Finite Element Modeling Of Shell Thickness Requirement Due To Local Loads On Spherical Pressure Vessels

Oludele ADEYEFA, Oluleke OLUWOLE

Abstract- Stresses caused by external local loads are major concern to design engineers of pressure vessels. The technique for analyzing local stresses and the method of handling these loadings to keep these stresses within prescribed limits of shell material allowable stress has been the focus of this research. Finite element method is employed as numerical tool for the investigation of local stresses at the point of attachments of legs on the shell of spherical pressure vessels. Shallow triangular element based on shallow shell formation was employed using area coordinate system. The element has six degrees of freedom at each corner node - five of which are the essential external degrees of freedom and the sixth is the nodal degree of freedom associated with in plane shell rotation. Set of equations resulting from finite element analysis were solved with computer programme code written in FORTRAN 90. The thickness requirements at the region of attachment of support legs were determined. Increase in thickness in this region was due to the local bending moment applied at the point of attachment of the supports. The obtained results were validated using ASME values. The results obtained with the methodology in this research shows no significant difference (P>0.05) with ASME values.

Index Terms- Liquefied Natural Gas, Finite Element Method, FORTRAN

1. INTRODUCTION

The most significant findings and solutions for the analysis of local loads are those developed by [1]. These investigations were carried out in the 50s and sponsored by the Pressure Vessel Research Committee of the Welding Research Council, WRC [1].

It is of note that American Society for Mechanical Engineers (ASME) Section VIIIDivision 1 does mentionlocal loads as part of the loads to be considered by design engineers when designing pressure vessels. It does not give the procedures for analyzing such loads and determination of shell thickness requirement to bring the effect of local loads to the acceptable stress limits. It is, therefore, expected of design engineers to use his engineering skills on the best methodology to use for the analysis of developed membrane shell stress due to local loadings.

[2] in their paper, analytical solutions for displacements and stresses in spherical shells over rectangular areas are developed. The analysis was based on spherical shallow shell equations, and solutions were obtained through the use of double Fourier series expressions to represent the displacement and loading terms. Three types of loading were considered: radial load, overturning moment and tangential shear. Study was carried out by [3] to investigate the collapse load of a spherical shell under axial loading on a central boss. The investigation was carried out using an existing lower-bound plastic limit analysis package and the ABAQUS finite element code. Results were obtained for various geometrical parameters and compared with previous analytical and experimental results. Analytical solutions for displacements, membrane stresses, and bending stresses in spherical shells due to local loadings over a rectangular area were developed by [4]. The three types of loading considered are radial load, overturning moment load, and tangential shear load.

Dynamic response and energy absorption of aluminum semi-spherical shells under axial loading using non-linear finite element techniques was treated by [5]. Aluminum and steel spherical shells of various radii and thicknesses were made by spinning. The influence of geometrical, material and loading parameters on the impact and quasi-static response was investigated using validated numerical models. Also the axial inward inversions of semi-spherical shells were investigated.

In designing of pressure vessels, there are various methods of reducing shell stresses due to local loads. These methods have some bearing on how local loads are

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ISSN 2229-5518

analyzed. Below are some of the methods available to pressure vessel design engineers to reduce shell stresses locally.

1. Increase the size of the attachment.
2. Increase the number of attachments.
3. Change the shape of the attachment to further distribute stresses.
4. Add reinforcing pads. Shell stresses must be investigated at the edge of the attachment to the pad as well as at the edge of the pad.
5. Increase shell thickness locally or as an entire shell

The local stress as outlined in this research work is due to external mechanical load – turning momentapplies on the spherical shell due to the support legs/columns.

2. METHODOLOGY

2.1. Finite Element Analysis

Finite element methodology adopted in this present research work is the one used in [6,7]. In their analysis shell thicknesses due to the internal applied pressure were determined. The results of their analysis gave acceptable results when compared with the American Society for Mechanical Engineers (ASME) Section VIII, Division 1 code requirements. The displacement functions and shell element gave acceptable results with few elements.

2.2. Model Assumptions

The below are the assumptions made in the course of this research work

1. The compressive force carries by each leg is determined by dividing spherical pressure vessel hydrotest weight with the number of legs/columns.

2. In the analysis, shell thickness requirement due to local load of one leg is representative of all other legs. Therefore, section of spherical shell with leg attachments is thickened by additional calculated thickness requirement.

3. The point of attachment of pressure vessel should be practically closed as possible to the centroid of the support legs as shown in **figure 1**. This is necessary to minimize the eccentricity, e, thereby reducing the resulting bending moment.

4. Tangential force acting on the spherical shell is equal to the reaction of leg supports due to the compressive force as in assumption one, if eccentricity, e, is zero as shown in **Fig. 2.**

5. The shell thicknesses due to the internal pressure had been determined using any applicable methods. Therefore, the starting shell thickness in the FE modeling is the shell thickness at the point of attachment of legs due to the internal pressure.

2.3. Displacement Functions

The use of shallow triangular element (Figure 3.) and "area coordinates" is adopted herein as in [6, 7]. Transverse displacement, w, as a polynomial function of third degree as given by [8]. Linear polynomial equations were then used to represent the membrane displacements u and v using area coordinates, resulting in a constant strain triangle for the membrane action. The assumed displacement equations are:-

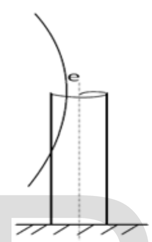

Fig. 1. Spherical shell supported with eccentricity, e

Fig. 2.Spherical shell supported with no eccentricity

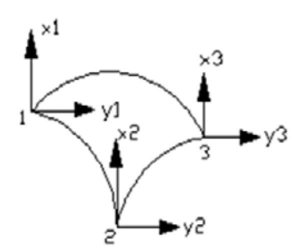

Fig. 3. Shallow Triangular Element

ISSN 2229-5518

$$u = a_1L_1 + a_2L_2 + a_3L_3$$
(1)

$$v = a_4L_1 + a_5L_2 + a_6L_3$$
(2)

$$w = a_7L_1 + a_8L_2 + a_9L_3 + a_{10}L_1L_2 + a_{11}L_2L_3 + a_{12}L_3L_1 + a_{13}\left[L_1^2L_2 + \frac{1}{2}L_1L_2L_3\left\{3(1-\mu_3)L_1 - (1-3\mu_3)L_2 + (1-3\mu_3)L_3\right\}\right] + a_{14}\left[L_1^2L_2 + \frac{1}{2}L_1L_2L_3\{3(1-\mu_3)L_1 - (1-3\mu_3)L_2 + (1-3\mu_3)L_3\}\right] + a_{15}\left[L_1^2L_2 + \frac{1}{2}L_1L_2L_3\{3(1-\mu_3)L_1 - (1-3\mu_3)L_2 + (1-3\mu_3)L_3\}\right]$$
(3)

$$\theta_x = \frac{\partial w}{\partial y}$$
(4)

$$\theta_y = -\frac{\partial w}{\partial x}$$
(5)

where

$$\mu_i = \frac{l_k^2 - l_j^2}{l_i^2}$$

and l_i is the length of the side opposite node i. The modified interpolation for displacement is taken as

$$\phi = Pa$$
(6)

to determine constants a_s, known displacements at nodes are substituted and the equations become

$$[a] = [C^{-1}][\delta]$$
(7)

Where[δ] is the nodal degrees of freedom, $[C^{-1}]$ is inverse of transformation matrix and[a] is vector of independent constants.

2.4. Strain-Displacement Equations

Strain-displacement relationships for shallow thin shells as given by [11] are simplified for the shallow shell and expressed as follows in curvilinear coordinates.

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{r},\ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{r},\ k_x = -\frac{\partial^2 w}{\partial x^2}$$
$$k_y = -\frac{\partial^2 w}{\partial y^2},\ k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \qquad (8)$$

The above strain equations (8) can be written in matrix form after necessary substitutions of u, v and w in equations (1, 2 and 3) into the above strain equations.

2.5. Stresses in a Curved Triangular Element

Stress varies from point to point along the shell profile and also through the thickness of the shell making it an unknown function of two variables [9]. It is represented as shown below [4]:

$$\sigma_b = \frac{6M}{t^2}\ \sigma_m = \frac{N}{t} \qquad (9)$$

Where: M is the moment per unit length, M and σ_b is the bending stress at the surface.

N is force per unit length and σ_m which is membrane stress.

2.6. Strain Energy

The strain energy equation for an isotropic linear shell as given by [9] was adopted in this work;

$$U = \iint_A \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{E}{2(1-v^2)}\begin{bmatrix}\varepsilon_x^2 + \varepsilon_y^2 + 2v\varepsilon_x\varepsilon_y \\ +\frac{1}{2}(1-v)\gamma_{xy}^2\end{bmatrix} d\varsigma dxdy \qquad (10)$$

Where, t = thickness of the shell, v = Poisson's ratio and E = Modulus of elasticity

ε and γ are the strain and shear strain notations.

After substitution for strains in the above expression and integration with respect to ς, the strain energy can be separated into the membrane energy U_m and the bending energy U_b.

$$U = U_m + U_b \qquad (11)$$

$$U_m = \frac{Et}{2(1-v^2)}\iint_A \begin{bmatrix} e_x^2 + e_y^2 + 2ve_xe_y \\ +\frac{1}{2}(1-v)e_{xy}^2\end{bmatrix} dxdy \qquad (12)$$

$$U_b = \frac{Et^3}{24(1-v^2)}\iint_A \begin{bmatrix} k_x^2 + k_y^2 + 2vk_xk_y \\ +\frac{1}{2}(1-v)k_{xy}^2\end{bmatrix} dxdy \qquad (13)$$

The potential energy, $\Phi = U - W$ where W represents the work done by the external load on the system. In the finite element method, the potential energy of a shell is expressed as:

$$\Phi = \sum_{k=1}^{n} \phi_k \qquad (14)$$

ISSN 2229-5518

where ϕ_k is the potential energy of the k[th] element.

2.7. Stiffness Matrix

$$k_m = t[C^{-1}]^T \iint_A B_m^T D_m B dA [C^{-1}] \qquad (15)$$

$$k_b = t[C^{-1}]^T \iint_A B_b^T D_b B dA [C^{-1}] \qquad (16)$$

k_m and k_b are element stiffness matrices due to membrane and bending stresses respectively
D_m and D_b are elasticity matrices for membrane and bending stresses respectively
B_m and B_b are strain matrices for membrane and bending stresses respectively
Therefore, element total stiffness matrix is

$$k = k_b + k_m \qquad (17)$$

The element stiffness matrices were then combined to give the system stiffness matrix. The stiffness matrices k_b and k_m in terms of area coordinates were using three Gauss quadrature points. To integrate explicitly, the integral equation below as it is in [8] was very useful.

$$\int_A L_1^a L_2^b L_3^c dA = \frac{a!b!c!}{(a+b+c+2)!} 2\Delta \qquad (18)$$

Where Δ is the area of triangular element
The element matrix above is 15 x 15 but there are six degrees of freedom per node. The in-plane rotation is the sixth degree of freedom. This rotation does not enter the minimization procedure and this is accounted for by simply inserting appropriate number of zeros into the stiffness matrix. The addition of zeros in the stiffness matrix leads program complexity due to singularity. In [8], a set of rotational stiffness coefficients was used in general shell problem for all elements. These were defined in such that the local coordinate overall equilibrium is not disturbed. This set of rotational stiffness matrix is adopted for this element.

$$\begin{bmatrix} M_{zi} \\ M_{zj} \\ M_{zm} \end{bmatrix} = \frac{1}{36}\alpha_n E t^n \Delta \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$$

The above is that proposed by [6] where the value of n is unity in the scaling value t^n. Since the term leads to stiffness that is in terms of rotational parameter the scaling indicated above permits values proportional to generate by bending rotations – namely t cubed.

2.8. Boundary Conditions

Before the system equations are ready for solution, they must be modified to account for the boundary conditions of the problem. For this system, it is assumed that displacements in all directions with the exception ofdirections at which bending moment and internal pressure are acting. Due to the symmetry nature of the system, the spherical vessel is divided longitudinal and meshed as shown in **Fig. 4.**

Fig. 4 shows the arrangement for a mesh with five elements in the spherical shell. In **Fig. 5,** first and third courses take corresponding calculated thickness of elements 1 and 5 respectively of **FGig. 5**. The second course takes the highest of calculated thickness of elements 2, 3 and 4 in **Fig. 5**.

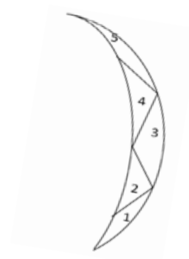

Fig. 4. 5-element Mesh

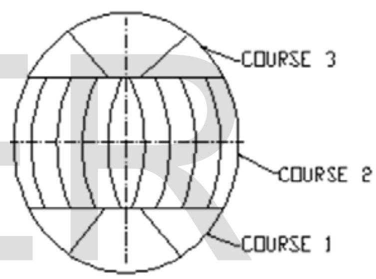

Fig. 5. 3-Course Version Spherical Vessel

3. LOCAL LOAD ANALYSIS

Case 1: Determination of Shell Thickness of a 13m Dia. Vessel storing compressed gas

Using a spherical pressure vessel with the following simulation parameters, the thickness of each element corresponding to the membrane stress developed at the centroid was determined. Membrane stresses at the centroid were programmed to be within the range of 0.0% and 2.0 % less than the spherical vessel construction material allowable stress given byASME standard. Results of the analyses are placed in Tables 8.1a and 8.1b. Data used for the analyses are:

Design Internal Pressure = 8.0MPa
Material of Construction = A516M Grade 70
Material Allowable Stress = 138 MPa
Specified Minimum Yield stress = 260 MPa
Poisson Ratio = 0.3

ISSN 2229-5518

Radius of Spherical Vessel = 6.5 m

The gas is Propane (C_3H_8) with density of 1.882kg/m3 at STD.

The leg is 12 inch Schedule - Standard.

TABLE 1
5-ELEMENT SPHERICAL MESH WITHOUT APPLIED LOCAL BENDING MOMENT

	t_{FEA}(mm)	t_{ASME} (mm)	% Deviation in Shell Thickness	Developed Membrane Stress (x 10^9 Pa)
Element 1	23.6	23.57	0.1273	0.1386
Element 2	23.6	23.57	0.1273	0.1376
Element 3	23.6	23.57	0.1273	0.1383
Element 4	23.6	23.57	0.1273	0.1362
Element 5	23.6	23.57	0.1273	0.1367

TABLE 2
5-ELEMENT SPHERICAL MESH WITH APPLIED LOCAL BENDING MOMENT – 787865Nm

	t_{FEA}(mm)	Developed Membrane Stress (x 10^9 Pa)	Factor of Safety
Element 1	23.72	0.1386	1.88
Element 2	23.72	0.1308	1.99
Element 3	24.40	0.1371	1.89
Element 4	23.72	0.1378	1.89
Element 5	23.72	0.1359	1.91

TABLE 3
5-ELEMENT SPHERICAL MESH WITH APPLIED LOCAL BENDING MOMENT – 78786Nm

	t_{FEA}(mm)	Developed Membrane Stress (x 10^9 Pa)	Factor of Safety
Element 1	23.72	0.1391	1.87
Element 2	23.72	0.1345	1.93
Element 3	24.00	0.1368	1.90
Element 4	23.72	0.1369	1.90
Element 5	23.72	0.1363	1.91

4. RESULTS AND DISCUSSIONS

Table 1 shows the FE thicknesses, calculated ASME thicknessess and developed membrane stress values for each element in a shallow spherical mesh without applied local bending moment. **Table 2** shows the same spherical shell values with applied local bending moment of 787865Nm. **Table 3** shows the same spherical shell values with reduced applied local bending moment of 78786 Nm. In **Table 1,** FEA and analytical thicknesses are given. The analytical values were calculated using ASME standard. In relating the obtained results to the 3-course spherical vessel as described in Fig. 5, the second course is thicker than the other courses because there is applied bending moment load due to the attachment of the supporting column. This is in order because in order to reduce the effect of the local load, one of the methods is to increase the shell thickness around the load.

Developed stresses in all the cases are stresses de veloped at the centroids of each element. In deter mining these stresses, the iteration is carried out by changing element thickness until corresponding de veloped membrane stress for each element is with a range of 0.0% and 2% less than the shell allowable stress. The reason in programming the allowable stress to be within the range of 0.0% and 2.% less than the spherical vessel construction material allowable stress could be seen here because highest developed membrane stress values fall not too far off from the shell material allowable stress. The percentage differences in the shell thicknesses of Table 1 are reasonable. . The FE model in these case studies show the possibility of obtaining reasonable results with few elements.

5. CONCLUSIONS

This study has investigated shell thickness requirement due to local loads acting on shell of spherical pressure vessels using the finite element method. Minimal shell thickness requirement due to local bending loads on spherical shell were determined for different cases. In the computer simulation, each element thickness has to be increased and developed membrane and bending stresses at the centroid of each element determined. These thicknesses are increased until the stress developed are within the range of less than 2% or equal to the shell material allowable stress as given by American Society of Engineers (ASME) material property part D.

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