EFFECT OF CHEMICAL REACTION AND THERMO–DIFFUSION ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A JEFFREY FLUID IN A CONCENTRIC CYLINDRICAL ANNULUS WITH NON–LINEAR DENSITY TEMPERATURE RELATION

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Abstract: The effect of non linear density temperature variation on mixed convective heat and mass transfer flow of a Jeffrey's fluid through a porous medium in a circular annulus region between the concentric porous cylinder \( r = a \) and \( r = b \) in the presence of heat sources have been investigated. The equations governing the flow heat and mass transfer have been solved by employing Gauss-Seidel iteration procedure. The effect of various governing parameters on the flow characteristics have been discussed graphically. The rate of heat and mass transfer are evaluated numerically for different variations.

Index Terms: Chemical reaction, Thermo-diffusion, Heat & Mass Transfer, Jeffrey fluid, non-linear density temperature.

1. Introduction

A large class of real fluids does not exhibit the linear relationship between stress and the rate of strain. Because of the non-linear dependence, the analysis of the behavior of the fluid motion of the non-Newtonian fluids tends to be much more complicated and subtle in comparison with that of the Newtonian fluids. In the literature, the mechanics of non-linear fluids presents special challenges to engineers, physicists and mathematicians since the non-linearity can manifest itself in a variety of ways. One of the simplest way in which the viscoelastic fluids have been classified is the methodology given by Rivlin and Ericksen [209] and Truesdell and Noll [267] who presents constitutive relations for the stress tensor as a function of the symmetric part of the velocity gradient and its higher (total) derivatives. In recent years there have been several studies [183,184,187,189,102] on flows of non-Newtonian fluids, not only because of their technological significance but also in the interesting mathematical features presented by the equations governing the flow. On the other hand, it is well known that the rheological properties of many fluids are not well modeled by the Navier–Stokes equations [107]. It is not possible to obtain a single equation exhibiting all properties of all non-Newtonian fluids from available literature. That is why several models of non-Newtonian fluids are proposed. Jeffery-six constant fluid is one of these models.

Chen and Yuh [41] have investigated the heat and mass transfer characteristics of natural convection flow along a vertical cylinder under the combined...
buoyancy effects of thermal and species diffusion. Sree evani [251] has investigated the convective heat and mass transfer through a porous medium in a cylindrical annulus under radial magnetic field with Soret effect. Prasad [169] has analyzed the convective heat and mass transfer through a porous cylindrical annulus in the presence of heat generating source under radial magnetic field. Sreenivas Reddy [250] has discussed the Soret effect on mixed convective heat and mass transfer through a porous cylindrical annulus. Ramakrishna Reddy [192] has analyzed the thermo-diffusion effect on mixed convection heat and mass transfer through a porous medium confined in a cylindrical annulus.

In all the above investigations, the variation of density is taken in the linear form

\[ \Delta \rho = -\rho \beta (AT) \]  \hspace{1cm} (A)

Where \( \beta \) is the co-efficient of thermal expansion and is 2.07 x 10\(^4\) (OC\(^{-1}\)). This is valid for temperature variation near 20\(^0\)c. But this analysis is not applicable to the study of the flow of water at 40\(^0\)c, the density of water is maximum at atmosphere pressure and the above relations \( (A) \) does not hold good. The modified form of \( (A) \) is applicable to water at 40\(^0\)c is given by

\[ \Delta \rho = -\rho \gamma (\Delta T)^2 \]  \hspace{1cm} (B)

where \( \gamma = 8 \times 10^{-6} (OC)^{-2} \). Taking this fact into account, Goren [87] showed in this case, similarity solutions for free convection flow of water at 40\(^0\)c past a semi-infinite vertical plate. Taking non-linear density temperature variation Sarojamma [219] has analyzed the hydromagnetic free convection flow in a cylindrical geometry.

In this chapter we discuss the effect of non linear density temperature variation on mixed convective heat and mass transfer flow of a Jeffrey’s fluid through a porous medium in a circular annulus in the presence of heat sources. The equations governing the flow heat and mass transfer have been solved by employing Gauss-Seidel iteration procedure. The effect of various governing parameters on the flow characteristics have been discussed graphically. The rate of heat and mass transfer are evaluated numerically for different variations.

2. Formulation of the Problem

We analyze the fully developed steady laminar free convective flow of a viscous, electrically conducting Jeffrey fluid through a porous medium confined in an annular region between two vertical co-axial porous circular pipes in the presence of heat generating sources. We choose the cylindrical polar coordinates system \( O (r, \theta, z) \) with the inner and outer cylinders at \( r = a \) and \( r = b \) respectively. The fluid is subjected to the influence of a radial magnetic field (\( H_0 / r \)). Pipes being sufficiently long, all the physical quantities are independent of the axial coordinate \( z \). The fluid is chosen to be of small conductivity so that the Magnetic Reynolds number is much smaller than unity and hence the induced magnetic field is negligible compared to the applied radial field. Also the motion being rotationally symmetric the azimuthal velocity \( V \) is zero. The equation of motion governing the MHD flow through porous medium are

\[ u_r + u / r = 0 \]  \hspace{1cm} (1)

\[ \rho e u u_r = -p_r + (\mu / (1+\lambda)) (u r r + u / r^2) \]

\[ \rho e w w_r = -p_w + (\mu / (1+\lambda)) (w r r + w / r) - \rho g - (\sigma \mu c H_0^2 a^2 / r^2 (1+\lambda)) w \]

\[ 0 = k_f (T_r + T_v / r + Q) \]  \hspace{1cm} (4)

\[ 0 = D(C_r + C_v / r) - k_f C = k_f (T_r + T_v / r) \]  \hspace{1cm} (5)

\[ \rho - \rho_{rc} = -\beta (T - T_e) - \beta (C - C_e) \]  \hspace{1cm} (6)

where \( (u, w) \) are the velocity components along \( O (r, z) \) directions respectively, \( \rho \) is the density of the fluid, \( p \) is the pressure, \( T \),

\[ \rho \]
C are the temperature and concentration, \( \mu \) is the coefficient of viscosity, \( C_p \) is the specific heat at constant pressure, \( k \) is the porous permeability, \( \sigma \) is the electrically conductivity, \( \mu_e \) is the magnetic permeability and \( p_r , T_e , C_e \) are density, temperature and concentration in the equilibrium state, \( k_i \) is the coefficient of thermal conductivity, \( D \) is the molecular diffusivity, \( \beta \) is the volumetric expansion with mass fraction, \( \kappa_i \) is chemical reaction coefficient, \( \kappa_{11} \) is cross diffusivity and \( Q \) is the strength of the heat generating source (suffices \( r \) and \( z \) indicates differentiation with respect to the variables).

The boundary conditions are

\[ w(a) = w(b) = 0 \quad (7a) \]
\[ T(a) = T_e \text{ and } T(b) = T_o \quad (7b) \]
\[ C(a) = C_i \text{ and } C(b) = C_o \quad (7c) \]

The equation of continuity gives

\[ \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r u) = 0 \quad (8) \]

In the hydrostatic state equation (3) gives

\[ -p_r \frac{g}{\beta} - p_e \frac{g}{\beta} + \frac{\partial}{\partial r} (\rho u \frac{g}{\beta}) = 0 \quad (9) \]

where \( p_r \) and \( p_e \) are the density and pressure in the static case and hence

\[ -p_g + p_e = \frac{1}{\beta} (\rho - \rho_e) g \frac{\partial}{\partial r} \rho \quad (10) \]

Where \( p_e \) is the dynamic pressure

Substituting (10) in (2) we find

\[ \frac{\partial P_g}{\partial r} = f (r) \quad (11) \]

Using the relations (8) – (11) in (1) – (4) the equations governing free convective heat transfer flow under no pressure gradient are

\[ w_r + (1 - \lambda) \frac{w}{r} \frac{\partial}{\partial r} (w^2) - (\frac{\partial}{\partial r} \frac{w}{r} \lambda) \frac{\partial}{\partial r} \frac{w}{r} + \frac{1}{\lambda} \frac{\partial}{\partial r} (\frac{w}{r}^2) \frac{\partial}{\partial r} \frac{w}{r} = 0 \quad (12) \]

\[ C_r = \frac{\partial C}{\partial r} \frac{\partial}{\partial r} C = \frac{C_0 - C_e}{C_0 - C_i} \quad (13) \]

Introducing the non-dimensional variables

\[ (r^*, w^*, \theta^*) \text{ as } \]
\[ r^* = r / a , w^* = w (a / v) , \theta^* = \frac{\theta - T_e}{T_i - T_e} , C^* = \frac{C - C_e}{C_i - C_e} \quad (15) \]

the equations (13) and (14) reduce to

\[ w_{r^*} + (1 - \lambda) \frac{w^*}{r^*} \frac{\partial}{\partial r^*} (w^{*2}) - (\frac{\partial}{\partial r^*} \frac{w^*}{r^*} \lambda) \frac{\partial}{\partial r^*} \frac{w^*}{r^*} + \frac{1}{\lambda} \frac{\partial}{\partial r^*} (\frac{w^*}{r^*^2}) \frac{\partial}{\partial r^*} \frac{w^*}{r^*} = 0 \quad (16) \]

\[ \theta_{r^*} + (1 - \lambda) \frac{\theta^*}{r^*} = 0 \quad (17) \]

\[ C_{r^*} = \frac{\partial C^*}{\partial r^*} \frac{\partial}{\partial r^*} C = \frac{C_0 - C_e}{C_i - C_e} \quad (18) \]

Where

\[ M = (\sigma \rho C_p H_o^2 a^2 / \rho v^2)^{1/2} \] (Hartmann number)
\[ G = (\beta g a^2 (T_i - T_e)^2 / \nu^2) \] (Grashoff number)
\[ \alpha = \alpha C / \nu \] (Suction parameter)
\[ D_{r^*} = (a^2 / k) \] (Darcy parameter)
\[ \nu = \nu C / \beta \] (Prandtl number)
\[ a = \frac{Q r^2}{\Delta t k_f} \] (Heat Source parameter)
\[ S_{C_r} = \frac{v}{D_1} \] (Schmidt number)
\[ N = \frac{\beta^* \lambda}{\rho \nu} \] (Buoyancy ratio)
\[ k_f = \frac{k_1 a^2}{D_1} \] (Chemical reaction parameter)
\[ \gamma_1 = \frac{\beta^* \Delta C}{\rho \Delta T} \] (Density ratio)
\[ S_0 = \frac{k_1 \Delta T}{k_f C_p \Delta C} \] (Soret parameter)
\[ s = \frac{b}{a} \] (Width of annular region)

The corresponding boundary conditions are

\[ w^* = 0 , \theta^* = 0 , C = 0 \quad \text{on } r = 1 \]
\[ w^* = 0 , \theta^* = 0 , C = 0 \quad \text{on } r = s \quad (19) \]

The differential equations involving \( \theta_{r^*}, \theta^* \), \( w_{r^*} \) and \( w^* \) are reduced to the following difference equations

\[ 1 - \frac{h(1 - \lambda) \Delta \theta_{r^*}}{2 \eta} \theta_{r^*+1} - (2 \theta_{r^*} + (1 + \frac{h(1 - \lambda) \Delta \theta_{r^*}}{2 \eta}) \theta_{r^*+1} + 2 \lambda^2 C_{r^*} = 0 \quad (20) \]

\[ 1 - \frac{h(1 - \lambda) \Delta C}{2 \eta} C_{r^*+1} - 2 C_{r^*} + (1 + \frac{h(1 - \lambda) \Delta C}{2 \eta}) C_{r^*+1} + (\frac{N S_0}{N}(N_{r^*+1} - 2 \theta_{r^*} - \theta_{r^*+1}) + 1) \frac{h(1 - \lambda) \Delta \theta}{2 \eta} \theta_{r^*+1} - \frac{h(1 - \lambda)}{2 \eta} \theta_{r^*+1} - 2 h^2 (D_{r^*}^{-1} + (\frac{M^2}{r^*^2} (1 + \lambda))) w_j + (1 + \frac{h(1 - \lambda)}{2 \eta} \theta_{r^*+1} = -G (1 + \lambda) \lambda^2 (\theta_{r^*} + N C_j) \quad (21) \]

Where \( h \) is the step length taken to be 0.05 together with the following conditions
All the above difference equations are solved using Gauss-Seidel iterative method to the fourth decimal accuracy.

3. Shear Stress, Nusselt Number and Sherwood Number

The shear stress on the pipe is given by

$$\tau' = \mu \left( \frac{\partial w}{\partial r} \right)_{r=a,b}$$

which in the non-dimensional form reduces to

$$\tau = \tau'(\mu^2 / a^2) = \left( \frac{w_r}{r} \right)_{r=1,a}$$

The heat transfer through the pipe to the flow per unit area of the pipe surface is given by

$$q = k_0 \left( \frac{\partial T}{\partial r} \right)_{r=a}$$

which in the non-dimensional form is

$$Nu = \frac{q a}{k_0 (T_1 - T_e)} = \left( \frac{\partial T}{\partial r} \right)_{r=1}$$

The mass transfer through the pipe to the flow per unit area of the pipe surface in the non-dimensional form is

$$Sh = \left( \frac{q \alpha}{D_0 (C_1 - C_e)} \right) = \left( \frac{\partial C}{\partial r} \right)_{r=1}$$

Particular Case

In the absence of thermo-diffusion ($S_0 = 0$) the results are in good agreement with that of Suresh Babu et al. [259(a)].

4. Results and Discussion

Figures (1 - 4) represents the axial velocity $w$ for different values of $S_0$, $k_r$, $\lambda_1$, $\gamma$. It is found that the axial flow is in the vertically downward direction and hence $w>0$ represents a reversal flow.

Fig (1) represents the effect of thermo-diffusion on $w$. It is found that higher the Soret effect larger $|w|$ in the flow region. The effect of chemical reaction $k_r$ on $w$ can be observed from Fig (2). It is found that $|w|$ depreciates in the degenerating chemical reaction case and enhances in the
generating chemical reaction case. The effect of Jeffrey parameter $\lambda_1$ on $w$ can be seen from Fig (3). It is found that $|w|$ experiences an enhancement with increasing Jeffrey parameter $\lambda_1$. Fig (4) represents the effect of non-linear density temperature variation ($\gamma$). It is found that the non-linearity in the density – temperature variation results in a depreciation in $|w|$ in the flow region.

The non-dimensional temperature distribution ($\theta$) is shown in Fig (5) for different parametric variations. We follow the convention that the non-dimensional temperature convection is positive or negative according as the actual temperature ($T$) is greater / lesser than equilibrium temperature ($T_e$). The effect of chemical reaction $kr$ on $\theta$ can be seen from Fig (5). It can be seen from the profiles that the actual temperature reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case. The effect of porosity of the boundary $\lambda$ can be observed from Fig (6). It is found that higher the suction parameter at the boundary larger the temperature in the flow region. Fig (7) represents $\theta$ with Jeffrey parameter $\lambda_1$. It is found that the actual temperature enhances with increase in $\lambda_1$.

Fig (8) represents $\theta$ with density ratio $\gamma$. It is found that a non-linearity in the density temperature relation results in a depreciation in the actual temperature.

The non-dimensional concentration ($C$) is exhibited in figures 9 and 10 for different parametric values. We follow the convention that the non-dimensional concentration is positive or negative according as the actual concentration is greater/ lesser than the equilibrium concentration ($C_{\infty}$).

Fig (9) represents the concentration with chemical reaction parameter $kr$. It can be seen from the profile that the actual concentration enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case. Fig (10) represents $C$ with Jeffrey parameter $\lambda_1$. An increase in $\lambda_1$ leads to a depreciation in the actual concentration.

From tables 1 & 2 we notice that $|\tau|$ enhances with increase in $So$. An increase in the density ratio $\gamma$ results in a depreciation in $|\tau|$ at $r = 1$ & 2.
From tables 3 & 4 we observed that the rate of heat transfer enhances at r=1 and reduces at r=2 in the degenerating chemical reaction case while in the generating chemical reaction case it reduces at r=1 and enhances at r=2. An increase in the density ratio γ enhances |Nu| at r=1 and reduces at r=2.

The variation of Sh with chemical reaction parameter k, shows that the rate of mass transfer reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case. Also an increase in the density ratio γ enhances |Sh| at r=1 and reduces at r=2.

5. Salient Features:

- An increase in γ deprecates the velocity and temperature whereas it enhances the actual concentration in the entire flow region.

- The axial velocity (w) and the temperature deprecates in the degenerating chemical reaction case and enhances in the generating chemical reaction case whereas concentration(C) enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case.

- Higher the thermo-diffusion larger |w| and θ in the flow region whereas the actual concentration reduces with increase in S0.

- |w| and θ experiences an enhancement with increasing Jeffrey parameter λ1 whereas it leads to a depreciation in the actual concentration.

- An increase in λ1 reduces |τ| and |Nu| at r=1 and enhances at r=2.

- An increase in S0 results in an enhancement in |τ| and |Nu| at both the cylinders.

- An increase in the density ratio γ reduces in a depreciation in |τ| at r=1 & 2 whereas it enhances |Nu| at r=1 and reduces at r=2

- |τ| reduces in the degenerating chemical reaction case and enhances in the generating chemical reaction case at both the cylinders whereas |Nu| enhances at r=1 and reduces at r=2 in the degenerating chemical reaction case while in the generating chemical reaction case it reduces at r=1 and enhances at r=2.

6. References:
