# Damping and Positioning Effects due to an Offshore Structure for Fluid Flow in Application to a Sink (Wave Breaker) 

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#### Abstract

Water waves are frequent phenomenon, some of which are high impact events (especially tsunamis) that cause a number of fatalities. Some of the dangers posed by waves include: coastal flooding, coastal erosion, onshore structures and beach destructions. Waves cause major damages and significant economic loss to a large section of coastlines. For instance, surface waves have caused serious damage to some sections of Fort Jesus which lies adjacent to the Indian Ocean. Against this backdrop, therefore, it is important to understand how different structures behave in presence of incoming incident waves. Hence this study seeks to analyze the incoming waves before an offshore (rectangular, cylindrical or spherical) structure with the aim of determining the most suitable position in application to a sink (surface wave breaker). Incident wave potential is obtained by separation of variables method. The wave velocity and acceleration will be obtained through direct differentiation of the velocity potential. The expected outcomes will be used to modify the sink (break water) with focus centered on wave height attenuation, the decline of wave speed and, most importantly, damping of fast propagating surface waves. Proper analysis of the waves would provide information for the determination of suitable shape and positioning of the wave breaker. This would help alleviate the threat posed to the critical coastal structures, coastal inhabitants, and coastal activities.


Furthermore, the study is expected to significantly shed light on ways of preventing or even stopping certain adversities caused by advancing water waves. The idea is to arrest through damping the destructive waves which strike the coastal areas, and consequently slow down their adverse effects.

Keywords: Incident surface wave velocity potential, velocity and acceleration, surface-fixed floating offshore structures, damping, suitable location.

## INTRODUCTION

The study of coastal and sea water management techniques is paramount in ensuring a friendly environment for the all parties involved. The protection of the ports and other onshore structures near the coastline from the incoming wave attack has always been a concern to researchers and ocean engineers (Karmakar et al., 2013). Suitable wave breakers are being erected in the ocean to provide protection to the near shore structures by reducing the wave height, speed, and energy of the incoming waves. Much of research has concentrated on bottom fixed and floating breakwaters for wave height attenuation but have failed to address the issue of suitable location. The flexible floating (surface) break waters are advantageous over the bottom fixed ones as they are cheap, reusable, rapidly deployable and removable (Cho and Kim, 1999).

Environmental constraints, poor bottom architecture, and deep ocean regions have been a challenge for durability of the breakwaters. Problems such as coastal flooding, erosion, million deaths and destruction of property worth billions every year are serious challenges that deserves a lot of attention if lives, economy, living standards among others are to be better and sustainable. Though much has been done, improved techniques coupled with suitable location need to be
addressed. Many studies have adopted floating surface-fixed vertical rectangular membrane as proposed by (McCartney et al., (1985).

Ngina et al., (2014) observed that, the wave velocity and acceleration of the incident wave is directly proportional to that of the surface-fixed structure and therefore, analysis of the incident wave characteristics is important to scientists and engineers in construction of floating wave breakers (sinks). However, the work done has not considered the best location in scattering or attenuating surface waves and hence, the present study aims at analyzing the surface incident wave motion for different water depths prior to their interaction with the floating rigid structure.

The analyzed results obtained when surface gravity waves interact with rectangular and circular cylindrical floating structures will be useful in establishing the position of a sink. However, the floating surface-piercing membranes may pose problems to water transportation. A solution to such induced problems may be arrived at by constructing removable structures to allow free movement of sea animals and marine vehicles. The proposed study will seek to investigate the wave propagation for different water depths with an aim of determining the most suitable location for construction of a sink (wave breaker).

Based on various researches, both rectangular and circular cylindrical structures form a vertical wall in presence of wave terrain, therefore, the analysis of the incident wave characteristics will be carried in the vicinity of a rectangular box. Small amplitude wave theory is used to derive the incident wave velocity potential. (Chakrabarti \& Gupta, 2007) assumed the velocity potential $\phi=(x, y, z, t)$ to take the form of power series in terms of perturbation parameter $\varepsilon$, which for this case represents the wave slope. For convenience the present study adopts similar assumptions. Once the velocity potential is known calculation of the pressure forces and moments on the floating surface fixed structure can be computed.

The study is motivated by erosive power of the surface waves along the coast, for instance, the area on the eastern side of Fort Jesus monument adjacent to the Indian Ocean has highly been eroded and very little has been done due to the hydrodynamic loads and dynamics of the ocean water waves. Furthermore, a large section of the Japan and U.S total population are considered coastal, and may be at risk of the destructive surface waves. The formulation below follows closely the Chakrabarti's approach but the solutions are quite different. For simplicity of calculations, the density of the fluid is always assumed to be constant (Koo and Kim, 2010).

## MATHEMATICAL FORMULATION

The study considered three-dimensional offshore obstacles. The sea floor was taken to be at a distance $h$ from the free surface and below by impervious bottom. This problem was analyzed in response to propagating regular waves with small amplitude $a$ as compared to the wavelength $\lambda$. In this work, small amplitude wave theory (Chakrabarti, 1987) is used to derive the velocity potential. Considering the reference point at $(x=0)$, the wave amplitude is given by $a=\frac{H}{2}$. Other important aspects that must be analyzed include wave elevation, wave velocity and acceleration on each spatial axis is obtained through direct differentiation of the velocity potential. The problem is investigated in the three-dimensional Cartesian coordinate system with $x-z$ being the horizontal plane and $y$-axis being vertically upward positive. The plane $y=0$ in this case represents the undisturbed free surface. $y=\eta(x, z, t)$, denotes the wave elevation. It is assumed that the time is travelling in the negative $x$-direction. The problem is governed by a variety of boundary conditions.

## Boundary conditions

When fluid particles are in contact with a stationary surface, they assume zero tangential velocity. These particles retard the motion of the particles in the adjoining fluid layer, which in turn retards the motion of particles in the next layer and so on, until at a certain distance from the surface where the effect becomes negligible (Rahman and Bhatta, 1993). Certain conditions are necessary for analyses of waves on a body. For the proposed study the bottom of the bathymetry is impermeable to water and therefore the vertical water velocity at the bottom must be zero at all times. This possibility is due to the fact that the distinction between fluid motions occurs due to the boundaries imposed on the fluid domain (Linton and Mclver, 2001). Assuming an infinitely long wave maker along the $x$-axis, generating waves in the $x-z$ plane; in order to solve Laplace equation, the following linearized conditions must be satisfied.

According to Newman et al., (2005) the relation which must hold at the boundary is given by;

$$
\begin{equation*}
w(x, z=-h t)=\frac{\partial \phi}{\partial z}(x, z=-h, t)=0 \tag{1.0}
\end{equation*}
$$

The equation above can be reduced to:

## Bottom and dynamic surface condition

The no flow condition at the bottom implies that flow velocity along the $z$-axis is zero.

$$
\begin{equation*}
v=\frac{\partial \phi}{\partial y}=0 \quad \text { On } y=-h \text { bottom (B.C) for horizontal boundary } \tag{1.1}
\end{equation*}
$$

The pressure $p$ which follows from Bernoulli's equation for unsteady flow is represented in equation (1.2),

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(u^{2}+v^{2}\right)+\frac{p}{\rho}+g y=0 \tag{1.2}
\end{equation*}
$$

Neglecting nonlinear term $\left(u^{2}+w^{2}\right)$ for small amplitude waves, the linearized form of the unsteady Bernoulli's equation is

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}+\frac{p}{\rho}+g y=0 \tag{1.3}
\end{equation*}
$$

the pressure above the free surface is assumed to be constant. This constant pressure is taken to be zero without loss of generality. If we consider a point on the surface by taking $y=n$ and $p=0$, equation (1.3) becomes

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}+g \eta=0 \tag{1.4}
\end{equation*}
$$

Making $\eta$ the subject, equation (1.4) becomes,
$\eta=-\frac{1}{g} \frac{\partial \phi}{\partial t}$
On $y=0 \quad$ dynamic free surface (B.C)

### 2.3.2 Kinematic free condition

This condition requires that the fluid particle at the surface to remain on the free surface at all times for as long as the motion is smooth and the wave doesn't break.

$$
\begin{equation*}
\frac{\partial \phi}{\partial y}=\frac{\partial \eta}{\partial t} \tag{1.6}
\end{equation*}
$$

$$
\text { On } x-z \text { kinematic free surface (B.C) }
$$

For small amplitude waves, $\eta(x, z, t)=0$ therefore $y=0$

### 2.3.3 Radiation condition

States that surface waves and hence the velocity potential, disappear at any distance away from the body. That is, $\nabla \phi=0$ as $R \rightarrow \infty$ (see Ngina et al., 2014).
$\lim _{R \rightarrow \infty} \sqrt{R}\left(\frac{\partial \phi}{\partial n}-i k \phi\right)=0$

## Governing equations

For an inviscid and incompressible flow, the divergence of the velocity vector field $v(x, y, z, t)$ is everywhere zero (Ngina et al., 2014); that is,
$\nabla \bullet V=0 \quad$ Where $v=[u, v, w]$
Due to irrotational nature of the incompressible fluid, there exists a velocity potential $\phi(x, y, z, t)$ such that the velocity components in $x, y, z$ direction is given as follows:
$u=\frac{\partial \phi}{\partial x}, v=\frac{\partial \phi}{\partial y}, w=\frac{\partial \phi}{\partial z}$
The velocity potential satisfies the Laplace equation

$$
\begin{equation*}
\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}=0 \tag{2.0}
\end{equation*}
$$

For the purpose of solving the Laplace equation, wave motion along the $z$-axis is assumed to be zero and this paper adopts the assumption. Equation (2.0) becomes
$\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial x^{2}}=0$
To solve the Laplace equation, kinematic and dynamic boundary conditions from equation (1.4) and (1.6) are used.

In addition, the no flow condition at the seabed on $z$-axis given by

$$
\begin{equation*}
v=\frac{\partial \phi}{\partial y}=0, y=-h \tag{2.2}
\end{equation*}
$$

The pressure at the surface is equal to the atmospheric pressure.

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^{2}+\frac{p}{\rho}+g y=0 \tag{2.3}
\end{equation*}
$$

Taking $y=\eta, p=0$ on the surface and ignoring the higher order powers we obtain

$$
\begin{equation*}
\left(\frac{\partial \phi}{\partial t}\right)_{y=0}+g \eta=0 \quad \text { Where } y=\eta(x, y, t) \tag{2.4}
\end{equation*}
$$

Consider a function $F(x, y, z, t)$ on the fluid field. From substantial derivative we have

$$
\begin{equation*}
\frac{D F}{D t}=\frac{\partial F}{\partial t}+v \cdot \nabla F \tag{2.5}
\end{equation*}
$$

It should be noted that a fluid particle on the free surface is assumed to remain on the surface, which implies that $\frac{D F}{D t}=0$,

$$
\begin{equation*}
\frac{D F}{D t}=\left(\frac{\partial F}{\partial t}+u \frac{\partial F}{\partial x}+w \frac{\partial F}{\partial z}-y\right)_{y=0}=0 \tag{2.6}
\end{equation*}
$$

The kinematic condition that the fluid particles cannot cross the air-water interface is obtained by equating the vertical speed of the free surface itself to that of a fluid particle in the free surface to get
$\frac{\partial \eta}{\partial t}+\frac{\partial \eta}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)_{y=\eta}+\frac{\partial \eta}{\partial y}\left(\frac{\partial \phi}{\partial y}\right)_{y=\eta}=\left(\frac{\partial \phi}{\partial y}\right)_{y=\eta}$
$\left(\frac{\partial \phi}{\partial t}\right)_{y=0}+g \eta=0$


From equation (2.3) we have;

From equation (2.7) we have;

$$
\begin{equation*}
\frac{\partial \eta}{\partial t}=\left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{2.9}
\end{equation*}
$$

Since $\phi$ can be redefined to contain $\eta$, equation (2.8) and (2.9) can be combined by differentiating equation (2.8) to produce,
$\frac{\partial^{2} \phi}{\partial t^{2}}+g \frac{\partial \eta}{\partial y}=0 \quad$ at $y=0$
When the velocity potential $\phi$ is oscillating harmonically in time with angular frequency $w$ we write equation (2.8) as
$-w^{2} \phi+g \frac{\partial \phi}{\partial y}=0, \quad$ On $y=0 \quad($ Faltinsen, 1990)

## Solution of velocity potential

The velocity potential with respect to a non-dimensional parameter $\varepsilon$ takes the form

$$
\begin{equation*}
\phi(x, y, z, t)=\sum_{n=1}^{\infty} \varepsilon^{n} \phi_{n} \tag{3.2}
\end{equation*}
$$

Similarly the free surface elevation $\eta(x, y, z, t)$, takes the form

$$
\begin{equation*}
\eta(x, y, z, t)=\sum_{n=1}^{\infty} \varepsilon^{n} n_{n} \tag{3.3}
\end{equation*}
$$

Where, the perturbation parameter $\varepsilon$ is the product of the wave amplitude $a$, the wave number $k$ and $n$ represents the nth order approximation (Chakrabarti \& Gupta, 2007).

The BVP is considered for the progressive wave with the speed, $c$ and the periodicity given by

$$
\begin{equation*}
\alpha=x-c t \tag{3.4}
\end{equation*}
$$

Equation (2.1) is solved by the separation of variable technique and the velocity potential to be obtained must satisfy all the boundary conditions. The velocity potential, $\phi$ is assumed to take the form
$\phi=Y(y) X(x)$
Substituting equation (3.5) into (2.1) we obtain
$Y(y) X(x)+Y(y) X(x)=0$
By separation of variables and equating the result with $k^{2}$ we obtain two differential equations
$Y(y)-k^{2} Y(y)=0$
$X(x)+k^{2} X(x)=0$
$k^{2}$ is a constant and is taken to be the wave number. On solving equations (3.7) and (3.8) we obtain

$$
\begin{align*}
& Y=A_{1} \cosh k y+A_{2} \sinh k y  \tag{3.9}\\
& X=A_{3} \cos k x+A_{4} \sin k x \tag{4.0}
\end{align*}
$$

At the reference point $X(x)=0$, time $t=0$ and hence from equation (4.0), it is clear that $A_{3}=0$.
Using the boundary condition 2.2 gives
$A_{2}=A_{1} \tanh k d$
Substituting equations (3.9), (4.0), and (4.1) in equation (3.5), and further taking into account the periodicity in equation (3.4), the velocity potential becomes

$$
\begin{equation*}
\phi=A_{5} \frac{\cosh k(y+d)}{\cosh k d} \sin [k(x-c t)] \tag{4.2}
\end{equation*}
$$

From equation (1.6) and applying $\eta=\frac{H}{2}$ at $x=0, y=0 t=0$ and from the relation $c=\frac{w}{k}$ we obtain

$$
\begin{equation*}
A_{5}=\frac{g H}{2 w} \tag{4.3}
\end{equation*}
$$

Since the problem adopted linear wave theory, the velocity potential expression for the first order degree $\phi=\varepsilon \phi$ becomes
$\phi=\frac{g H}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \sin [k(x-c t)]$
In three dimensions, the velocity potential in equation (4.4) can be expanded to take the form

$$
\begin{equation*}
\phi=\frac{g H}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \exp i[\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{4.5}
\end{equation*}
$$

Taking the real part of the velocity potential $[\operatorname{Re}(\phi)]$, equation (4.5) takes the form

$$
\begin{equation*}
\operatorname{Re}(\phi)=-\frac{g H}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \sin [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{4.6}
\end{equation*}
$$

Equation (4.5) is the required velocity potential. The velocity potential obtained is used to derive equations that govern wave characteristics (pressure field, surface profile, wave celerity and wave acceleration and deceleration)

## Surface profile

Having obtained the velocity potential, wave elevation can be obtained from the fact that

$$
\begin{align*}
& \eta=-\frac{1}{g} \frac{\partial \phi}{\partial t}  \tag{4.7}\\
& \eta=\frac{H}{2} \frac{\cosh k(y+d)}{\cosh k d} \cos [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{4.8}
\end{align*}
$$

Wave elevation is an important aspect because it is used in determination of the upward movement of any immersed or floating structure though in this paper, upward movement will not be of great interest. Linear theory that governs wave problem requires amplitude of any structural motions be small relative to other length scales (Ngina et al., 2014). Hence wave elevation serves a great deal in guiding the construction of water sinks (wave breakers).

## Wave celerity

The horizontal component of velocity forms a fundamental aspect of this paper, but for purposes of comparisons and generalizations velocity components in each spatial axis is given as:

Along $x$-axis
$u(x, y, z, t)=-\frac{k g H \cos \beta}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \cos [\{k(x \cos \beta+z \sin \beta)-w t\}]$
Along $y$-axis

$$
\begin{equation*}
v(x, y, z, t)=\frac{-k g H}{2 w} \frac{\sinh k(y+d)}{\cosh k d} \sin [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.0}
\end{equation*}
$$

Along $z$-axis

$$
\begin{equation*}
w(x, y, z, t)=-\frac{k g H \sin \beta}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \cos [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.1}
\end{equation*}
$$

## Pressure field

The moments and forces on the surface fixed floating barge is obtained from the expression of pressure given as

$$
\begin{equation*}
p(x, y, z, t)=-\frac{\rho H}{2 w} \frac{\cosh k(y+d)}{\cosh k d} \cos [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.2}
\end{equation*}
$$

## Wave acceleration

The acceleration of the fluid particles is equivalent to that of the floating body structure (Faltinsen, 1990).

Along $x$-axis

$$
\begin{equation*}
\boldsymbol{u}_{t}(x, y, z, t)=-\frac{k g H \cos \beta}{2} \frac{\cosh k(y+d)}{\cosh k d} \sin [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.3}
\end{equation*}
$$

Along $y$-axis

$$
\begin{equation*}
\nu_{t}(x, y, z, t)=\frac{k g H \sin \beta}{2} \frac{\cosh k(y+d)}{\cosh k d} \cos [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.4}
\end{equation*}
$$

Along $z$-axis

$$
\begin{equation*}
w_{t}(x, y, z, t)=-\frac{k g H \sin \beta}{2} \frac{\cosh k(y+d)}{\cosh k d} \sin [\{k(x \cos \beta+z \sin \beta)-w t\}] \tag{5.5}
\end{equation*}
$$




Figure 1: Schematic diagram representing a rectangular box in presence of surface waves
The paper seeks to analyze the incident wave characteristics and behavior such as wave-induced force on the structure. It is important to note that:

1. When the wave height, water depth and wave period is known, the wave is fully defined and all of its characteristics can be calculated.
2. By inserting the velocity potential in the equation (3.0); the DSBC, differentiating and rearranging we obtain the dispersion relation

$$
\begin{equation*}
C=\sqrt{\frac{g \lambda}{2 \pi} \tanh \frac{2 \pi d}{\lambda}} \tag{5.9}
\end{equation*}
$$

Equation (5.9) indicates that for high amplitude waves the wave speed is dependent of the wave height.

Letting $\quad C=\frac{\omega}{k}$
For small wave events (in shallow water)

$$
\begin{equation*}
C=\sqrt{\frac{g}{k} \tanh k h} \tag{6.1}
\end{equation*}
$$

For large wave events (in deep water)

$$
\begin{equation*}
C=\sqrt{\frac{g}{k}\left(1+\varepsilon^{2}\right)} \tag{6.2}
\end{equation*}
$$

It is observed that, the horizontal component of velocity is constant from the water surface to the bottom in shallow water, and it can be concluded that the best suited location for a sink should be some distance away from the shore. This is due to the fact that the hyperbolic functions in equation (4.5) principally means that the horizontal velocity component decrease with increase in the water
depth. For a given wave height, water depth, and period it is possible to quantify the energy level or flux (rate of transmission) useful in designing a sink.

## Results and discussions



Figure 2: Graph showing the relationship between wave elevation, velocity and acceleration in $x$ direction for water depth $h=4.5$.


Figure 3: Graph showing the relationship between wave elevation, velocity and acceleration in $x$ direction for water depth $h=14.5$.


Figure 4: Graph showing the relationship between wave elevation, velocity and acceleration in $x$ direction for water depth $h=4.5$.


Figure 5: Graph showing the relationship between wave elevation, velocity and acceleration in $x$ direction for water depth $h=4.5$.


Figure 6: Graph showing the relationship between wave elevation, velocity and acceleration in $x$ direction for deep water sections $h>14.5$.

In order to modify or adjust the wave breaker (sink) for effective wave damping, analysis of the changes in the characteristics of a surface wave as it propagates from the deep sea to the shore is of paramount importance. The energy concomitant with the propagating wave is mostly dissipated at the air-water boundary and in shallower water, at the boundary between the water and the bathymetry (Coastal Engineering Research council, 1975). Therefore, analysis of the incident wave characteristics before or on the floating rigid structure for different water depths is quite important for scientists and engineers for designing the most effective wave breakers.

As observed by Ngina et al., (2014), the hyperbolic functions in the velocity potential give the exponential decay of the magnitude of each spatial velocity components in respect to increase of distance below the free surface. It was noted that the acceleration of the surface wave is directly proportional to that of surface fixed structure. The horizontal component of velocity and acceleration forms the centre of interest in this paper due to the fact that the location of wave breakers rely solely on the impacts caused by progressive waves.

Figure 2 indicates the relationship between elevation, velocity and acceleration in $x$ direction (wave propagation direction). It can be deduced that, for finite water depth, displacement precedes the wave velocity and acceleration. It can also be observed that, elevation and velocity are out of phase by 180 degrees, while elevation and acceleration are out of phase by 90 degrees. Ngina (2014) noted that, when elevation is at the maximum, the wave velocity is at minimum. For finite depth, the horizontal velocity is higher compared to both elevation and acceleration. In deep sections, the wave exhibit high wave frequency which on the other hand leads to high horizontal accelerations as compared to the wave velocities and displacement. Ngina (2014) observed decrease in horizontal component of acceleration with decrease in water depth. It is further observed that at the breaking point of the wave (at shore), frictional drag at the water-seabed boundary results to decline in motion of the water molecules just above the sea floor. The phenomenon causes sudden forward movement of water particles which highly impact on the
floating offshore structure. In such cases, the wave characters have varying impacts on the offshore floating structures and in such situations these structures are destroyed and rendered useless within a short period of time.

## Conclusion

It is therefore not advisable to locate wave breakers near the breaking point. For many decades, this confusion has caused location problems to engineers and naval architects. Moreover, it can be deduced that, in between the source and the breaking point the wave exhibit a rather calm motion in spite of increased wave frequency and wave amplitude. The suggested floating structures can be fixed at some distance away from the breaking point. The exact distance can be determined by the engineers depending on the nature of the wave. It can be observed that, for large wave events, the effect of the wave propagation could be highly catastrophic. In such happenings, the floating breakwater should be modified, strengthened or moored to increase its effectiveness. The transmission coefficient is not zero, and therefore the values of the damping coefficients are more than zero and may provide information on construction of multiple breakwaters (Ghassemi \& Yari, 2011).

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