Economic Load Dispatch of Power System Using Grey Wolf Optimization with Constriction Factor

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Abstract: In this paper, we propose a new meta-heuristic, nature inspired technique known as Grey Wolf Optimization to solve Economic Load Dispatch (ELD) problems of thermal power units considering transmission losses, and constraints such as ramp rate limits and prohibited operating zones. Grey Wolf Optimization Algorithm (GWOA) is a relatively new optimization technique. Mathematical models of this algorithm demonstrate the efficiency, quality of solution and convergence speed of the method and successful application of the algorithm on economic load dispatch problems. Simulation results found that the proposed approach outperforms several other existing optimization techniques in terms quality of solution obtained and computational efficiency. Results also be confirmed the robustness of the proposed methodology.

Keywords: Economic load dispatch, Grey Wolf optimization technique, prohibited operating zone, quadratic cost function, ramp rate limits,

1. Introduction

Economic load dispatch (ELD) is applied in electric power utilities is to provide high-quality, reliable power supply to the consumers at the lowest possible tariff. It can be defined in normal condition the operation of generation facilities is to produce electrical power at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities. It is an important role in electrical power system operation for allocating generation among the committed units such that the imposed constraints and the energy requirement are satisfied. The characteristics of fuel cost for modern generating units are highly nonlinear with demand for solution techniques having no restrictions on to the shape of the fuel cost curves. For science and engineering, many optimization techniques are developed for used in ELD problem to accomplish to the main goal. But the calculus-based methods [1] are not fulfillment to solving ELD problems, as these techniques are required smooth, differentiable objective function. Another method which is Linear programming method [2] is speedy and reliable but it has some drawback related with the piecewise linear cost approximation. In power system small improvements in the unit output scheduling can give significant cost savings. So the dynamic programming approach was proposed by Wood and Wollenberg [3] to solve ELD problems but this technique does not impose any restriction on the nature of the cost curves, but suffers from the “curse of dimensionality” or local optimality and larger simulation time. In order to overcome this problem in current years, several attempts have been made to solve ELD with intelligent and modern technique which is meta-heuristic algorithm is helpful for solution of complex ELD problems they are Genetic algorithm [4], particle swarm optimization [5], Simulated Annealing (SA)[6], Artificial Neural Networks [7], Differential evolution [8], Tabu search [9], Evolutionary Programming (EP) [10], Ant colony optimization [11], Artificial immune system (AIS) [12], Bacterial Foraging Algorithm (BFA) [13], Biogeography-based Optimization (BBO) [14] etc. This mentioned method may confirm to be very effective for solving nonlinear ELD problems without any restriction on the shape of the cost curves. They often provide a fast, reasonable nearly global optimal solution but these methods do not always assurance global best solutions, they often achieve a fast and near global optimal solution. In recent years, different hybridization and modification of GA, EP, PSO, DE, BBO like improved GA with multiplier updating (IGA-MU) [15] directional search genetic algorithm (DSGA) [16], hybrid genetic algorithm (GA)-pattern search (PS)-sequential quadratic programming (SQP) (GA-PS-SQP) [17], improved fast evolutionary programming (IFEP) [18], new PSO with local random search (NPSO_LRS) [19], adaptive PSO (APSO) [20], self-organizing hierarchical PSO (SOH-PSO) [21], improved coordinated aggregation based PSO (ICA-PSO) [22], improved PSO [23], combined particle swarm optimization with real-valued mutation (CBPSO-RVM)[24], DE with generator of chaos sequences and
2. MATHEMATICAL MODELING OF THE ELD PROBLEM

The traditional formulation of the ELD problem is to minimize the fuel cost of generations for both convex and non-convex nonlinear constrained optimization problem. In this section, ELD problems have been formulated and solved by GWOA. These are presented below:

2.1. 2.1.2  ELD with quadratic cost function, ramp rate limit, prohibited operating zone and transmission loss

The overall objective function $F_T$ of ELD problem may be written as

$$F_T = \min \sum_{i=1}^{N} F_i(P_i) = \min \left( \sum_{i=1}^{N} \left( a_i + b_i P_i + c_i P_i^2 \right) \right)$$  \hspace{1cm} (1)

Where, $F_i(P_i)$ is fuel cost function of the $i^{th}$ generator, and is usually expressed as a quadratic polynomial function; $N$ is the total number of committed generators; $a_i$, $b_i$, and $c_i$ are the cost coefficients of the $i^{th}$ generator; $P_i$ is the generated power of the $i^{th}$ generator. The ELD problem consists in minimizing $F_T$ subject to following constraints:

2.1.1. Real Power balance constraint:

$$\sum_{i=1}^{N} P_i - (P_D + P_L) = 0$$  \hspace{1cm} (2)

Where, $P_D$ is the total load demand by consumer; $P_L$ is the total transmission loss in power system; Calculation of $P_L$ using the $B$- coefficients matrix is expressed as:

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} P_j + \sum_{r=1}^{l} B_{ro} P_r + B_{bo}$$  \hspace{1cm} (3)

2.1.2 Generation capacity constraint:

There is a limit on the amount of active power generation. For normal condition, real power output of each generator is restricted by lower and upper bounds as follows:

$$P_i^\text{min} \leq P_i \leq P_i^\text{max}$$  \hspace{1cm} (4)

Where $P_i^\text{min}$ and $P_i^\text{max}$ are the minimum and maximum power generated by generator $i^{th}$ unit, respectively.

2.1.3 Ramp Rate Limit Constraint:

The power $P_i$ generated by the $i^{th}$ generator in certain interval could not exceed that of previous interval $P_{i0}$ by more than a certain amount $U_{Ri}$ is the up-ramp rate limit and neither may it be less than that of previous interval by more than some amount $D_{Ri}$ the down-ramp rate limit of the generator. These give rise to the following constraints.

As generation increases,

$$P_i - P_{i0} \leq U_{Ri} \hspace{1cm} (5)$$

As generation decreases,

$$P_{i0} - P_i \leq D_{Ri} \hspace{1cm} (6)$$

Modified generation limits after considering ramp rate limits are given below

$$\max(P_i^\text{min}, P_{i0} - D_{Ri}) \leq P_i \leq \min(P_i^\text{max}, P_{i0} + U_{Ri}) \hspace{1cm} (7)$$

2.1.4 Prohibited Operating Zone:

The prohibited operating zones are the range of output power of any generator where the operation causes undue vibration of the turbine shaft. Generally such vibration occurs at the point of opening or closing of the steam valve which might cause damage to the shaft and bearings. It is difficult to determine the exact prohibited zone by actual testing or from operating records. Normally operation is avoided in such regions.

Hence mathematically the feasible operating zones of unit can be described as follows:

$$P_{i,j,0}^m \leq P_i \leq P_{i,j,0}^p$$  \hspace{1cm} (8)

Where $j$ represents the number of prohibited operating zones of unit $i$. $P_{i,j,0}^m$ is the upper and $P_{i,j,0}^p$ is the lower limit of $j^{th}$ prohibited operating zone of $i^{th}$ unit. $n_j$ is the total number of prohibited operating zone of the $i^{th}$ unit.

2.2 Calculation for slack generator:

Let $N$ committed generating units are delivering their power output maintaining the power balance constraint (2) and the respective capacity constraints of (4) and/or (7), (8). It is assumed that the power loadings of first ($N$-1) generators are known, the power level of $N^{th}$ generator (called Slack Generator) is given by

$$P_s = P_p + P_L - \sum_{i=1}^{N-1} P_i$$  \hspace{1cm} (9)

The transmission loss $P_L$ which is a function of all the generator outputs including the slack generator, is given by

$$P_L = \sum_{i=1}^{N-1} \sum_{j=1}^{N} P_i P_j + 2P_s \sum_{i=1}^{N} B_{ii} P_i + B_{bo} P_s^2 + \sum_{i=1}^{N} B_{ro} P_i + B_{bo} P_s + B_{bo}$$
The complete population structure is provided in the mathematical models as follows:

\[ \verb|X| = B_{\infty} \]

\[ Y = \left( 2 \sum_{i=1}^{N-1} B_{i} P_{i} + B_{\infty} - 1 \right) \]

\[ z = \left( P_{0} + \sum_{i=1}^{N-1} B_{i} P_{i} + \sum_{i=1}^{N-1} B_{i} P_{i} - \sum_{i=1}^{N-1} P_{i} + B_{\infty} \right) \]

The positive roots of the equation are obtained as

\[ P_{x} = \frac{-Y \pm \sqrt{Y^{2} - 4XZ}}{2X}, \text{ Where } Y^{2} - 4XZ \geq 0 \]

In order to satisfy the equality constraint (9), the positive root of quadratic equation violates operation limit constraint of (4) at the initialization process of the algorithm, then Generation value of first \( (N-1) \) generators is reinitialized until the positive root satisfies the operation limit and other constraints (if any).

3. GREY WOLF OPTIMIZATION ALGORITHM (GWOA)

This section presents an interesting new optimization algorithm called grey wolf optimization algorithm (GWOA) which was proposed by Mirjalili and Mirjalili [30]. This technique based on behaviour of grey wolf in searching and hunting of their quarry. The leaders of the group, a male and a female are called alpha (\( \alpha \)). The next levels of grey wolves, which are subordinate wolves mainly provide help to the leaders for decision making or in other activities, are called beta (\( \beta \)). The third level of grey wolves dominate the omega which are known as delta (\( \delta \)). The last rank of the grey wolves is called omega (\( \omega \)), which surrenders to all the other governing wolves. The proposed technique (GWOA) is provided in the mathematical models as follows:

A. Social hierarchy: If we draw a mathematical model of this algorithm, we consider social hierarchy of the grey wolves; here alpha (\( \alpha \)) has best fittest solution. The second best solution beta (\( \beta \)) and third best solution delta (\( \delta \)) belong to the grey wolf family. The omega (\( \omega \)) is the last candidate solution. Therefore alpha (\( \alpha \)), beta (\( \beta \)) and delta (\( \delta \)) helped wolves to the grey wolf family.

Encircling Prey: The wolf during hunting period tends to encircle their prey. The following equations express the behavior [30]:

\[ \verb|D| = \left[ \vec{C}_{i} \vec{X}_{p}(t) - \vec{X}(t) \right] \]

\[ \vec{X}(t+1) = \vec{X}_{p}(t) - \vec{A} \cdot \vec{D} \]

Where \( \vec{A} \) and \( \vec{C} \) are the coefficient vectors, \( \vec{X}_{p} \) is the prey’s position vector, \( \vec{X} \) denotes the grey wolf’s position vector and ‘\( t \)’ is the current iteration.

The mathematical calculation of vectors \( \vec{A} \) and \( \vec{C} \) is done as follows [30]:

\[ \vec{A} = 2 \vec{a} \vec{r}_{1} \vec{a} \]

\[ \vec{C} = 2 \vec{r}_{2} \]

Where values of ‘\( \vec{a} \)’ are linearly reduced from 2 to 0 during the course of iterations and \( r_{1}, r_{2} \) are arbitrary vectors in the gap [0, 1].

B. Hunting: Alpha, beta and delta guided the entire group with their better knowledge about the potential location of prey. The other agents update their positions according to the best search agent’s position. The update of their agents can be expressed as follows [30]

\[ \vec{X}_{i}(t+1) = \vec{X}_{i}(t) - \vec{A}_{i} \vec{D}_{i} \]

C. Search for prey and attacking prey This is the final position of this algorithm. Under this circumstance they will be in a random position within a circle which is defined by the positions of the alpha, beta, and delta in the search space. In fact alpha, beta, and delta estimate the victim’s position and other wolves update their positions randomly around the victim.

3.2. METHODOLOGY

Since the appraisal variables for ELD problem are real power output for each generator but here they are used to represent the wolf’s population structure where \( P \) is total population for each group and \( n \) is number of groups. The whole population is \( n \cdot P \). Each individual population structure represents the real power output for each generator and also fulfillment (4). For initialization purpose we can choose the number of generating units \( N \) and total number of population structure \( PopSize \).

By the following the complete population structure is represented as

\[ P = P = \left[ P_{1}, P_{2}, P_{3}, ..., P_{PopSize} \right] \]

Where, \( i = 1, 2, ..., \text{PopSize} \).

Here the population set is \( P \) which is each individual element of the population structure of matrix. This population set \( P \) is initialized randomly within the
fulfillment real power balance constant and generator capacity constant. This initialization is based on (4), (7) and (8) when consider ramp rate limit, prohibited operating zone. For this initialization process, choose no. of generating units N and also specify maximum and minimum capacity of each generator, power demand, B-coefficients matrix for calculation of transmission loss.

In ELD problem here fitness value is fuel cost of generation for all the generators which calculated using (1) for the system having quadratic fuel cost characteristic. This Eq. (1) applies to determine performance evolution of ELD problem until the optimum cost is achieved. The technique will continue until the maximum no. of iteration is met and the optimum result is obtained.

4. RESULTS AND DISCUSSIONS

Proposed GWOA has been applied for solving ELD problem in a test system and its performance has been compared to several other optimization technique like GA [5], CTPSO [23] and PSO [5, 23] for verifying its feasibility. The essential codes has been written in MATLAB-7 language and executed on a 2.0 GHz Intel Pentium (R) Dual Core personal computer with 1-GB RAM.

4.1. Description of the Test System

Test System 1: In this paper, 15 generating units with ramp rate limit and prohibited operating zones constraints has been considered and transmission loss has been also included in this problem. Consumer power demands are 2630 MW and the characteristics of the fifteen thermal units are given in Tables 1 and 2. The loss coefficients are given in [5]. This paper results obtained from proposed GWOA, PSO [5] and different versions of PSO [23] and other method are presented in this paper. Their best results are shown in Table 3. Minimum, average and maximum fuel costs obtained by GWOA and different versions of PSO [23], over 50 trials are presented in Table 4.

### Table No. 01

<table>
<thead>
<tr>
<th>Generating Unit Data</th>
<th>Consumer Power Demands are 2630 MW</th>
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<tr>
<td>Unit</td>
<td>$P_i^{\text{fuel}}$ (MW)</td>
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<tr>
<td>1</td>
<td>150</td>
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### Table No. 02

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<th>Prohibited Zones of Generating Units</th>
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<tr>
<td>Unit</td>
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### Table No. 03

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<th>Best Power Output for 15-Generators System (P$_0$=2630MW)</th>
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### Table No. 04

| Comparison Between Different Methods Taken After 50 Trials (15-Generators System) |
|-------------------------------|-----------------|-----------------|-----------------|------------------|----------------|
| Methods | Generation Cost ($/hr.) | Time/Iteration (Sec) | No. of hits to minimum solution |
| MAX | Min. | Average |
| CTPSO [23] | 32704.4514 | 22.5 | NA* |
| CSPSO [23] | 32704.4514 | 16.1 | NA |
| OPCPSO [23] | 32704.4514 | 85.1 | NA |
| OPCPSO [23] | 32704.4514 | 16.2 | NA |

* NA: Data

4.2. Comparative study:

1) Solution Quality: Table 3 represents comparable studies of GWOA algorithm with other optimization methods and it is found that GWOA algorithm reaches the best result of fuel costs in power system. Table 4 gives comparative...
studies for minimum, maximum and average values for different technique with other method. From these results it is observed that the performance of GWOA is better, in terms of quality of solutions obtained.

2) Computational Efficiency: Table 3, shows 15 units while comparing it with other technique and the cost is found to be 32701.870839 $/hr., The cost is compared to the results obtained by many previously developed techniques and it is found to be lesser. The time taken by GWOA to achieve minimum fuel costs is quite less compared to that obtained by many other techniques. These are shown in Table 4. These outputs prove significantly better computational efficiency of GWOA.

3) Robustness: Performance of any heuristic algorithms cannot be judged by the results of a single run. Normally their performance is judged after running the programs of those algorithms for certain number of trials. A large numbers of trials with different initializations of population size should be made to obtain a useful conclusion about the performance of the technique. An algorithm is said to be robust, if it gives consistent result during all trials. This performance is much superior compared to many other algorithms, presented in the different literatures in table 4. Therefore, the above results establish the enhanced capability of GWOA to achieve superior quality solutions, in a computational efficient and robust way.

CONCLUSION

In this paper, we present a newly developed GWOA method, which is very flexible, quite efficient, rarely gets trapped in global minima. It does not require computationally expensive derivatives, and is quite easy. It has been successfully implemented to solve different constraints, ELD problems. This simulation results proved that the performance of GWOA is better as compared to that of several previously developed optimization techniques. Therefore GWOA process can be considered as one of the powerful tool to solve ELD problem. In future, GWOA can also be tried for solution of complex hydrothermal scheduling, optimal power flow problems and dynamic ELD, in the search for good characteristics results.

REFERENCES


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