Direct Torque Control of Induction Motor Based On Space Vector Modulation with Adaptive Stator Flux Observer

Sajida Shaik ¹, Naseeb Khatoon ²

¹ (1. Electrical and Electronics Engineering, Audishankara Institute of Technology, India, Email id: shaik.sajida2001@gmail.com) &
² (2. Electrical and Electronics Engineering, Nalla Malla Reddy Engineering College, India, Email id: naseeb.kht@gmail.com)

ABSTRACT This paper describes a combination of direct torque control (DTC) and space vector modulation (SVM) for an adjustable speed sensorless induction motor (IM) drive. The motor drive is supplied by a two-level SV/PWM inverter. The inverter reference voltage is obtained based on input-output feedback linearization control, using the IM model in the stator D–Q axes reference frame with stator current and flux vectors components as state variables. Moreover, a robust full-order adaptive stator flux observer is designed for a speed sensorless DTC-SVM system and a new speed-adaptive law is given. By designing the observer gain matrix based on state feedback H∞ control theory, the stability and robustness of the observer systems is ensured. Finally, the effectiveness and validity of the proposed control approach is verified by simulation results.

Index Terms—Adaptive stator flux observer, direct torque control, feedback linearization, robust, space vector modulation.

I. INTRODUCTION

DIRECT TORQUE CONTROL (DTC) abandons the stator current control philosophy, characteristic of field oriented control (FOC) and achieves bang bang torque and flux control by directly modifying the stator voltage in accordance with the torque and flux errors. So, it presents a good tracking for both electromagnetic torque and stator flux [1]. DTC is characterized by fast dynamic response, structural simplicity, and strong robustness in the face of parameter uncertainties and perturbations.

One of the disadvantages of conventional DTC is high torque ripple [2]. Several techniques have been developed to reduce the torque ripple. One of them is duty ratio control method. In duty ratio control, a selected output voltage vector is applied for a portion of one sampling period, and a zero voltage vector is applied for the rest of the period. The pulse duration of output voltage vector can be determined by a fuzzy logic controller [3]. In [4], torque-ripple minimum condition during one sampling period is obtained from instantaneous torque variation equations. The pulse duration of output voltage vector is determined by the torque-ripple minimum condition. These improvements can greatly reduce the torque ripple, but they increase the complexity of DTC algorithm. An alternative method to reduce the ripples is based on space vector modulation (SVM) technique [5], [6].

Direct torque control based on space vector modulation (DTC-SVM) preserve DTC transient merits, furthermore, produce better quality steady-state performance in a wide speed range. At each cycle period, SVM technique is used to obtain the reference voltage space vector to exactly compensate the flux and torque errors. The torque ripple of DTC-SVM in low speed can be significantly improved. In this paper, SVM-DTC technique based on input-output linearization control scheme for induction machine drives is developed. Furthermore, a robust full-order speed adaptive stator flux observer is designed for a speed sensorless DTC-SVM system and a speed-adaptive law is given. The observer gain matrix, which is obtained by solving linear matrix inequality, can improve the robustness of the adaptive observer gain in [7]. The stability of the speed adaptive stator flux observer is also guaranteed by the gain matrix in very low speed. The proposed control algorithms are verified by extensive simulation results.

II. DTC-SVM BASED ON INPUT-OUTPUT LINEARIZATION

A. Model of Induction Motor

Under assumption of linearity of the magnetic circuit neglecting the iron loss, a three-phase IM model in a stationary D–Q axes reference with stator currents and flux are assumed as state variables, is expressed by

\[ i_D = - \left( \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} \right) i_D - \omega_r i_Q + \frac{\psi_D}{\sigma L_{rr}} + \frac{\omega_r}{\sigma L_s} + \frac{u_D}{\sigma L_s} \] (1)
\begin{align}
\dot{i}_Q &= -\frac{R_s}{L_s}i_Q - \frac{R_r}{L_r}i_D - \omega_s i_D + \frac{\omega_m \psi_Q}{L_s} + \frac{\psi_Q}{\sigma L_s} + \frac{u_Q}{\sigma L_s} \\
\dot{\psi}_D &= u_D - R_s i_D \\
\dot{\psi}_Q &= u_Q - R_r i_Q
\end{align}

(2)

where \(\psi_D\), \(\psi_Q\), \(u_D\), \(u_Q\), \(i_D\), and \(i_Q\) are respectively the D - Q axes of the stator flux, stator voltage and stator current vector components, \(\omega_m\) is the rotor electrical angular speed, \(L_s\), \(L_r\), and \(L_m\) are the stator, rotor, and magnetizing inductances, respectively, and \(\sigma = 1 - \left(\frac{L_m}{L_s L_r}\right)\) and \(R_s, R_r\) are the stator and rotor resistances respectively.

The electromagnetic torque \(T_e\) in the induction motor can be expressed as

\[ T_e = p_n \psi_s^2 x_s = p_n (\psi_D i_Q - \psi_Q i_D) \]

(5)

where \(p_n\) is the number of pole pairs.

**B. DTC-SVM Based on Input-Output Linearization**

The DTC-SVM scheme is developed based on the IM torque and the square of stator flux modulus as the system outputs, stator voltage components defined as system control inputs and stator currents as measurable state variables.

Let the system output be

\[
\begin{align}
y_1 &= T_e = p_n (\psi_D i_Q - \psi_Q i_D) \\
y_2 &= \psi_s^2 x_s = \psi_D^2 + \psi_Q^2
\end{align}
\]

(6)

Define the controller objectives \(e_1\) and \(e_2\) as

\[
\begin{align}
e_1 &= T_e = T_{eref} \\
e_2 &= \psi_s^2 = \psi_{sref}^2
\end{align}
\]

(8)

Where \(T_{eref}\) and \(\psi_{sref}\) are reference values of electromagnetic torque and stator flux, respectively.

According to (1)-(5), the time derivative of \(e\) is as

\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} = \begin{bmatrix} g_1 & d \\
g_2 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\
e_2 \end{bmatrix}
\]

(10)

Where

\[
g_1 = p_n \left[ C (\psi_D i_Q - \psi_Q i_D) + \omega_m (\psi_D i_D + \psi_Q i_D) - \frac{\omega_s}{\sigma L_s} \psi_s^2 \right] \\
g_2 = -2R_s (\psi_D i_Q + \psi_Q i_D)
\]

\[
D = \begin{bmatrix}
\left( i_Q - \frac{\psi_Q}{\sigma L_s} \right) - \left( i_D - \frac{\psi_D}{\sigma L_s} \right) \\
2\psi_D \\
2\psi_Q
\end{bmatrix}
\]

\[
C = -\left( \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} \right)
\]

According to \(i_s = \left( \psi_s / \sigma L_s \right) - \left( L_m / \sigma L_r \right) \psi_r\), the characteristic determinant of \(D\) is as follows:

\[
\det(D) = -4L_m \left( \frac{\sigma L_s}{\sigma L_r} \right) p_n |\psi_s| |\psi_r| \cos(\psi_s, \psi_r)
\]

(11)

From (11), \(D\) is a nonsingular matrix since the inner product of stator flux vector and rotor flux vector cannot be physically zero.

Based on input-output feedback linearization [8], the following control inputs are introduced:

\[
\begin{bmatrix}
u_D^* \\
u_Q^*
\end{bmatrix} = \text{inv}(D) \begin{bmatrix}
g_1 + u_x \\
g_2 + u_y
\end{bmatrix}
\]

(12)

Where \(u_x\) and \(u_y\) are the auxiliary control inputs and are defined based on the pole placement concept of the linear control systems so that

\[
\begin{align}
u_x &= -c_1 e_1 \\
u_y &= -c_2 e_2
\end{align}
\]

(13)

where \(c_1\) and \(c_2\) are positive constants.

**III. SPEED ADAPTIVE STATOR FLUX OBSERVER**

**A. Speed Adaptive Stator Flux Observer**

Using the IM model of (1)-(4), the speed adaptive stator flux observer is introduced:

\[
x = Ax + Bu \\
i_s = Cx
\]

(14)

Where

\[
x = \begin{bmatrix} i_D \\
i_Q \\
\psi_D \\
\psi_Q \end{bmatrix}^T, U = \begin{bmatrix} u_D \\
u_Q \end{bmatrix}^T, \\
i_s = \begin{bmatrix} i_D \\
i_Q \end{bmatrix}^T, B = \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix}, C = \begin{bmatrix} I & 0 \end{bmatrix},
\]

\[
A = A_0 + \Delta A_R, A = \begin{bmatrix} \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} & I \\
0 & - \frac{1}{\sigma L_r} \end{bmatrix}, A_{\omega} = \begin{bmatrix} -\frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} & I \\
0 & - \frac{1}{\sigma L_r} \end{bmatrix} + \begin{bmatrix} \frac{\Delta R_s}{\sigma L_s} & \frac{\Delta R_r}{\sigma L_r} \end{bmatrix} + \begin{bmatrix} \Delta R_s & \Delta R_r \end{bmatrix}
\]

(15)

the uncertain parameters in matrix \(A\) are split in two parts: one corresponding to nominal or constant operation and the second to unknown behavior. \(R_{s0}\) and \(R_{r0}\) are nominal value of stator resistance and rotor resistance, \(\Delta R_s\) and \(\Delta R_r\) are
stator resistance and rotor resistance uncertainties, respectively.

The state observer, which estimates the state current and the stator flux together, is given by the following equation:

\[
\frac{dx}{dy} = (A_0 + \Delta A_R + \omega_r A_\omega)x + Bu + H(i_s - i_d) \tag{15}
\]

Where \( x = (i_d, i_q, \psi_D, \psi_Q) \) are estimated values of the state variable and \( H \) is the observer gain matrix.

Supposing state error is \( e \), i.e., \( e = \hat{x} - x \), so

\[
\frac{d}{dt}(e) = \frac{d}{dt}(\hat{x}) - \frac{d}{dt}(x) = (A_0 + HC + \Delta A_R + \omega_r A_\omega)e + \Delta \omega_r A_\omega \dot{x}. \tag{16}
\]

In order to derive the adaptive scheme, Lyapunov theorem is utilized. Now, let us define the following Lyapunov function:

\[
V = e^T e + (\dot{\omega}_r - \omega_r)^2 / \lambda. \tag{17}
\]

The time derivative \( V \) of is as follows:

\[
\frac{dV}{dt} = e^T [(A_0 + HC + \Delta A_R + \omega_r A_\omega)^T + (A_0 + HC + \Delta A_R + \omega_r A_\omega)]e + \Delta \omega_r (\hat{x}^T A_\omega^T e + e^T A_\omega \hat{x}) + \frac{2}{\lambda} (\dot{\omega}_r - \omega_r) \frac{d\omega_r}{dt}. \tag{18}
\]

Let

\[
\Delta \omega_r (\hat{x}^T A_\omega^T e + e^T A_\omega \hat{x}) + \frac{2}{\lambda} \Delta \omega_r \frac{d\omega_r}{dt} = 0 \tag{19}
\]

if we select observer gain matrix \( H \) so that the validity of the inequality

\[
e^T [(A_0 + HC + \Delta A_R + \omega_r A_\omega)^T + (A_0 + HC + \Delta A_R + \omega_r A_\omega)]e < 0 \tag{20}
\]

can be guaranteed, the state observer is stable.

The adaptive scheme for speed estimation is given by

\[
\dot{\omega}_r = \left( K_p + \frac{K_i}{\lambda} \right) (\psi_S^T) J (i_s - i_d) \tag{21}
\]

B. Observer Gain Matrix Computation

Let’s introduce a theorem about quadratic stability of uncertainty system before design the observer gain matrix.

Lemma: Uncertainty system

\[
\dot{x}(t) = (A_0 + \Delta A(t)) x(t), \quad x(0) = x_0 \tag{22}
\]

is quadratic stable, if and only if \( A_0 \) is stable and

\[
||F(sI - A_0)^{-1} E||_\infty < 1 \tag{23}
\]

Where \( A_0 \) is nominal matrix, which is supposed to be well known, \( \Delta A = E3F \) represent the uncertainties on \( A \) due to unmodeled behavior or parameter drift, \( E \) and \( F \) are the uncertainty structure matrices of the system, \( \delta \) is uncertainty coefficient.
If $\Delta A_R$ is also written as $\Delta A_R = \mathbf{E}\mathbf{F}$ so system (16) is quadratic stable, if and only if $A_0 + \omega_r A_{\omega} + HC$ is stable and

$$||F(sI - A_0 - \omega_r A_{\omega} - HC)^{-1}E||_{\infty} < 1$$  \hspace{1cm} (24)

Supposing $\mathbf{K} = HC$, quadratic stability problem of system (16) can be transformed to static state feedback $H_{\infty}$ control problem for the system as Fig 1.

$$G(s) = \begin{bmatrix} A_0 + \omega_r A_{\omega} & E & I \\ F & 0 & 0 \\ I & 0 & 0 \end{bmatrix}$$ \hspace{1cm} (25)

A state-space realization of Fig. 1 is as (25)

As system (25), there will be a state feedback controller $\mathbf{K}$, if and only if there are positive definite matrix $\mathbf{X}$ and $\mathbf{W}$ to make linear matrix inequality (26) is satisfied

$$\begin{bmatrix} AX + W + (AX + W)^T & E & (FX)^T \\ E^T & -1 & 0 \\ FX & 0 & -1 \end{bmatrix} < 0,$$ \hspace{1cm} (26)

If $\mathbf{X}^*$ and $\mathbf{W}^*$ is a feasible solution to linear matrix inequality (26), then $\mathbf{u} = \mathbf{W}^*(\mathbf{X}^*)^{-1}\mathbf{x}$ is a state feedback $H_{\infty}$ controller of system (25). So, $\mathbf{K} = \mathbf{W}^*(\mathbf{X}^*)^{-1}$ The observer gain matrix can be obtained from $\mathbf{H} = \mathbf{KC}^{-1}$.

<table>
<thead>
<tr>
<th>TABLE I PARAMETERS OF IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETERS</td>
</tr>
<tr>
<td>Rotor Resistance $R_r$ (Ω)</td>
</tr>
<tr>
<td>Stator Resistance $R_s$ (Ω)</td>
</tr>
<tr>
<td>Stator Inductance $L_s$ (H)</td>
</tr>
<tr>
<td>Rotor Inductance $L_r$ (H)</td>
</tr>
<tr>
<td>Magnetizing Inductance $L_m$ (H)</td>
</tr>
<tr>
<td>Base Frequency $f$ (Hz)</td>
</tr>
<tr>
<td>Number Of Poles</td>
</tr>
<tr>
<td>Rated Power $P$ (KW)</td>
</tr>
<tr>
<td>Rated Voltage $V$ (V)</td>
</tr>
<tr>
<td>Rated current $I$ (A)</td>
</tr>
<tr>
<td>Rated Speed $N$ (r/min)</td>
</tr>
<tr>
<td>Stator Flux Linkage $\Psi_s$ (wb)</td>
</tr>
</tbody>
</table>

IV. SIMULATIONS

To verify the DTC-SVM scheme based on input-output linearization and adaptive observer, simulations are performed in this section. The block diagram of the proposed system is shown in Fig. 2. The parameters of the induction motor used in simulation research are as Table I.

The reference stator flux used is 0.8 Wb and the command speed value is 50 rpm in both two systems. The speed and torque response curves of conventional DTC and proposed DTC-SVM are shown Fig. 3 and Fig. 4. At startup, the system is unloaded, the load torque is changed to 2 Nm at
t=0.3sec, then the load torque is changed from 2 Nm to 1 Nm at t=0.6sec. The stator flux observer curves are shown in Figs. 5 and 6. Compared with conventional DTC, the DTC-SVM has much smaller torque ripple. From Figs. 5 and 6, it can be seen that the adaptive observer can estimate the stator flux well and truly.

V. CONCLUSION

A novel DTC-SVM scheme has been developed for the IM drive system, which is on the basis of input-output linearization control. In this control method, a SVPWM inverter is used to feed the motor, the stator voltage vector is obtained to fully compensate the stator flux and torque errors. Furthermore, a robust full-order adaptive flux observer is designed for a speed sensorless DTC-SVM system. The stator flux and speed are estimated synchronously. By designing the constant observer gain matrix based on state feedback $H^\infty$ control theory, the robustness and stability of the observer systems is ensured. Therefore, the proposed sensorless drive system is capable of steadily working in very low speed, has much smaller torque ripple and exhibits good dynamic and steady-state performance.

REFERENCES
