[\mu_{R_{PS}}(P_q, S_i); \mu_{R_{SD}^{c}}(S_i, D_j) \right] \right\}, \quad (14)$$

Where m* is the number of symptoms exhibited by the patient Pq that occur in the definition of D., and a + ,B = 1.00. At present, we generally take α = 0.09 and, β = 0.91, i.e., the strength of confirmation has ten times more influence than the frequency of occurrence on the value of PND.

6. **Explanation of Diagnostic Results**: The physician's acceptance of CADIAG-2's diagnoses depends strongly on the ability of CADIAG-2 to explain its diagnostic output. On request, the information supporting confirmed diagnoses, excluded diagnoses, and possible diagnoses is presented; this takes the form of the names of the medical entities, their definitions, their measured and fuzzy values, and their relationships to the diagnostic output.

7. **Proposals for Further Examination of the Patient**: One of the main objectives of CADIAG-2 is to provide iterative consultations, starting with simple, easy-to-examine, and cheap data. A number of possible diagnoses can usually be inferred from these data, and further examinations are then necessary to confirm or exclude these hypotheses. CADIAG-2 uses the medical information stored in its knowledge base to propose what form these further examinations should take. The symptoms for further study are clearly those that would confirm or exclude a particular diagnosis. Additionally, those symptoms which enhance the position of the possible diagnosis in the ranked list of all possible diagnoses are also indicated.

V. CONCLUSION

In this paper, we have an CADIAG-2 (computer-assisted diagnosis), an expert system especially designed for internal medicine, which is presently being clinically tested. The medical expert system CADIAG2, which uses fuzzy set theory to formalize medical relationships and fuzzy logic to model the diagnostic process. We have two results generated by CADIAG-2 are

A. **Rheumatic Diseases:** CADIAG-2/RHEUMA has undergone partial tests with data from patients at a rheumatological hospital . A study of 400 patients with rheumatoid arthritis, gout, Bechterew's disease, Sjogren’s disease, systemic lupus erythematosus, Reiter's disease, and scleroderma showed that CADIAG-2 obtained the correct diagnosis in 94.5 percent of the cases considered.

B. **Pancreatic Diseases:** CADIAG 2/PANCREAS was tested with data from 47 patients. The discharge diagnoses of these patients were assumed to be correct. Pancreatic cancer was confirmed three times. Confirmation was aided by the existence of a result "Specific abnormal pancreatic biopsy," which has a strength of confirmation μu = 1.00 for pancreatic cancer.

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