Fin Spacing Optimization for Isothermal Rectangular Polished Aluminum Fins on a Vertical Base

Parimal S. Bhambare, Dr. Dinesh K. Kaithari, Ganesh S. Kharote

Abstract— The effect of fin spacing on the steady state heat transfer for an array rectangular aluminum fins on a vertical base has been investigated in the present work. The heat transfer rate from the fin array was experimentally studied by varying the fin spacing. Fin height (H), length (L) and thickness (t) are kept constant during experimentation while the fin spacing (S), heat input (Q) and number of fins have been varied as: 5 – 22.5 mm, 30 – 150 W and 5 – 15 respectively. Total 11 fin configurations have been tested during the experimentation. Radiation network analysis is used to estimate the radiation heat transfer rate and it is subtracted from the total heat input to estimate the convection heat transfer rate. Convective heat transfer coefficient, the average Nusselt number and Grashoff number have been estimated based on the analysis. Experimental results shows that for the given configuration of rectangular aluminum fin array on vertical base the optimum fin spacing is observed to be 6.9 mm (S/H = 0.046) for 12 fins configuration. Based on the analysis of the experimental data an expression between the average Nusselt number and aspect ratio (S/H) has been presented in the paper with overall error between 11-17%.

Index Terms: rectangular fins, fin spacing, heat transfer coefficient, Nusselt number, Grashof number

1 INTRODUCTION

Fins are extended surfaces on a heat transfer surface to enhance the heat transfer rate from the surface by convection. Convection heat transfer from a surface can be improved by the following ways:

1. Increase the temperature gradient between the surface and the environment;
2. Increase the heat transfer coefficient;
3. Increase the surface area.

The first two options may not be feasible but increase in surface area can be an economical solution to enhance the heat transfer rate from the surface. Fin arrays are used to improve the heat transfer rate and the fin spacing is an important parameter. The closely packed fin array has greater surface area thereby apparently feel more heat transfer rate from such fin arrays but truth is that the closely packed fin array results in reduction in heat transfer rate owing to the reduction in heat transfer coefficient. More spacing between the fins will result in higher heat transfer coefficient but with lesser area resulting in lower heat transfer rate. Hence optimum fin spacing is very important while designing multiple fin arrays in order to obtain maximum heat transfer rate. The heat generation especially in electronic components is a very common problem in the electronic systems leading to failure of system due to overheating. There are several important practical applications of natural convection from fin arrays such as radiators in cars, extended surfaces on the engine cylinder of two wheeler automobiles, CPU heat sinks of computers, heat exchangers in power plants and also in hydrogen fuel cells.

Different studies have been carried out in the field towards the optimization of the parameters as mentioned above. However, the studies are carried out and reported for the limited ranges of the fin configurations. The various heat transfer parameters such as average Nusselt number and average heat transfer coefficient for rectangular fin array with different number of fins, aspect ratios (spacing/height) and heat inputs are investigated.

1.1 Literature review

The heat transfer between isothermal vertical flat plates was investigated by Elenbaas [1] and investigated the effect of heat transfer coefficient with respect to plate spacing. For higher spacing between the plates the heat transfer coefficient approached the value of single plate whereas for closed packing the coefficient values approach that of fully developed laminar flow. So the spacing between the fins has a strong influence on the heat dissipation from plate fin arrays.

Welling and Wooldridge [2] found that horizontally based vertical fin configuration is not more useful compared to the vertically based vertical fin configuration because of its relatively poor ability to dissipate heat. Horizontally based vertical fin configuration is not more useful compared to the vertically based vertical fin configuration because of its relatively poor ability to dissipate heat as found by Jones and Sparrow Smith [3]. For example, in case of aluminum fins, the heat transfer is mainly due to the natural convection because the contribution of radiation to the total heat transfer is low due to low emissivity of aluminum. Rohsenow and Bar-Cohen [4] has provided the formula for evaluating optimum fin spacing for rectangular fin array in terms of characteristic length and Rayleigh number. They also

---

*Mr. Parimal S. Bhambare, Senior Lecturer, Caledonian College of Engineering, Muscat, Sultanate of Oman. E-mail: parimal.bhambare@gmail.com.*

*Dr. Dinesh K. Kaithari, Senior Lecturer, Caledonian College of Engineering, Muscat, Sultanate of Oman. E-mail: dineshkk36@yahoo.co.in.*

*Mr. Ganesh S. Kharote, Lecturer, Caledonian College of Engineering, Muscat, Sultanate of Oman. E-mail: gskytiet@gmail.com.*
provide expression to evaluate the convection heat transfer coefficient in terms of thermal conductivity of the fluid in contact with the fin surface and the optimal fin spacing. All the fluid properties are evaluated at the film temperature.

Theoretical investigation of natural convection heat transfer from horizontal fin arrays has been done by Baskaya et al. [5], where the effect of fin spacing, fin length, fin height and temperature difference between fin and surroundings on the heat transfer rate using finite volume method. It was shown that the optimum performance can’t be achieved only with variation of one or two parameters but the interaction between all the parameters needs to be considered. The results obtained were presented in the form of graphs with optimum values and correlations and the results were compared with experimental values from literature wherever possible.

Bar–Cohen et al. [6] analyzed the heat transfer characteristics such as total heat dissipation, heat dissipation per unit mass and space claim specific heat dissipation of rectangular fin array with optimum spacing and least material. The thermal performance of various materials such as aluminum, copper and magnesium was analyzed and magnesium was found to be efficient in terms of material utilization. The fin aspect ratios is found to be out of range of conventional manufacturing techniques that leads to scope of new manufacturing techniques in order to achieve the manufacture of optimal spaced fin arrays. Due to the simple construction, cheaper cost and effective cooling capability the rectangular fins are found to be more suitable among the all geometrical configurations as per Yazicioğlu and Yüncü [7]. Yazicioğlu and Yüncü [7] developed expression to predict optimal fin spacing for vertical rectangular fins protruding from a vertical rectangular base based on the experimental results available in literature. The fin spacing ranges from 2.85 mm to 85.5 mm whereas the temperature difference between the base and the ambient was from 14°C to 162°C. The range of fin thickness chosen for the analysis was from 1 mm to 19 mm. Width of the rectangular plate was from 180 mm to 250 mm. Natural convection heat transfer from rectangular fin arrays with notch at the center and without notch have been investigated experimentally by Shivdas and Hemant [9] and the heat transfer coefficient for various heat inputs are obtained. Unlike the usual fin material as aluminum, the fin material selected is copper in order to enhance the heat transfer rate as part of modernization. It is observed that the heat transfer coefficient increases with heat input and the heat transfer coefficient improved as a result of notch provided when compared with the case without notch for the same heat input.

Review on experimental and numerical studies pertaining to the natural, mixed and forced convective heat transfer through rectangular fin arrays was carried out by Daund et al. [10]. Review included the work of researchers who developed correlations between various parameters. Ashish and Mangesh [11] had performed numerical investigation for the heat transfer from rectangular fin arrays mounted on a circular base. The effect of 3 geometric parameters such as fin length, fin height and the number of fins as well as the heat flux on the thermal resistance and the average heat transfer coefficient was investigated numerically. The chimney flow pattern was considered where the outside air gets heated as they pass over the fins and emerge out by raising from inner region of the fins. It was observed that the thermal resistance and heat transfer coefficient decreased with an increase in fin length, fin height and the number of fins. Experimental investigation for the natural convection heat transfer from vertical aluminum fins was performed by Abbas and Ali [12]. Thickness, fin spacing and fin length chosen for analysis were 4 mm, 10 mm and 300 mm respectively. Three different fin heights (10 mm, 25 mm and 45 mm) were selected and the heat input ranges from 68 W to 716 W. The convection heat transfer coefficient, convection heat transfer rate, Nusselt number and Rayleigh number from fin arrays was found to depend on the fin height and found to be maximum for a fin height of 45 mm and minimum value for a fin height of 10 mm. An empirical correlation was developed between Nusselt number, Rayleigh number and fin height with a maximum variation of ±10%.

2 EXPERIMENTAL ANALYSIS
2.1 Experimental Set-up and Procedure

The experimental set-up used for the analysis consists of different components namely rectangular fins, base plate, heater, fin array, vertical enclosure, dimmerstat, voltmeter, ammeter and a non contact type infrared temperature sensor.

The vertical enclosure is constructed with plywood enclosing four sides screwed using set screws on the welded mild steel frame. The top and the bottom of the enclosure have been kept open in order to reduce extraneous air-current effect for maintaining the condition of natural convection inside the enclosure. The dimensions of the frame are kept large enough ensuring the ambient conditions inside the enclosure. A rectangular tungsten heater is sandwiched between the two base plates (material: aluminum, size: length (W) = 100 mm, height (H) = 150 mm and thickness (tp) = 5 mm, as the heat-generating surfaces). The fin array is constructed by joining the required number of polished aluminum fins on this heat generating surface using an appropriate quick adhesive having very low thermal contact resistance. The fin dimensions used during the experimentation: length (L) = 30 mm; height (H) = 150 mm and thickness (t) = 2 mm, are kept constant during the experimentation [8]. The spacing between the fins has been calculated at each stage by varying the number of fins attached on the base from 5 to 15. The fin spacing with number of fins has been shown in Table 1 as below. The constant vertical height, H = 150 mm of the fin was selected as a characteristic dimension for making different governing parameters dimensionless. Figure 1 show the electrical connections used during the experimentation.

Fin array is freely suspended by using M.S. strip welded at about 10” from top of the enclosure by using the rods. This strip is drilled with number of holes to suspend the fin array at required position by using acrylic suspension strips. Acrylic suspension strips are also drilled with holes at various positions for suspension of fin array.

Two thin G.I. strips with holes drilled at various positions are also used for the suspension of fin array. For measuring the
steady state temperatures, initially base temperature at various heat inputs without fins and then steady state temperatures at various defined points of the fins, a non-contact type infrared-based temperature-measuring instrument is used. The supply of input power in steps of 30 W is adjusted with the help of dimmerstat and system is allowed to attain the steady state temperature at each stage. At steady state, six temperatures at different locations on the fin are recorded using the non-contact type temperature sensor, whereas the ambient temperature was noted by using the mercury thermometer. Five electrical heat inputs: 30, 60, 90, 120 and 150 W were used to give 5 different temperature scenarios. Fin array with required number of fins is now suspended vertically in the enclosure.

<table>
<thead>
<tr>
<th>Number of fins (n)</th>
<th>Fin Spacing (S), mm</th>
<th>Number of Channels (n-1)</th>
<th>S/H ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>22.50</td>
<td>4</td>
<td>0.150</td>
</tr>
<tr>
<td>6</td>
<td>17.60</td>
<td>5</td>
<td>0.117</td>
</tr>
<tr>
<td>7</td>
<td>14.30</td>
<td>6</td>
<td>0.095</td>
</tr>
<tr>
<td>8</td>
<td>12.00</td>
<td>7</td>
<td>0.080</td>
</tr>
<tr>
<td>9</td>
<td>10.50</td>
<td>8</td>
<td>0.070</td>
</tr>
<tr>
<td>10</td>
<td>8.89</td>
<td>9</td>
<td>0.059</td>
</tr>
<tr>
<td>11</td>
<td>7.80</td>
<td>10</td>
<td>0.052</td>
</tr>
<tr>
<td>12</td>
<td>6.90</td>
<td>11</td>
<td>0.046</td>
</tr>
<tr>
<td>13</td>
<td>6.17</td>
<td>12</td>
<td>0.041</td>
</tr>
<tr>
<td>14</td>
<td>5.54</td>
<td>13</td>
<td>0.036</td>
</tr>
<tr>
<td>15</td>
<td>5.00</td>
<td>14</td>
<td>0.033</td>
</tr>
</tbody>
</table>

The same procedure is followed again to record the temperatures at six different points. The experimental work is carried for the different fin assembly having number of fins, N = 5, 6, 7,...15. Figure 2 (a) and (b) shows the details of the fin array and its dimensions. All the instruments used during the experiments are well calibrated.

### 2.2. Heat Transfer analysis:

The total heat transfer from the fin array at steady state is by two modes.

These modes are convection and radiation heat transfer. For evaluating the convection heat transfer we need to calculate heat transfer radiation mode and subtract the same from the total heat input given to the fin array.
2.2.1 Heat transfer by radiation (Qr):

In order to evaluate the heat transferred by radiation mode, one channel of fin array has been considered for the analysis as shown in Figure 3 above.

Following assumptions are considered for radiation heat transfer analysis:

- Fin surfaces and fin base are diffuse-gray bodies at average fin surface temperature measured during the experimentation and openings are blackbodies at ambient temperature, Ta.
- Emissivity, ε is considered constant during the calculation. The value of emissivity for polished Aluminum is taken as 0.045 [12].
- Fin temperature and the base-plate temperature are considered as equal. Since the fin material has high conductivity (~237 W/ (m·K)), the temperature variation within the material is quite low. Therefore, it does not affect the radiation analysis significantly.
- Properties of the surfaces are uniform.
- Air, which fills up the openings, is assumed to be non-absorbing and non-emitting.

The radiation heat transfer rate from the fin array used in the experimentation can be expressed as,

\[ Q_r = 2(n(Q_{11} + Q_{1n}) + (n - 1)Q_{1n} + \epsilon \cdot n \cdot A_r \cdot \sigma \cdot t \cdot (2L + H)(Ta^4 - T_{atm}^4) \]  

\[ A_r = 2\left[ (n - 1)S \cdot H + n(2H \cdot L + 2t \cdot L + t \cdot H) + t_p(W + H) \right] \]  

Fin spacing, S is calculated as,

\[ S = \frac{(100 - N \times t)}{N - 1} \]  

Here, \( Q_{11}, Q_{1n} \) and \( Q_{1n} \) are the net radiation heat transfer from surfaces 1, 3 and 5 respectively. n (number of fins), H, L, t, W, tp, are the geometric parameters of the fin array, \( \sigma = 5.67 \times 10^{-8} \) is the Stefan Boltzmann constant. Since surfaces 1 and 3 are identical, the net radiative heat transfer from surfaces 1 and 3 are equal. Therefore, equation 2.1 becomes,

\[ Q_r = 2Q_{1n} = 2(2nQ_{11} + (n - 1)Q_{1n} + \epsilon \cdot n \cdot A_r \cdot \sigma \cdot t \cdot (2L + H)(Ta^4 - T_{atm}^4)) \]  

\[ Q_{r1} \text{ and } Q_{r5} \text{ are evaluated using the radiation heat transfer network analysis as follows:} \]

\[ Q_{r1} = \frac{E_{b1} - J_1}{\epsilon A_1} = \sum_{j=1}^{6} \frac{J_k - J_j}{(A_k F_{kj})^{-1}} \]

\[ Q_{r5} = \frac{E_{b5} - J_5}{\epsilon A_5} = \sum_{j=1}^{6} \frac{J_5 - J_j}{(A_5 F_{5j})^{-1}} \]  

\[ E_{b1} \text{ and } E_{b5} \text{ are the blackbody radiosities, } J_1 \text{ are the radiosities of the gray surfaces. } F_{kj} \text{ and } F_{5j} \text{ are the view factors while } A_1 \text{ and } A_5 \text{ are the surface areas for surface 1 and 5 respectively.} \]

The black body radiosities \( E_{b1} \) and \( E_{b5} \), and the view factors \( J_2, J_3, J_4 \) of the openings can be calculated as,

\[ E_{b1} = E_{b5} = \sigma T_a^4 \]

\[ J_2 = J_4 = J_6 = \sigma T_{atm}^4 \]  

For the two geometries shown in Figure 4(a) and 4(b) the view factors required are calculated using the following relations [12],
Where, $X = \frac{S}{H}$ and $Y = \frac{H}{L}$; $P = \frac{H}{S}$ and $R = \frac{L}{S}$. For calculating the view factors and radiosities $J_1$ to $J_6$ and $E_b$ to $E_b\delta$, excel sheets are developed using above equations 1 to 10.

### 2.2.2 Convection Heat Transfer:

Based on the above equations 1 to 10 the rate of radiation heat transfer is calculated and it is subtracted from the total heat input, to calculate the convection heat transfer rate as shown in equation (11) below,

$$Q_c = Q_{in} - Q_r$$

The convection heat transfer rate is given as,

$$Q_c = h \cdot A_c (T_a - T_{atm})$$

Here $A_c$ is the heat transfer area for convection which is taken as equal to $A_r$, the radiation heat transfer area. Convection heat transfer coefficient ($h$) is calculated using above equation 12.

Average Nusselt number ($Nu_a$), the Grashof number ($Gr$) and Rayleigh number ($RaL$) are further estimated as below,

$$Nu_a = \frac{h \cdot H}{k_{air}}$$

$$Gr = \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\theta^2}$$

$$Ra = Gr \cdot Pr$$

$$\beta = \frac{1}{T_a}$$

$$\Delta T = T_a - T_{atm}$$

Thermal conductivity, kinematic viscosity and Prandtl number for the air are calculated at mean film temperature ($T_m$) using the relations available in the literature [14],

$$T_m = \frac{T_a + T_{atm}}{2}$$

$T_a$ and $T_{atm}$ are the average fin and the ambient temperatures in Kelvin.

### 3 RESULTS AND DISCUSSION

The variation of average convective heat transfer coefficient ($ha$), average Nusselt number ($Nua$), temperature difference and the Grashof number with $S/H$ for different heat inputs, Qin have been plotted. These graphs are shown in Figures 5 to 10.
Thereafter these factors decreases and becomes almost flat after $S/H = 0.041$ ($S = 6.1 \text{ mm}$ and $n = 13$) with increasing fin spacing for all heat inputs. Values of $N_u a$ and $h_a$ are less at lower fin spacing, this is due to the increased interference to the flow of air through the fin channels at lower fin spacing. At these lower fin spacing, air could not flow freely through the fin channel as compared to larger fins spacing channel and thus $h_a$ and $N_u a$ values are observed to be lower.

Grashof number and average temperature difference $(T_a - T_{atm})$ decreases initially with increasing fin spacing and is the least at $S/H = 0.046$ ($S = 6.9 \text{ mm}$ and $n = 12$), thereafter these factors continuously increases with increasing fin spacing.

The increase in the average temperature difference at higher fin spacing is in agreement with the Newton’s law cooling wherein the average heat transfer coefficient is directly proportional to temperature difference. Further similar observations are also noted for the variation of radiation heat transfer $(Q_r)$ and Rayleigh number with $S/H$ as shown in Figure 9 and 10.

Figure 10 shows the variation of Rayleigh number with the aspect ratio $S/H$ for the present analysis. The equation of the curve between average Nusselt number and aspect ratio is obtained by the least square regression using curve expert software with overall error varying between 11 - 17%; it is given as below,

$$N_u a = 83.2675 \left(\frac{S}{H}\right)^{0.136295}$$

(19)

The increase in average temperature difference and Grashof number with decreasing fin spacing is due to the higher surface temperatures observed on account of reduction in the values of average convective heat transfer coefficient and the Nusselt number at these lower fin spacing.
4 CONCLUSION

This work presented here presents the experimental analysis of natural convection heat transfer from rectangular aluminum fins on a vertical base. From the experimental results it could be seen that fin spacing affects the heat transfer rates, both convection and radiation. The variation of different convection heat transfer parameters with aspect ratio, S/H have been presented here from Figures 5 to 10.

Experimental results shown that the heat transfer in all the cases of fin arrays at different heat input, is increasing with increase in number of fins. This shows that addition of fins on the base enhances the rate of heat transfer.

From the results plotted, following conclusions can be made:

- At all the heat inputs, fin spacing reduces with increasing number of fins. This causes the decrease in the average temperature difference, the Grashof number and Rayleigh number. Their values are least at S/H = 0.046 (S = 6.9 mm, n = 12). Further with increase in number of fins, their values increases till n =15. Similar observations are seen for the radiation heat transfer rate.

- Nusselt numbers and convective heat transfer coefficients for all the cases of heat inputs increases with aspect ratio S/H initially. Their values are maximum at S/H = 0.046 (S = 6.9 mm, n = 12). With further increase in aspect ratio, convective heat transfer coefficient decreases continuously and becomes flat after S/H = 0.041.

- Detailed radiation heat transfer analysis using radiation network analysis for the fin array has been presented in the paper. Contribution of radiation heat transfer observed to be more at lower number of fins. With increasing number of fins, Nusselt number increases and the average temperature difference decreases and hence radiation heat transfer reduces. For the present case of polished aluminum fins with very low emissivity values, at all the cases of fin arrays with different heat inputs, radiation heat transfer have little effect (< 5%) compared with natural convection heat transfer. Natural convection heat transfer dominates the process.

- In the present case of the rectangular fin array, S/H = 0.046 (S = 6.9 mm, number of fins, n = 12) corresponding to the maximum values of Nua and ha is an optimum fin spacing. It is an important parameter for the design of the fin array.

- Based on the experimental results an empirical relation between the average Nusselt number and aspect ratio which is in good agreement with the experimental results (error < 17%).

REFERENCES