Effect the Beam Section Shape for Different Materials on Buckling Load Using Finite Element Method

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Abstract— Buckling loads are critical loads where certain types of structures become unstable. So, this study focuses on finding the effect of the beam section shape on buckling load using finite element software ANSYS R.15. The buckling loads for Rectangular (Rec.), U-channel (Uch.), Solid circle (Scir.) and Hollow circle (Hcir.) cross-section beams are calculated using Eigenvalue and Non-linear analysis for Carbon Steel, Titanium and Aluminum materials. A comparison made between the two methods and the different percentage is not exceeds of 0.27%. The results shows that the Buckling loads are strongly depend on the material properties and the beam shape geometry. Buckling loads are directly proportion with modulus of elasticity, thickness, outer to inner radius ratio and all beam geometry parameters except the beam length.

Index Terms : Buckling load, cross-section, ANSYS R.15, Eigenvalue analysis, Non-linear analysis, Euler.

1 INTRODUCTION

Buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Mathematical analysis of buckling often makes use of an axial load eccentricity that introduces a secondary bending moment, which is not a part of the primary applied forces to which the member is subjected [1]. As an applied load is increased on a member, such as a beam, it will ultimately become large enough to cause the member to become unstable and is said to have buckled. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of the member's load-carrying capacity. If the deformations that follow buckling are not catastrophic the member will continue to carry the load that caused it to buckle. If the buckled member is part of a larger assemblage of components such as a building, any load applied to the structure beyond that which caused the member to buckle will be redistributed within the structure [2]. Buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At a certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium: an undeformed state or a laterally-deformed state[3].

Beam buckling is a curious and unique subject. It is perhaps the only area of structural mechanics in which failure is not related to the strength of the material. A beam buckling analysis consists of determining the maximum load a beam can support before it collapses. But for long beams, the collapse has nothing to do with material yield. It is instead governed by the beam's stiffness, both material and geometric [3&4].

Euler Buckling Theory is the classical theory presented in textbooks and classrooms. It begins simply by noting that the internal bending moment in a loaded and deformed beam is (−Py) where P is the compressive load and y is the beam deflection. So insert (−Py) in form the beam bending equation

E.I.y''=M …….. (1)

Where, E modulus of elasticity, I area moment of inertia and M bending moment, [4, 5&6].

In 1757, mathematician L. Euler derived a formula that gives the maximum axial load that a long, slender, ideal beam can carry without buckling. An ideal beam is one that is perfectly straight, homogeneous, and free from initial stress. The maximum load, sometimes called the critical load, causes the beam to be in a state of unstable equilibrium; that is, the introduction of the slightest lateral force will cause the beam to fail by buckling. The formula derived by Euler for beams with no consideration for lateral forces is given below. However, if lateral forces are taken into consideration the value of critical load remains approximately the same [5&6].

\[ F = \frac{E.I. \pi^2}{(K.L)^2} \] \hspace{1cm} (2)

where, F maximum or critical force (vertical load on beam), L unsupported length of beam, Kbeam effective length factor. K.value depends on the conditions of end support of the beam, as follows

K= 2.0 for one end fixed and the other end free to move laterally
K= 1.0 for both ends pinned (hinged, free to rotate),
K= 0.699 for one end fixed and the other end pinned,
K= 0.5 for both ends fixed,

Therefore, some researchers interested with the field of buckling and its effects on the different sides of critical loads to obtain the optimized opinions to design some structures in
mechanical engineering.

Lee and Kim (2001) [1] studied buckling of an axially thin-walled laminated composite. Avcar (2014) [2] was study elastic buckling of Carbon Steel columns with three different cross sections, i.e. square, rectangle and circle cross sections, and two different boundary conditions, i.e. Fixed-Free (F-F) and Pinned-Pinned (P-P) boundary conditions, under axial compression has been investigated. Elnashai and Elghazouli (1993) [3] studied the Performance of composite Carbon Steel/concrete members under earthquake loading. Part i: analytical model, so, the model is calibrated and compared with experimental data from cyclic and pseudo-dynamic tests conducted by the writers on a new ductile partially-encased composite beam-column.


The aim of the present paper search is to investigate the effect of the section shape for different material beams on buckling load under axial loading using finite element software ANSYS R.15.

2 Materials and Methods

Beams under axial loading for different materials and section shapes are studied in this paper.

2.1 Specimens Material

The material properties of the beam specimens are taken from Kulkarni [11] and reported in Table 1.

<table>
<thead>
<tr>
<th>Type of material</th>
<th>Modulus of elasticity (Mpa)</th>
<th>Poison’s ratio</th>
<th>Density (Kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Steel</td>
<td>2.02 e5</td>
<td>0.292</td>
<td>7820</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.71 e5</td>
<td>0.334</td>
<td>2730</td>
</tr>
<tr>
<td>Titanium</td>
<td>1.135 e5</td>
<td>0.32*</td>
<td>4480</td>
</tr>
</tbody>
</table>

2.2 Numerical Solution

Buckling loads are calculated numerically using finite element software ANSYS R14.5 with PLANE188 element as a discretization element. Because of symmetry, half model as shown in Figure (1) is used in all cases. Rectangular (Rec.), U-channel (Uch.), Solid circle (Scir.) and Hollow circle (Hcir.) ANSYS models are shown in Figure 2 with the mesh, elements and boundary conditions while Figure 3 illustrates the dimensions for these cross-sections.
Generally, there are two procedures for buckling analysis as follow:

a) Eigenvalue buckling analysis

In this analysis, the structural eigenvalues for the given system loading are computed. It is known as classical Euler buckling analysis.

b) Non-Linear buckling analysis

This analysis based on gradually increases the applied load until a load level is found whereby the structure becomes unstable (a very small increase in the load will cause very large deflections).

2.2.1 BEAM188 Element Description

BEAM188 is based on Timoshenko beam theory which includes shear- deformation effects. The element is a linear, quadratic, or cubic two-node beam element in 3D. BEAM188 has six or seven degrees of freedom at each node. These include translations in the x, y, and z directions and rotations about the x, y, and z directions. This element is well-suited for linear, large rotation, and/or large strain Non-Linear applications [12]. The geometry, node locations, and the coordinate system for this element are shown in Figure 4.

![Figure 4 BEAM188 element type with the geometry, node locations and the coordinate system, Ansys help [12].](image)

2.3 Validation Test

Timoshenko [13] calculated the critical buckling load for a slender bar with hinged ends subjected to axial load (P). The information data of this bar as below:

Length (L) = 200 in, area (A) = 0.25 in², width (b) = 0.5 in, thickness (t) = 0.5 in, axial load (P) = 1lb, modulus of elasticity (E) = 30e6 Psi and poisons ratio (υ) = 0.3.

Table 3 reported the theoretical and numerical results.
The comparison between the two methods shows that the validity of the ANSYS R15 is very good due to the different percentage is not exceeds of 0.005 %.

**TABLE 3** A comparison between theoretical and numerical solution.

<table>
<thead>
<tr>
<th>Timoshenko [13]</th>
<th>ANSYS R.15</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_c$</td>
<td>38.553</td>
<td>38.551</td>
</tr>
</tbody>
</table>

### 3. Results and Discussions

Buckling load values are numerically calculated using ANSYS R15 for three different materials, four cross-section types and the two analysis approaches (Eigenvalue and Non-Linear analysis) with different cases reported in Table 1. To compute the required results using two mentioned analysis in a faster and accuracy way, programs are written with APDL (Ansys Parameter Design Language). Figures 5 & 6 show APDL flow chart for Eigenvalue and Non-Linear analysis, respectively.

#### 3.1 Eigenvalue analysis

**3.1.1 Rec. Cross - Section**

Figures 7&8 illustrate the variations of critical buckling loads ($P_c$) with $L$ and $t$ for Rec. type parameters, respectively. Three different materials (Carbon Steel, Titanium and Aluminum) (case study 1 & 2) are used. These Figures explain that $P_c$ values are reversed proportion with the beam length and direct proportion with parameter. In all mentioned Figures, it is found that the $P_c$ values for Carbon Steel are more than that of Titanium and Aluminum.

**Figure 7** Variation of Rec. beam length with critical load for different materials.

**Figure 8** Variation of Rec. beam thickness with critical load for different materials.

#### 3.1.2 Scir. Cross - Section

The variations of $P_c$ with $L$ and $R$ for Scir. type are represented in Figures 9 & 10, respectively (case study 3 & 4). It can be seen that, $P_c$ values are increase with decrease $L$ and increase $R$. The $P_c$ values for Aluminum are less than that of Titanium and Carbon Steel.

**Figure 9** Variation of Scir. beam length with critical load for different materials.

**Figure 10** Variation of Scir. Beam radius with critical load for different materials.

#### 3.1.3 Hcir. Cross - Section

Figure 11&12 explain the variations of $P_c$ with $L$ and $Ro/Ri$ ratio, respectively for Hcir. type (case study 5 & 6). It’s clear that increase $L$ and decrease $Ro/Ri$ ratio lead to decrease the $P_c$ values especially when we used Aluminum material.
3.1.4 Uch. Cross - Section

The variation of numerically computed \( P_c \) for the three of Uch. Different material specimens in 7, 8& 9 cases are shown in Figures13-15, respectively. It can be seen that, the increment in \( P_c \) values are become when decrease \( L \) and increase \( w_3 \) and \( t_1 \). As the same of all above mentioned Figures, \( P_c \) values for Carbon Steel specimens are greater than that of Titanium and Aluminum specimens.

3.2 Non-Linear analysis

Using Non-Linear analysis solution, Figures 16a, 17a, and 18a represent the variation of the Rec. beam deflection with the applied load gradually increases with time until a critical buckling load is found (when occurs a very large deflection). Case study number 10 for Carbon Steel, Titanium and Aluminum materials are shown in these Figures, respectively. It can be seen (at the same conditions) that the \( P_c \) values calculated using this analysis approximately the same with the values calculated using eigenvalue analysis. Table 4 shows \( P_c \) calculated using these two approaches with differences is not exceeding 0.27%.
TABLE 4 Buckling load values calculated using Eigenvalue and Non-Linear analysis.

<table>
<thead>
<tr>
<th>Beam Section type</th>
<th>Parameters (mm)</th>
<th>Solution type</th>
<th>Critical buckling load (Pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t=10</td>
<td></td>
<td>Carbon Steel</td>
</tr>
<tr>
<td>Rec.</td>
<td>L=100</td>
<td>Eigenvalue</td>
<td>39302</td>
</tr>
<tr>
<td></td>
<td>b=10</td>
<td>Nonlinear</td>
<td>39200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Difference %</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Furthermore, Figures 16b, 17b and 18b graphically illustrated deformed and un-deformed shape for the Rec. beam specimens represent the case study number 10.

Figure 16 a) Variation of Rec. Aluminum deflection with the applied load gradually increases with time b) Graphically illustrated deformed and un-deformed shape due to buckling load.

Figure 17 a) Variation of Rec. Steel deflection with the applied load gradually increases with time. c) Graphically illustrated deformed and un-deformed shape due to buckling load.
4. Conclusions

In light of the results of the current study can be reached the following conclusions:

1) Buckling loads are strongly depends on the material properties and the shape geometry of the beam.
2) Buckling loads are directly proportion with modulus of elasticity, thickness, outer to inner radius ratio and all beam geometry parameters except the beam length.
3) $P_c$ values are calculated from the Eigenvalue analysis and Non-Linear analysis for different cases are approximately same and the difference percentage is not exceeds of 0.27%.
4) Numerical solution is suitable to calculate $P_c$ due to it is used for all cases and with regular and irregular shapes.

References

12] ANSYS help.
Appendex:

Figure (5) APDL Flow chart for Eigenvalue buckling analysis

Figure (6) APDL Flow chart for Non-Linear buckling analysis