

Dynamic Model Analysis of a DC Motor in MATLAB

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Abstract— In this paper, modeling of a DC motor is performed by using generalized equation in MATLAB and also analysis of its performance through a speed control using Proportional Integral Derivative controller. Basically DC motors have some important application like electric traction, robot manipulators, and industrial applications. Due to the simplicity in operation conventional Proportional Integral Derivative controller is used as a control strategy. Here performance analysis of the modeled DC motor is performed through a speed control by programming in MATLAB using Proportional Integral Derivative controller.

Index Terms— DC Motor, Dynamic Model, MATLAB, Modeling, PID Controller.

1 INTRODUCTION

THE DC motor acts as an energy conversion actuator that converts electrical energy (of source) into mechanical energy (for load). These motors are extensively applied for robotic manipulations, cutting tools, electrical tractions, etc. The torque-speed characteristics of DC motors are most compatible with most mechanical loads. Hence DC motors are always a good ground for advanced control algorithm.

The control characteristics of these motors have resulted to their immense use and hence control of their speed are required. Speed of a DC motor depends on supply voltage, armature resistance and field flux produced by the field current. The methods to control speed of these motors are armature voltage control, armature resistance control and field flux control.

PID (Proportional Integral Derivative) controllers are extensively operated in control applications in industries due to their mathematical modeling, easy operation, good robustness, high reliability, elimination of steady state error. As a result, PID controllers are used to control the speed of a DC motors.

In this paper, modeling of a DC motor is performed and also its performance is analysed through a speed control using PID controller.

2 OBJECTIVES OF THIS WORK

The main objectives of this paper:

- 1) By using generalized equation in MATLAB, modelling of a DC motor is done.
- 2) By using PID controller to analyze the performance by controlling the speed of modeled DC motor.

3 MODELING OF DC MOTOR

As The DC motor model is implemented by mathematical expression. This implemented model is analysed by techniques of time domain and frequency domain analysis in MATLAB simulation and the advance control analysis of hi-tech problems are solved by the help of this model.

3.1 Physical System

Figure 1 depicts the electrical circuit and free body of a closed loop system of DC motor.

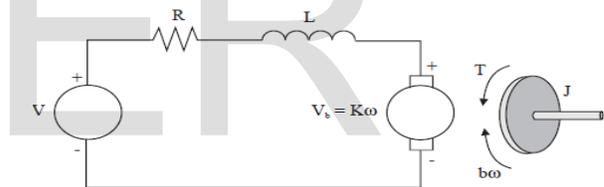


Figure 1: Free body diagram of armature of a DC motor

Here, V is the input armature voltage. The DC motor is analysed by either state space model or Laplace transform. In this paper, we discussed about the dynamic model of dc motor by state space equations as well as Laplace transform equations.

3.1 State Space Modelling

DC Above mention dc motor can be analyzed by its state space equation. With the increase in motor torque T , the armature current i , also increases by a constant K_t . Also With the increase in motor back e , the rotational velocity θ , also increases by a constant K_e .

Then We get,

$$\begin{aligned} T &= K_t i \\ e &= K_e \dot{\theta} \end{aligned} \quad (1)$$

Where

K_t is armature constant, K_e is motor constant, $\dot{\theta}$ or ω is angular velocity.

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By combination of Newton's law with Kirchoff's law from Figure 1:

$$\begin{aligned}
 J \ddot{\theta} + b \dot{\theta} &= K i \\
 L \frac{di}{dt} + Ri &= V - K \dot{\theta}
 \end{aligned}
 \tag{2}$$

Where

J stands for moment of inertia, b stands for friction coefficient, L is armature inductance, R stands for armature resistance, V stands for input voltage, K stands for electromotive force constant, stands for angular acceleration.

The rotating speed and current act as state variables and voltage (V) as input and the output is angular velocity, in state space.

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V$$

$$\dot{\theta} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix}$$
(3)

3.3 Laplace Transform Expression

In control system, analysis and design of technologies are applied for development in real-time. The analysis of DC motor can be done easily by Laplace transform.

The Laplace transform of Equation (2) is given as:

$$Js^2\theta(s) + bs\theta(s) = KI(s)$$
(2.1)

$$LsI(s) + RI(s) = V(s) - Ks\theta(s)$$
(2.2)

Here s is the Laplace operator, $\theta(s)$ represents angular displacement in s domain. From Equation (2.2) I(s) is given as:

$$I(s) = \frac{V(s) - Ks\theta(s)}{R + sL}$$
(2.3)

Which is substituted in Equation (2.1) to obtain:

$$Js^2\theta(s) + bs\theta(s) = K \frac{V(s) - Ks\theta(s)}{R + sL}$$
(2.4)

This Laplace Equation for the DC motor is implemented by the block diagram in Figure 2.

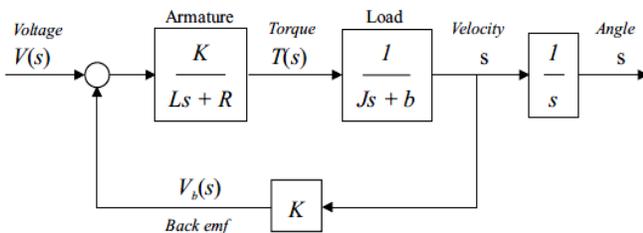


Figure 2: Modeled block diagram of a DC motor

From Equation (2.4), the transfer function (the ratio of $\theta(s)$ to voltage $V(s)$) is:

$$G_A(s) = \frac{\theta(s)}{V(s)} = \frac{K}{s[(R + Ls)(Js + b) + K^2]}$$
(2.5)

The transfer function of the angular velocity $\omega(s)$ to the input voltage $V(s)$ is (referring Figure 2):

$$G_v(s) = \frac{\omega(s)}{V(s)} = \frac{K}{(R + Ls)(Js + b) + K^2}$$
(2.6)

4 ANALYSIS OF DC MOTOR IN MATLAB

4.1 MATLAB Representation

In this paper, the dynamic modeling of DC motor is implemented using Laplace transform expression. The transfer function from Laplace transform expressions is put into MATLAB m-file by defining the polynomial. Furthermore, function conv (A, B), which results in the product of the polynomials. The transfer function is put in MATLAB using Voltage as input and velocity and angle as output.

4.2 Simulation Results

Here two types of analysis i.e. Time-Domain Analysis and Frequency-Domain Analysis is depicted. These two responses are analysed in the modeled DC motor by MATLAB m-file.

(A) Time-Domain Responses

Here the step and impulse response of the modeled DC motor is depicted from Figure 3 to 6.

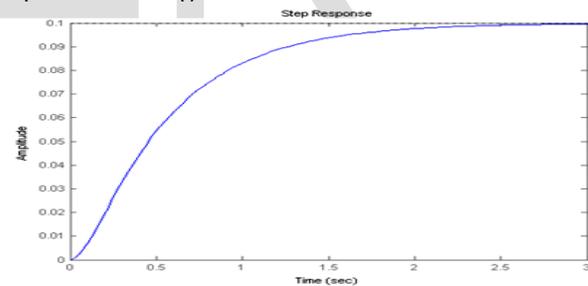


Figure 3: Step response of velocity

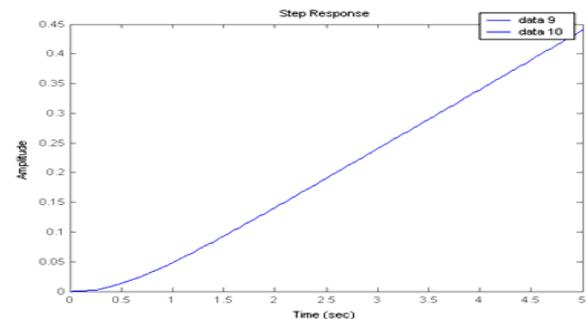


Figure 4: Step response of angle

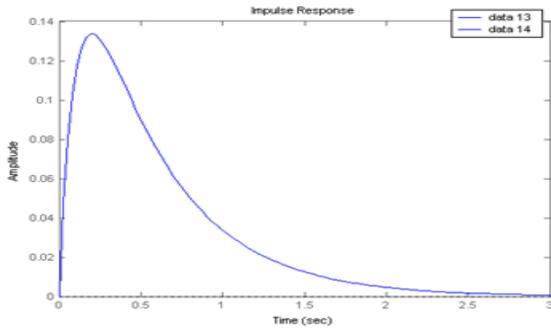


Figure 5: Impulse response of velocity

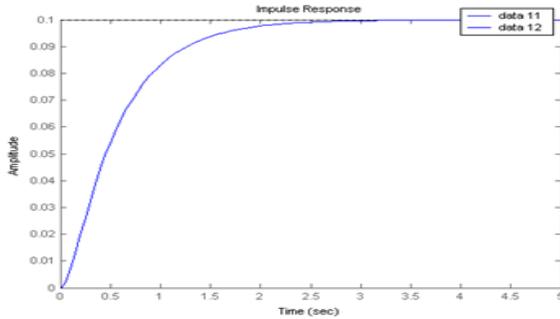


Figure 6: Impulse response of angle

(B) Frequency-Domain Responses

Here the bode response of the modeled DC motor is depicted from Figure 7 and 8.

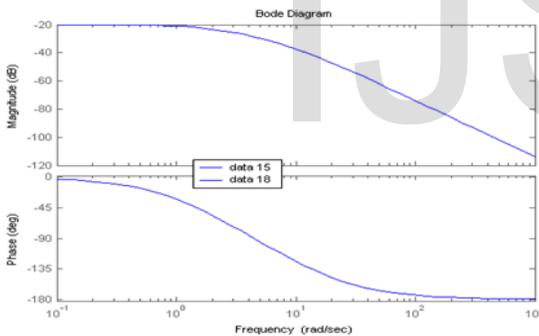


Figure 7: Bode response of velocity

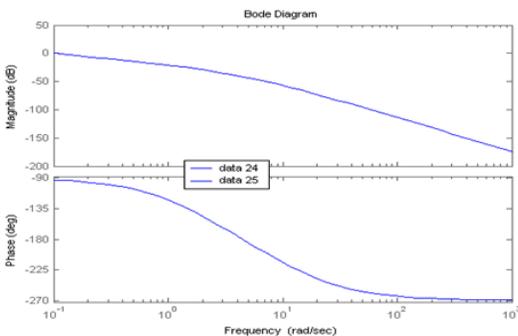


Figure 8: Bode response of angle

4.3 Performance Analysis

The performance of the DC motor whose modeling is done here is analysed by using a PID controller for speed control test. The block diagram of PID controller with closed loop is depicted in Figure 9.

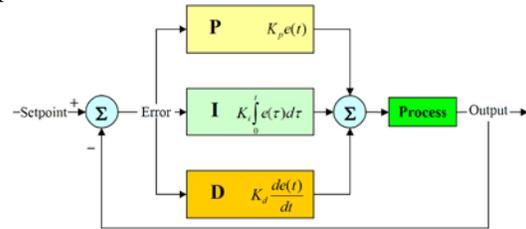


Figure 9: Block diagram of closed loop PID controller

By applying Laplace's transform the transfer function $C(s)$ of a PID controller with step response is can be written as:

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$$

$$= \frac{K_d s^2 + K_p s + K_i}{s} \tag{2.7}$$

Here, U = controller output

E = controller input

K_p = proportional gain

K_i = integral gain

K_d = derivative gain.

Now with necessary PID commands the program is made to run and the graph obtained is depicted in Figure 10.

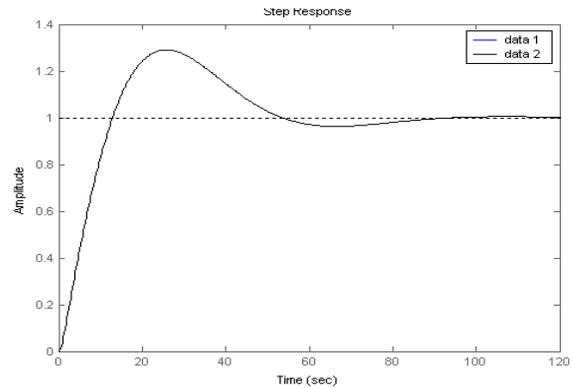


Figure 10: Closed loop step response of a DC motor using PID controller

5 RESULT AND DISCUSSION

A model of a DC motor using the generalized equation in MATLAB is obtained and the time domain and frequency domain responses for both step, impulse and bode plot is respectively discussed.

To do the modeling and simulation of DC motor, the following steps are followed:

- 1) The DC motor circuit diagram is represented.
- 2) The system equations are created.
- 3) The transfer function is calculated.
- 4) Then the model block diagram is prepared.
- 5) The is done to simulate the model in programming MATLAB.
- 6) And finally the performance analysis is carried out.

The simulation parameters are set and the simulation is made to run to see the step response. For the definition of the motor constants the given values for the physical quantities are assumed in Table 1:

TABLE 1: Motor Constants

Symbol	Description	Value with unit
J	Moment of Inertia of the rotor	0.01 kg.m ² /s ²
B	Damping ratio	0.1 Nms
K=K _e =K _t	Electromotive force constant	0.01 Nm/Amp
R	Electrical Resistance	1 Ω
L	Electrical Inductance	0.5 H

Table 2 shows the result of the step and impulse responses.

TABLE 2: Results of step & impulse response

Response	Output Variable	Settling Time	Overshoot	Steady-State Error
Step	Velocity	2.07 (Sec)	2.22e - 014 (%)	1.14 (Sec)
	Angle	NA	NA	NA
Impulse	Velocity	2.27(Sec)	NA	NA
	Angle	2.07(Sec)	NA	NA

4 CONCLUSION

This paper is all about DC motor modeling using generalized equation in MATLAB simulation and to analysis the performance of this modeled DC motor using PID controller.

Here the time domain analysis for both step and impulse signal is discussed. The frequency domain analysis is made by bode plot.

5 FUTURE SCOPE

Following areas of further study are considered interesting:

- 1) To analysis the performance of higher order speed control to other non linear systems may be attempted.
- 2) To reduce error of PID controller for dc motor may

be attempted.

- 3) To control the speed by SEPIC converter when DC supply is Solar energy.

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