Fractal Character of Drying Paint Films
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Abstract — With the progress in new materials and colloidal suspensions there has been progress in development of surface coating paints and oil paints. In this process of evaluating the performance of such paints under different conditions, the drying properties and formation of the final paint coats has been subject of interest. The properties of the dried paint film, in addition to its intrinsic properties depend on the substrate and the conditions under which the film is deposited. It has been shown that the porosity of substrate and the presence of moisture adversely affect the film properties and its strength. Under severe cases pealing of film and pitting of the surface is also caused by the seepage or percolation of moisture from the substrate below the paint coating. The thickness of the paint film and the rate of drying of the paint also determine the final structure and texture of the coat. Once the paint is applied, the process of drying commences and proceeds gradually, however as the upper surface tend to be little harder as the film solidifies, the thinner and solvents present in the paint finds it difficult to leave the free surface and get evaporated. As a result the rate of drying gets retarded as the top most layer of the paint solidified and become harder and harder. If the entire paint layer is not sufficiently dry at this stage, the physical stress present in the drying layer causes the layer to undergo deformation to arrive at a stable state. Such a drying of relatively thicker paint films is found to exhibit wrinkling of the paint coatings. We studied the drying process of thick paint films under different conditions and tried to estimate the irregularity of shape in relation to structure and texture associated with the dried film using the concept of fractal dimensions. The patterns formed exhibit broader wrinkles with relatively higher fractal dimensions for thick coats in contrast to those using thinner coat showing finer patterns with lower fractal dimensions indicating higher degree of complexity associated with the shape with lower thickness, results are presented.

Keywords: Paint Film, wrinkling, Fractal, Fractal dimension, Box counting.

1 INTRODUCTION

Irregular shapes can best be characterized and quantified using the concept of fractals and fractal dimension [1 to 4]. The irregular patterns [5] developed during the drying of thicker oil paint films are developed under different conditions and characterized using the concept of fractal dimensions [6 to 8]. Two different types of substrates were studied namely steel sheet and wooden planks, same quality of teak wood planks were used for the study. Different thickness of paint layers were tried, too thick layer of paint tends to flow and develop tendency of accumulating at some region, therefore too thick films were avoided. During the experiments, the surface to be painted was laid horizontally to see to it that the paint layer does not get affected by the slow flowing of the viscous paint.

To control the drying rate, controlled convection was used and all the experiments were conducted under controlled temperature conditions[9, 10]. It is observed that on wooden surface the fluids present in the oil paint are absorbed in the porous for wooden surface and drying is relatively faster as compared to coatings on the steel sheets. This faster removal of solvent and fluids in coatings on wooden surfaces results in relatively faster drying of the oil paint, however the formation or wrinkles on the drying surface film[11,12] is found to be limited and less as compared to that on steel sheets under similar conditions.

2 Effect of Thickness of Coat:

Fig. 1 shows two images obtained from the drying paint coated on horizontally laid steel sheet. Fig. 1 A corresponds to application of a thick layer of oil paint on horizontal steel sheet allowed to dry under normal conditions. It shows thick streaks of wrinkled oil paint surface, it is interesting to note that these streaks possess self avoiding behavior that is characteristic of
Diffusion Limited Aggregation (DLA) [13 to 16] which is governed by Laplace’s equation which in present case is related to the flow conditions and properties of fluid. For the conditions like drying of paint coat, the fluid parameters and the flow conditions change both temporally and spatially and the precise estimation of these values is not feasible in general. The development of irregular patterns exhibiting fractal character is a consequence of these competing processes like random flow conditions and drying and sticking tendency of particles and molecules leading to growth of fractal patterns.

Most of the patterns resulting from DLA exhibit self similarity and scale invariance and thus are fractal in character. Fig. 1 B is similar to Fig. 1 A except in that the surface of the metal plate used was not perfectly flat and had a hump. It is observed that those portions where there was a hump, the layer of oil paint remained thinner due to flow to the nearby low lying areas thereby increasing the thickness of the coat. The thicker layer regions show broader lines and patterns as compared to those in the thinner layer regions.

The images were analysed for self similarity and scale invariance using box counting technique. It was observed that the patterns possess self similarity and scale invariance and power law is obeyed. For box counting the image was analysed using boxes of different size (r) and the number of boxes required to completely cover the pattern (N) was found using a computer program[15, 16]. Plot of log(N) versus log(r) is shown in Fig. 2. It is clearly seen that all the points lie along a straight line indicating that the power law holds and self similarity and scale invariance exist. The fractal dimension estimated from the plot is 1.757 for pattern of Fig. 1 A and 1.778 for that of Fig. 1 B. A higher fractal dimension indicates higher degree of complexity associated with the pattern in terms of structure and texture and a crowded pattern. The raised portion in the hump region has lower paint thickness and the pattern formed has finer details in this region.

![Fig. 1 Showing the effect of the thickness of the paint layer on drying. Thicker paint regions tend to exhibit wider wrinkles (A) as compared to thinner regions (B right side part).](image)

![Fig. 2 Plot of log(N) versus log(r) for patterns of Fig. 1 A and B. Points plotted are data from box counting and the line joining the points is the least square fit to these data points. The fractal character of the drying paint films shown in Fig. 1 A and B is confirmed by the scale invariance](image)
and self similarity as the plot of log(N) versus log(r) is a straight line. Points plotted are from the actual results of box-counting and the line joining these points is the least square fit applied to these points. The equation of the best fitting line is given in the inset and the value of R² is also close to unity (about 0.995) indicating that the points are represented by a straight line and the fractal dimension is found from the slope of the straight line.

A typical pattern obtained using a moderate layer of oil paint on a horizontal steel sheet is shown in Fig. 3. As the thickness of the drying layer was relatively less, the wrinkling pattern has finer details of structure and texture. In contrast to Fig. 1, here the pattern formed has thinner lines and possess similar character of self avoiding nature and two lines never cross. In most of the cases these streaks show continuity and zigzag shapes. Inspection of the pattern indicates that the thickness of these tracks formed is more or less identical over the entire region.

Fig. 3 Showing the more or less uniform pattern for uniform thickness of the drying paint.

Fig. 4 Plot of log(N) versus log(r) for patterns of Fig. 3. Points plotted are data from box counting and the line joining the points is the least square fit to these data points.

The results of box-counting for the pattern shown in Fig. 3 are presented in Fig. 4. The points plotted are actual data points from results of box-counting and the line joining the points is the least square fit straight line to this data. The points fairly lie along a straight line as is seen from the value of R², which is more 0.99 (Table - 1) and close to unity indicating the goodness of fit. This confirms the presence of self similarity and scale invariance of the pattern and that the pattern is a fractal with a fractal dimension of 1.8132. The fractal dimension in this case is higher than both the earlier patterns. This indicates that the pattern obtained has more of complexity of structure and texture associated with it and the pattern is more crowded structure than the earlier patterns of Fig. 1 A and B.

Fig. 5 Showing the more or less uniform pattern for uniform thickness of the drying paint but thicker paint layer as compared to that in Fig.3.

Fig. 5 is a paint drying pattern obtained using oil paint on a steel sheet and is identical to Fig. 3 except in that the oil paint coating thickness is more than that used in the case of Fig. 3. The effect of increased thickness is seen in the form of thicker streaks and distribution of the pattern. The geometric structure is also relatively less complex as compared to that of Fig. 3. This also exhibits wrinkles of uniform thickness and line structure.
continuity, the self avoiding character is also clearly visible.

\[ y = -1.8152x + 5.0278 \]
\[ R^2 = 0.9961 \]

![Graph showing log(N) versus log(r) for patterns of Fig. 5. All the points lie along a straight line and the resulting fractal dimension is 1.8152 indicating higher degree of complexity of shape.](image)

The pattern was analysed by using box counting for fractal dimensions and the results of box counting are presented in Fig. 8.

It is observed that for porous surfaces like that of wood, the drying patterns differ from those obtained on metallic plates. Also fresh wooden surface with limited or no primer applied indicate faster drying and relatively reduced wrinkling of drying paint surface. Properly treated wooden surfaces with suitable primer applied exhibit slower drying compared to raw wooden surfaces, as a consequence there is less of wrinkling on the dried paint surface and this character resembles that of metal surfaces. To study the difference caused by application of primer to a wooden surface, a patch of primer was applied to the wooden surface and after drying, it was given a thick coat of oil paint (including portions without primer). It was observed that in the middle, where there is a undercoat of primer, limited wrinkling is seen as compared to neighboring regions where thick wrinkles are found on either sides of the primer patch. There were few defective points on the wooden surface, non uniformity of coating and drying pattern is visible at those sites.
The fractal character of the pattern shown in Fig. 7 is established from Fig. 8 as all the points lie along a straight line with a value of $R^2$ close to unity indicating the goodness of the fit. The fractal dimension obtained from plot of Fig. 8 is 1.8621 indicating that the complexity of shape associated with the pattern is high and it possesses details of structure and texture.

The presence of discontinuities and irregularities in the surface gets reflected in the final surface of the coating. Fig. 9, 10 A and 10 B are examples of selected defective surfaces. Fig. 9 has a joint of wooden planks and a Screw head and few smaller defects. Fig. 10 A shows a region selected containing a screw head and a nail and Fig. 10 B has a defective corner with a charred corner.

In most of the cases streaks in the wrinkling patterns appear to originate from discontinuities or unevenness of the surface of the substrate being coated. In Fig. 9, the lines are seen originating from regions of unevenness. The broad ridge is the joint between the two planks of the substrate. In Fig. 10 A, the entire pattern looks as if originating from the round head of the screw seen as a circular patch. Similarly in Fig. 10 B, the tree like branching pattern seems to originate from the upper left corner with non uniformity.

Fig. 9 Showing different patterns in different regions of thickness of the drying paint, the wrinkle streaks appear to originate from the discontinuity seen as horizontal broad ridge around joint of planks.

Fig. 9, 10 A and 10 B were analysed for self similarity and scale invariance using box counting technique and the results are presented in Fig. 11 as a plot of log (N) versus log(r). For all the three patterns the data points are represented by three different lines with different slopes. The fractal dimensions obtained for the three patterns are 1.8088, 1.7603 and 1.8867 respectively. Dense populated patterns with crowded and dense structure exhibit higher fractal dimensions. The fractal dimension also represents the degree of complexity associated with the pattern, higher the fractal dimension, more is the associated complexity.

Fig. 10 Showing presence of irregularities and discontinuity in the substrate used, in A the radial pattern appears to originate from the round patch (head of screw) and in B from upper left corner.
Results and discussion

The fractal characterization of the patterns developed by thick layer of drying paint shows wrinkling on the drying surface with stripes and bands showing self avoiding tendency that is a characteristic of the DLA fractal patterns. The results of box counting show that self similarity and scale invariance is present over more than one order of magnitude for all the patterns presented in Fig. 1, 3, 5, 7, 9 and 10. The log(N) versus log(R) plot is a straight line with R² value of more than 0.99 (close to unity) indicating the goodness of fit and thus confirming scale invariance, self-similarity and fractal character. The fractal dimensions estimated are listed in Table 1.

<table>
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<th>S.No.</th>
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