Diquark Approach to Calculating the Width of Theta pentaquark state

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Abstract—Now it is well known that the multiquarks cannot be a simple n-quark states in ground state, because they would freely recombine and decay in baryons and mesons, with a very broad decay width. Lattice QCD (LOCD) determinations of hadrons charge distributions do not support the concept of substantial u - d scalar diquark clustering as an appropriate description of the internal structure of the states, Thus vector diquarks maybe more favourable. In our previous paper we calculated the mass of one interesting multiquark state ( Theta pentaquark, \( \Theta^+ \)) by using of vector diquarks. In this paper we use our diquark approach for calculating its decay width using mass calculation results simultaneously. The limit on the \( \Theta^+ \) width is very low [20] \( \Gamma_\Theta \approx 1 \text{MeV} \ll \Gamma_B \) for conventional baryon resonances and in quark model this picture is less clear. By using of vector diquark ideas in the chiral limit diquark correlations in the relativistic region and imposing HF interactions between quarks in a vector diquark we have calculated the mass and decay width of Theta pentaquark state simultaneously.

Keywords – Pentaquark, Diquark, Decay Width, QCD, Quark Mass, Hyperfine Interactions.

1 INTRODUCTION

The year 2003 will be remembered as a renaissance of hadron spectroscopy at the earlys of that year (LEPS) collaboration, T. Nakano et al, reported the first evidence of a sharp resonance \( Z^+ \) renamed to \( \Theta^+ \) at \( M_\Theta = 1.54 \pm 0.01 \text{GeV} \) with a width smaller than \( \Gamma_\Theta < 25 \text{GeV} \). The experiment performed at the Spring - 8 facility in Japan and this particle was identified in the \( K^+N \) invariant mass spectrum in the photon production reaction \( \gamma n \rightarrow K^+ + \Theta^+ \), which was induced by a Spring - 8 tagged photon beam of energy up to \( 2.4 \text{GeV} \)[1].

The existence of \( \Theta^+ \) was soon confirmed by various groups in several photo nuclear reactions [2] including V . V Barmin et al [3]. ITEP (DIANA)[4], JLAB (CLAS)[5], ELSA (SAPHIR)[6],... et al[7].

Since 2003, January there have been several reports of exotics and Because of the observation of such states in various reaction channels, the existence of pentaquark baryons now becomes widely accepted. [8] Such states are believed to belong to a multiplet of states where the possible observability of the other members has to be worked out. This discovery has triggered an intense experimental and theoretical activity to understand the structure of the state[9].

With the conventional constituent quark model, the conservation rules guarantee that it has a strangeness \( S = 1 \), baryon number \( B = 1 \), and charge \( Q = 1 \), thus the hypercharge is \( Y = B + S = 2 \) and the third component of isospin is \( I = 0 \). No corresponding \( pK^+ \) ( \( I = 1 \)) state is observed at the same mass, due to absence of a \( \Theta^{++} \) in the channel \( \gamma p \rightarrow pK^+K^- \) and thus the isospin of \( \Theta^+ \) is the same \( I = 0 \) and it also seems important that no \( S = 1 \) baryon states has been observed below the NK threshold, and this state seems to be the ground state [10].
width (\( \Gamma \approx 10 \text{ MeV} \)) is exceptionally narrow as for a hadron resonance located at 110 MeV a bove the NK threshold usually referred to narrow width puzzle[16].

There is no direct measurement of its spin, S and Isospin, I and its angular momentum, J and parity, P are different in various theoretical works, however most of them postulated its angular momentum, J to be J = 1/2 but the possibility of J = 3/2 and S = 1/2 and P = + is rather plausible[17]. If some of theoretical models is correct, there should be new pentaquark states waiting for discovery[18]. Are these new states exist ?. The answer is “experiment” but in the experiments we see a variety of mass for \( \Theta^+ \), are they same particle and the differences are due to experimental errors ?.

If \( \Theta^+ \) confirmed and established, a new landscape of multiquark hadrons is emerging from the horizon, we must answer what the underlying dynamics leading to its low mass and narrow width simultaneously in a special production mechanism?.

The behavior of QCD in the low-energy is nonperturbative and the SU(3) colour group structure is non-abelian. However besides conventional mesons and baryons, QCD itself does not exclude the existence of the nonconventional states such as glueballs (gg, ggg, …), hybrid mesons (qqg) and other multiquark states (qqqq, qqqqq)[19].

Can low-energy QCD describe the underlying dynamical forces between quarks and gluons in multiquark states and generate their mass and width correctly?. It is very difficult to calculate the whole hadrons' spectrum from first principles in QCD. Under such a circumstance various models which are QCD- based or incorporate some important properties of QCD were proposed to explain the hadron spectrum and other low-energy properties[20]. Whether the \( \Theta^+ \) exist or not, but it is still of interest to see what QCD has to say on the subject.

Now it is well known that the multiquarks cannot be a simple n-quark states in ground state, because they would freely recombine and decay in baryons and mesons, with a very broad decay width.

If we discuss about an correlated perturbative chiral quark model (C P \( \chi \) QM) with explicit symmetry on configuration, the hyperfine interactions (FS, CS) would be considered on the n-quark subsystems (diquarks, …) and a special form of confining potential.

Although the diquark approach in our model is extendable to all possible multiquark states, we consider the \( \Theta^+ \) pentaquark as one example of multiquark states and use the diquark approach for calculating its mass and decay width simultaneously. The limit on the \( \Theta^+ \) width is very low \( \Gamma \approx 1 \text{ MeV} \ll \Gamma_\Theta \) for conventional baryon resonances and in quark model this picture is less clear.

We have calculated the mass of \( \Theta^+ \) pentaquark in our previous paper, now in this paper we use our diquark approach for calculating its decay width using mass calculation results simultaneously.

### 2 NARROW WIDTH PUZZLE

The pentaquark width (\( \Gamma < 10 \text{ MeV} \)) is exceptionally narrow as for a hadron resonance located at 110 MeV above the NK threshold both \( \Theta^+ \) and \( \Xi^- \) (\( \Xi^- \)) is the most peculiar feature of this new resonance. In other words is there mysterious selection rule which is absent from the conventional hadron interaction?. Since there is no known selection rule from symmetry to make the width naturally small, the narrow width should have dynamical origin. Can low-energy QCD describe the underlying dynamical forces between quarks and gluons in such states and generate their mass and width simultaneously correctly?

For comparison there is S = 1 hyperon \( \Lambda (1520) \)\( D_{03} \) state with \( J^P = 3/2^- \) and in the same mass region as the \( \Theta^+ \), has dominant two- body decay D - wave with final states NK, with a smaller phase space and higher partial wave, and its width is 15.6 MeV[23], while \( \Theta^+ \) -wave has a total width less than several MeV, corresponding to negative or positive parity. \( \Lambda (1520) \) decay to KN through D - wave and its width is 7 MeV also \( \Delta (1600) \) decay to K N through P -wave and its width is 100 MeV, this two decays need \( q\bar{q} \) creation and \( \Theta^+ \) is in the same phase space but its width is smaller than several MeV, there is a puzzle.

The question is the origin of narrow width of the pentaquark which is the most peculiar feature of this new resonance. In other words is there mysterious selection rule which is absent from the conventional hadron interaction?. Since there is no known selection rule from symmetry to make the width naturally small, the narrow width should have dynamical origin. Can low-energy QCD describe the underlying dynamical forces between quarks and gluons in such states and generate their mass and width simultaneously correctly?

There have been several attempts to explain the narrow width[24], from combinational suppression from the spin - flavor and color factors or from the special - spatial structure due to diquark correlations and from the theories which describe the behavior of quarks and gluons like chiral soliton and instanton liquid models. Also Several attempts performed on confinement, however no definite conclusion has been reached yet. For the \( \Theta^+ \) the most efficient decay mechanism is for the 5 quarks to regroup with each other into a three-quark baryon and a meson that is in contrast to the \( 3P_0 \) decay models of the ordinary hadrons[25].

In order to refine our understanding of quark dynamics at low energy where it is not perturbative we review some general features of the dynamics of a K N resonance, \( \Theta^+ \) lies about 100 MeV above \( K^+ N \) threshold at a center of mass...
momentum $K = 270$ MeV, the characteristic parameter $K_R$ is about 6.4 if we use a typical Range $R = 1$ F for this interaction. Assuming isospin zero with $K_R = 1.4$ only the $S$ or $P$ wave is likely and the spin $S = 1/2$ is pleasurable.

QCD features are:
1- Because of low center of mass momentum and no other hadronic channels coupling to $K^+ N$ below $K\Delta$ threshold at 1725 MeV we are at nonrelativistic region.
2- $\Theta^+$ is an exotic particle and in its scattering to $K^+ N$ there is no quark-antiquark annihilation graphs, thus we no have confined states that couple by qq annihilation.
3- The wave function of $K^+$ and $N$ or final states differ from the $\Theta^+$ in space, color, and spin.

Thus one can choose the potential Range to be 1 F and decrease the mass of $\Theta^+$ by some additional dynamics beyond nonrelativistic potential scattering, for example: hyperfine interactions such as flavour- spin and color- spin interactions between quarks inside the $\Theta^+$ and confinement effects. Although this is our understanding of quark dynamics at low energy there is various attempts to refine it.

### 3 Pentaquark States

Effective field theories are not just models, they represent very general principles such as analyticity, unitarity, cluster decomposition of quantum field theory and the symmetries of the systems.

The chiral perturbation theory ($\chi$PT), for example, represents the low-energy behavior of QCD (at least in the meson sector). With a conventional $\chi$PT if we consider group theoretical clustering between quarks and hyperfine QCD interactions between them, we have an correlated $\chi$PT between quarks or correlated $\chi$QM. In other case we are working with non-correlated theories.

In fact, because of unconstrained number of degrees of freedom the uncorrelated five-body approaches lead to a larger number of degrees of possible configurations of constituents than correlated ones. On the other hand uncorrelated models cover a wide spectrum of possibilities for the possible pentaquark structure of the $\Theta^+$ baryon. Moreover, in this treatment the quark fermi statistics can be imposed strictly, while in the correlated approaches it is only exactly fulfilled when the diquark is really a pointlike particle.

In a conventional model the baryons are described by their valence quarks as relativistic fermions $\psi(x)$ moving in an external field (static potential):

$$V_{\text{eff}}(r) = S(r) + y^0 V(r)$$

for example a confinement potential with

$$r = |\vec{x}|, \quad S(r) = c r, \quad V(r) = \text{constant} = V_0$$

The valence quark core is supplemented in the flavor SU(3) version by a cloud of goldstone bosons $(\pi, K, \eta) \Phi_i(x)$ according to the chiral symmetry requirement and in addition by quantum fluctuations of gluon field $A^a_\mu(x)$.

Treating also goldstone field as small fluctuations around $\Phi_i(x)$, one can drive the linearized effective lagrangian:

$$L_{\text{eff}}(x) = \bar{\psi}(x)(i - V_{\text{eff}}(r))\psi(x) + \frac{1}{2}\sum_{i=1}^3[\bar{\psi}_i\Phi_i(x)]^2 - \frac{1}{3}g_s^2 F^{\mu\nu} - \bar{\psi}(x)(S(r)i\gamma^5 \Phi_i(x) + g_s^2 A^a_\mu(x)\gamma^\mu \gamma^5 \Phi_i(x) + L_{\chi SB}(x))$$

where $F = 88$ MeV is the pion decay constant in the chiral limit, $g_s$ is the quark gluon coupling constant, $A^a_\mu(x)$ is the quantum component of gluon field and $F_{\mu\nu}$ is its conventional field strength tensor.

$$P = \pi^\pm, \pi^0, K^\pm, K^0, R^0, \eta.$$ The term $L_{\chi SB}(x)$ contins the mass contributions both for quarks and mesons, which explicitly break chiral symmetry.

$$L_{\chi SB}(x) = -\bar{\psi}(x)\mathcal{M}\psi(x) - \frac{\check{g}}{2} F_{\mu\nu}[\Phi^2(x), \mathcal{M}]$$

here, $\mathcal{M} = \text{diag}\{m_u, m_d, m_s\}$ is the mass matrix of current quarks and $B = -(0|\bar{u}u|0)/F^2$ is the quark condensate constant.

Perturbation theory is formulated by the expansion respect to $\bar{\Phi}(x)/F \sim 1/\sqrt{n_c}$ and all calculations are performed at one loop or at order of accuracy $o(1/F^2, \bar{m}, m_s)$.

The explicit form of the ground state quark wave function is set up as:

$$u_{\sigma}(\bar{r}) = \left(\frac{g_\sigma}{\sqrt{f_\sigma}}\right)(\bar{r}) Y_{\sigma}^0(\bar{r}) X_\sigma X_c$$

and for an antiquark we have:
By using of the interaction lagrangian, i.e. the 4-th term of the total lagrangian of Eq (3).

The interaction lagrangian includes effects of the meson cloud and gluon corrections to the baryon by applying Wicks theorem with appropriate propagators for quarks, mesons and gluons. Following equation we can find the energy shift of pentaquark valence particles interacting with pseudoscalar mesons and quantum gluon fields.

\[ \Delta m_B = \sum_{\alpha} \sum_{\lambda_i} \lambda_i \bar{\lambda}_i \langle \rho_0 | \tilde{\lambda}_i \bar{\lambda}_i \sigma_i | \rho \rangle \]  

(7)

and \( c \) refers to connected graphs only.

According to this calculations the contribution of pion-exchange colour-spin interaction \((FS)\) between two quarks and between a quark and an antiquark to the pentaquark mass shift is proportional to:

\[ \langle B | \sum_{i<j} \sum_{\alpha=1}^3 \lambda_i \bar{\lambda}_j \sigma_i \bar{\sigma}_j | B \rangle \]  

(8)

and

\[ \langle B | \sum_{i=1} | \tilde{\lambda}_i \bar{\lambda}_i \sigma_i \bar{\sigma}_i | B \rangle \]  

(9)

and the contribution of gluon exchange colour-spin interaction \((CS)\) between two quarks to the pentaquark mass shift is proportional to:

\[ \langle B | \sum_{i<j} \tilde{\lambda}_i \bar{\lambda}_j \sigma_i \bar{\sigma}_j | B \rangle \]  

(10)

If we discuss on an correlated P\(x\)QM with explicit symmetry on configuration, this hyperfin interactions \((FS,CS)\) would be considered on the four-quark subsystems which is used by many clustered quark models and a special form of confining potential. In such models the parameterizations of the confining potential and the effective quark-gluon coupling, are set up and constrained such as to give a reasonable fit to mass shifts in multiplets (octet or decuplet) sector of conventional baryons. In fact, in such models \((\chi PQM)\) we keep only the terms which are of leading order in \(N_c\) in the power counting expansions and our calculation is a systematic expansion in powers of \(\delta m\), and we are unable to complete the perturbative calculation, because our calculational methods does not allow us to include representations beyond first a few ones.

From the restricted calculations, we see that the mixings among representations are large. It implies that the inclusion of more representations is important, and possible breakdown of the perturbative treatment.

Here, pentaquark states are studied in a correlated perturbative chiral quark model \((CPQM)\) Hamiltonian. Any pentaquark state can be formally decomposed in combinations of simpler colour-singlet clusters, the energy of the state is computed with the state matrix elements of the Hamiltonian.

In our previous paper we suggested that the \(\Theta^+\) state is a composed state of two vector diquarks and a single antiquark, the spatially wave functions of these diquarks has a p-wave and a s-wave in angular momentum in the first and second versions of our model respectively.

\[ [\{Q Q\}^{l=1} \rho, \gamma] \]  

(11)

At the first we constructed the total OCFS symmetry contribution of quarks. Then by using of diquark ideas in the chiral limit diquark correlations in the relativistic region and imposing HF interactions between quarks in a diquark, we led to introducing a conventional Hamiltonian and by considering its solution for the first version of our model we led to \(\Theta^+\) pentaquark mass, \(M(\Theta^+) = 1540\) MeV.

4 Decay Width

In order to capture correctly the physics of QCD between the confinement scale and the chiral symmetry breaking scale we have used of the diquark chiral effective theory. In our approach we considered \(l=1\) angular momentum between vector diquarks in an intracting confining potential. Since the diquark masses are smaller than the constituents, they are stable against decay near mass shell, in such a configuration, the diquarks are nearby and "tunneling" of one of the quarks between the two diquarks may take place.

We assume that in \(\Theta^+ \rightarrow K^+ N\) decay a d quark tunnels from a diquark ud to the other diquark to form a nucleon udd and an off-shell u quark, which is annihilated by the antibar strange quark. (If u were to tunnel, the decay is to \(K^0\)P with a comparable decay width).

The decay width is therefor given as:

\[ \Gamma = \lim_{v \to 0} \alpha(\bar{s} + \Phi_{ud} + \phi_{ud} \rightarrow K^+ + n) v |\psi(0)|^2 \]  

(12)

where \( v \) is the velocity of \(\bar{s}\) in the rest frame of the target diquark and \(\psi\) is the 1S wave function of the quark-diquark inside the pentaquark.

The differential cross section for the annihilation process is then:

\[ d\sigma = \frac{(2\pi)^4 |M|^2}{4 \sqrt{(p_1 p_2)^2 - m^2} M^2_{ud}} \frac{d\Phi(p_1 + p_2; k_1, k_2)}{4 e^{-2S_0}} \]  

(13)

in which \(e^{-2S_0}\) is tunneling probability.
If we insert annihilation amplitude $|\mathcal{M}|$ and phase space $d\Phi$ and integrate over the $d\Phi$ and taking $\nu \to 0$ we find:

$$\Gamma_{\Theta^+} = 5.0 \frac{e^{-2s_0} g^2 f_\Theta^2}{8\pi f_\pi^2} |\psi(0)|^2$$  \hspace{1cm} (14)

by using the WKB approximation for $e^{-2s_0}$

$$e^{-s_0} = \left(\int d^4x e^{-2s_0} \right) \approx e^{-\Delta E r_0},$$  \hspace{1cm} (15)

where $\Delta E = (M_u + M_d) - M_{ud}$

and for $r_0$ we have:

$$M_{\Theta^+} = 2M_{ud} + m_s + \frac{2}{M_{ud}}.$$  \hspace{1cm} (16)

where the third contribution is the rotational energy of diquarks in a $p$-wave. The $1S$ wave function of the quark - diquark at the origin can be written as:

$$\psi(0) = \frac{\frac{2}{a_0^{3/2}}}{\sqrt{4\pi}}$$  \hspace{1cm} (17)

where $a_0$ is the Bohr radius of the quark - diquark bound state.

Assuming they are non-relativistic we get by the dimensional analysis $a_0 \approx (2mB)^{-1/2}$

where $m = 250$ MeV is the reduced mass and $B$ is the binding energy of quark - diquark bound state. Taking $B = 100 - 200$ MeV, comparable to the pentaquark binding energy, $g^2 = 3.03$ and $g_A = 0.75$ from the quark model, one can find the $\Theta^+$ width using Eq (20) as:

$$\Gamma_{\Theta^+} = 1.30 \text{MeV}.$$  \hspace{1cm} (18)

which is unusually narrow and comparable with the recent experimental limit [22].

5 Conclusions

Our theoretical results on the mass and width of $\Theta^+$ are in agreement with many experimental limits and one can use the vector diquark approach for simultaneously calculating the mass and width of other multiquark states, for example those multiplet states which we have introduced in Eq (2) for pentaquarks in which they have vector diquarks. Also one can use our vector diquark approach for calculating the mass and width of other multiquark states such as tetraquarks and dibaryons, see our papers at ref, [3,4].

5 REFERENCES


Cover page

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