Cyclic Stress Analysis of a Rocket Engine Thrust Chamber using Chaboche Constitutive Model

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Abstract—High performance rockets are developed using cryogenic technology. High thrust cryogenic rocket engines operating at elevated temperatures and pressures are the backbone of such rockets. The thrust chambers of such engines which produce the thrust for the propulsion of the rocket can be considered as structural elements. Often double walled construction is employed for these chambers for better cooling and enhanced performance. The double walled rocket engine thrust chamber investigated here has its hot inner wall fabricated out of a high thermal conductive material like copper alloy and outer wall made of stainless steel. Inner wall is subjected to high thermal and pressure loads during operation of engine due to which it will be in the plastic regime. Major reasons for the failure of such thrust chambers are low cycle fatigue, creep and thermal ratcheting. This occurs due to repeated or cyclic loads. Elasto plastic material models are required to simulate the above effects through a cyclic stress analysis. Stress analysis of the thrust chamber is required to assess the soundness of its design and for optimization. This paper gives the details of cyclic stress analysis carried out for the thrust chamber using the Chaboche nonlinear kinematic hardening plasticity model. Two dimensional finite element model of a cross section of the thrust chamber is chosen for carrying out stress analysis. Loading include pressure and thermal loads. Analysis is done for two successive hot tests. Each hot test consists of six stages of operation starting from pre chilling, operation and finally back to ambient condition.

Keywords—Thrust chamber; Chaboche model; Copper alloy; cyclic stress analysis

1. INTRODUCTION

Thrust chamber is one of the main components of a cryogenic rocket engine. It is the subsystem of an engine where the propellants are injected, mixed and burned to form hot gas products which are accelerated and ejected at high velocity. The thrust chamber considered in this study is double walled and regeneratively cooled using Liquid Hydrogen. The engine works on Liquid Oxygen (LOX) and Liquid Hydrogen (LH₂) oxidizer/fuel combination. The inner wall of the thrust chamber is made up of a special copper alloy whereas the outer wall is fabricated from stainless steel. Copper alloy exhibits cyclic hardening behavior under symmetric cyclic loading conditions. Chaboche model combined with Multilinear isotropic hardening model can be used to predict the cyclic hardening behaviour [1] for simulating the actual behavior of the structure. During operation, the inner wall experiences severe thermal and pressure loads. The inner copper wall has to take care of two contradictory functional requirements. The wall thickness has to be optimised to offer least resistance for heat transfer rate and thereby limit thermal gradients. The inner wall also should have sufficient thickness to withstand the pressure and mechanical loads exerted by coolant pressure and combustion gas pressure. In order to meet both these requirements, the inner wall is designed to operate at stresses beyond the elastic limit. Usually an engine has to undergo repeated hot tests before putting to actual use in the flight. Hence cyclic stress analysis of the thrust chamber is of paramount importance so that its structural integrity during flight is ensured. Stress analysis predicts the manner in which a component will perform structurally under anticipated working conditions. The goal is to design it with sufficient, but not excessive, strength in every detail.
low cycle fatigue, creep and ratcheting [2-6]. Chaboche constitutive model is a nonlinear kinematic hardening model [7-11] which when combined with the classical isotropic hardening model can show better stress strain behaviour compared to conventional plasticity models for cyclic loading. It has the additional capability to capture cyclic hardening of the material and ratcheting effects.

Fig. 2. Dog house effect

A. Plasticity models

Development of models for in-elastic behavior of materials has been an area of substantial development over the past 20-30 years and is still a very active research area. New models are being developed even now. Today’s finite element analysis codes provide excellent models for the analysis of plastic deformation of metallic materials, even though the latest models are yet to be implemented.

Plasticity models provide a mathematical relationship that characterizes the elasto-plastic response of materials. The choice of plasticity model depends on the experimental data available to fit the material constants. The basic requirements of a plasticity model are:

• Yield criterion
• Flow rule
• Hardening rule

Conventional plasticity models are classified as:

• Linear isotropic hardening models
• Linear kinematic hardening models
• Nonlinear isotropic hardening models
• Nonlinear kinematic hardening models

B. Linear isotropic hardening models

These models are appropriate for large strain, proportional loading situations. They are less preferred for cyclic loading since isotropic hardening model alone is incapable of describing cyclic behaviour that shows repeated cyclic deformation. However these models are capable of simulating complex cyclic behaviour when combined with kinematic hardening models.

C. Linear kinematic hardening models

They follow a linear hardening curve in cyclic loading situations. The hardening rule is given by

\[ d\alpha_i = cde_{ij} \]  \hspace{1cm} (1)

where

\[ d\alpha_i = \text{incremental back stress} \]
\[ de_{ij} = \text{incremental plastic strain} \]
\[ c = \text{material parameter} \]

They can describe stable loops in cyclic loading, including the Bauschinger effect. However, for a prescribed uniaxial stress cycle with a mean nonzero stress, they fail to distinguish between shapes of the loading and reverse loading hysteresis curves and consequently produce a closed loop with no ratchetting.

D. Non linear kinematic hardening models

They follow a smooth non linear hardening curve in cyclic loading situations. The hardening rule is given by

\[ d\alpha_i = \frac{2}{3}cde_{ij} - \gamma d\rho \]  \hspace{1cm} (2)

where

\[ \alpha = \text{back stress} \]
\[ dp = \text{accumulated plastic strain} \]
\[ \gamma = \text{material parameter} \]

They can simulate ratchetting and shakedown effects in an FEA simulation. Nonlinear kinematic hardening implies a shift (or movement) of the yield surface along a nonlinear path. It is similar to linear kinematic hardening except for the fact that the evolution law has a non linear term called recall term. Non linear kinematic hardening does not have a linear relationship between hardening and plastic strain. The non linear term is associated with the translation of the yield surface.

II. CHABOCHE MODEL

The Chaboche model is a type of non linear kinematic hardening model commonly used to simulate the plastic deformation of metals. Chaboche model is based on the well known von Mises yield criterion. The yield function is given by:

\[ F = \frac{3}{2} \left[ \{S\} - \{\alpha\} \right] ^T [M] \left[ \{S\} - \{\alpha\} \right] - R = 0 \]  \hspace{1cm} (3)

where

\[ \{S\} = \text{deviatoric stress tensor} \]
\[ \{\alpha\} = \text{back stress tensor} \]
\[ [M] = \text{matrix containing information on different yield strengths in different directions} \]
\[ R = \text{initial yield stress} \]
Experimental data and a special curve fitting tool are used to determine a set of material parameters for the Chaboche model in ANSYS (Version 14.5) general purpose finite element analysis code [12]. A third order Chaboche model is generally used, as it provides sufficient variation to calibrate the non linear behavior of the metal.

Chaboche model is expressed as [7, 8]

$$\alpha = \sum_{i=1}^{n} \alpha_i$$  \hspace{1cm} (4)

where

$$\alpha_i = \frac{2}{3} c_i \varepsilon_p^{\gamma} - \gamma_i \alpha \Delta \varepsilon$$  \hspace{1cm} (5)

$\varepsilon_p$ = plastic strain  
$c$, $\gamma$ = Chaboche material parameters

The first term in the equation is the hardening modulus and the second term is called the "recall term" which produces the nonlinear effect. The recall term incorporates the "fading memory effect" of the strain path and essentially makes the rule nonlinear in nature. The material parameter, $\gamma_i$, controls the rate at which hardening modulus decreases with increasing plastic strain.

Most of the above parameters can be evaluated from a stable hysteresis loop of the material obtained from a low cycle fatigue test at the required temperature and strain range. A stabilized hysteresis curve can be divided into three critical segments: the initial high modulus at the onset of yielding, the constant modulus segment at a higher strain range and the transient non linear segment (knee of the hysteresis curve). Chaboche proposed to use three decomposed hardening rules to improve the simulation of the hysteresis loops in these three segments. He suggested that the first rule ($\alpha_1$) should start hardening with a large modulus and stabilize very quickly. The second rule ($\alpha_2$) should simulate the transient non linear portion of the stable hysteresis curve. Finally, the third rule ($\alpha_3$) should be a linear hardening rule ($\gamma_3=0$) to represent the subsequent linear part of the ratcheting curve at a high strain range. The resulting yield surface center is

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3$$  \hspace{1cm} (6)

"Fig. 3", shows the details of third order Chaboche model drawn in the stress-strain space [13]. Ratchetting predictions can be improved by introducing a slight nonlinearity in the third rule by assigning a small value to $\gamma_3$, keeping other parameters the same. This small value does not introduce any noticeable change in the strain controlled stable hysteresis loop simulation. A non zero $\gamma_3$ does not have any effect on $\alpha_3$, but it changes the course of $\alpha_3$ and thereby of $\alpha_2$, which improves the uniaxial ratcheting simulation and prevents shakedown. The higher the value of $\gamma_3$, the third rule would reach its limiting state and, consequently, the earlier the steady rate of ratcheting would start.

III. CHABOCHE MODELING OF COPPER ALLOY

The copper alloy being investigated is a Cu-Cr-Zr-Ti alloy which has high thermal conductivity and high ductility. It is being used in the inner wall of the double walled rocket thrust chamber in the cryogenic rocket engine, being investigated in this study.

In order to evaluate the Chaboche parameters of the copper alloy at 900 K, curve fitting based on a nonlinear regression method is carried out using ANSYS implemented in a PC. As the first step for this analysis, it is required to clear the noise of the obtained strain controlled hysteresis loop from an LCF test. The test is conducted on the specimen at a strain rate of 3e-3 mm/mm/s and the strain range is kept as ± 0.75%. A smooth graph free from noise is required for determination of these parameters. "Fig. 4", shows the raw hysteresis loop and “Fig. 5", the idealized one after removing the noise.

A. Determination of Chaboche parameters

A third order Chaboche model is chosen for the study as it is reported to be sufficient for practical purposes. It requires six material parameters $C_1$, $\gamma_1$, $C_2$, $\gamma_2$, $C_3$, $\gamma_3$ and the initial yield stress $\sigma_0$. The parameters must be determined so that the model closely matches the material behaviour. The method for doing so is to evaluate initial values of the parameters data from the above loop and use that data with the curve fitting tool to determine final parameters that minimize the error between the data and the model predictions.

In this study, the initial values of all parameters, except $\gamma_3$, are determined from the loop in “Fig.5".
C1 should be a large value to match the plastic modulus at yielding. It is obtained from the slope of the initial linear portion of the hysteresis loop and is 330000 (N/mm²). C2 is obtained from the slope of the plastic portion and is 577 (N/mm²). C3 is obtained from the slope of the curvilinear portion and is 16000 (N/mm²). “Fig. 6”, shows the process of determination of C1, C2, and C3.

Chaboche curve fitting determines the final material constants by relating the experimental data to the Chaboche nonlinear kinematic hardening model. Curve fitting is performed interactively in ANSYS. The fitting method used by fitting tool is an iterative process and requires initial values for each of the parameters. The success and quality of the fit depends on how far the initial values are from the values that give a good fit. For fitting purposes, the individual components of back stress for the model (γ1, γ2 and γ3) are chosen to represent the three regions of the hysteresis loop.

In each region the corresponding C1 is chosen to approximately match the plastic modulus. The γi parameters are chosen to accommodate the history dependence [12] defined by

\[
\alpha_i = \frac{C_i}{y_i} \left[1 - 2\exp\left(-y_i (\varepsilon_{i}^p - (\varepsilon_{L}^p))\right)\right]
\]

where

\[\varepsilon_{i}^p = \text{plastic strain in the uniaxial direction}\]
\[\varepsilon_{L}^p = \text{strain limit of the hysteresis loop}\]

and the relationship [12] defined by

\[
\frac{C_1}{\gamma_1} + \frac{C_2}{\gamma_2} + \sigma_0 = \sigma_i - \frac{C_1}{2} \left[\varepsilon_i^p - \left(-\varepsilon_{L}^p\right)\right]
\]

where

\[\sigma_0 = \text{initial yield stress}\]
\[\sigma_i = \text{uniaxial yield stress}\]
\[\gamma_1 = \text{should be large enough so that the exponential term quickly diminishes, } \gamma_2 = \text{is calculated from the chosen } C_2\text{ and the ratio } C_2/\gamma_2\text{ that satisfies equation given above. } \gamma_3\text{ is assigned a small positive value since it does not enter the above equations.}\]

The initial yield stress of the material is fixed and does not affect the error minimization performed by the curve fitting tool. Curve fitting is done by trial and error until a correct match with the experimental data is obtained. A plot obtained as a result of performing curve fitting inside ANSYS is shown in “Fig. 7” which shows a good fit. Table 1 shows the initial and final parameters for the alloy.

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IV. CYCLIC STRESS ANALYSIS OF A BLOCK

Cyclic stress analysis of a simple cubical block of unit dimension is carried out using a single SOLID185 element (8
noded brick element with 3 dof per node viz. UX, UY and UZ) with quarter symmetry boundary conditions and uniaxial loading in the Y direction. The purpose of this study is to try out various metal plasticity models in ANSYS to choose the best one suited for cyclic stress analysis. Elastic properties for copper alloy are Young’s modulus of 110660 N/mm² and Poisson’s ratio of 0.3. "Fig. 8", shows the finite element model of the block.

Only a single element is used in the model to reduce solution time. Analysis is done using MISO (Multilinear Isotropic Hardening), KINH (Multilinear Kinematic Hardening), Chaboche and combined Chaboche + Multilinear Isotropic hardening models (CHAB+MISO). Analysis was done for cyclic displacement loading.

Results showed that MISO model could not capture nonlinear nature of cyclic stress strain curve but the model was found successful in capturing the cyclic hardening/softening behavior. "Fig. 9" shows the axial stress-strain graph for the block using MISO model. "Fig. 10" shows the axial stress-strain plot using KINH model. This model showed only linear cyclic response and no cyclic hardening/softening was evident.

When Chaboche model was applied on the block, cyclic response captured was non-linear but the phenomenon of cyclic hardening/softening was not observed. "Fig. 11" shows the plot for Chaboche model. Actual cyclic response was predicted when combined CHAB+MISO model was used. This model captured the non-linearity and cyclic hardening/softening behavior beautifully. "Fig. 12" shows the axial stress-strain plot when combined CHAB+MISO model was used.
V. CYCLIC STRESS ANALYSIS OF THRUST CHAMBER

The thrust chamber is the subsystem of a rocket engine in which combustion occurs. "Fig. 13," shows the geometrical details of the chamber. Analysis of thrust chamber cross section is carried out using MISO, KINH and MISO+CHAB models for inner wall and BKIN model for outer wall. Two dimensional plane strain finite element model of the cross section at throat is employed for analysis of the chamber. "Fig. 14," shows cross section model. Only one half of one coolant channel is modeled and analyzed to exploit cyclic symmetry.

Analysis is done for two successive hot tests. Initially a thermal conduction analysis is done with the specified wall surface temperatures for the inner and outer walls to get the steady state temperature distribution during the hot run at all interior nodal points as shown in "Fig. 15." For these nodal temperatures and the specified gas pressure and coolant pressure loads, a cyclic stress analysis is subsequently carried out. Thermo - mechanical loads expected on the chamber during one complete cycle of operation of the engine [1] are applied in 7 separate load steps as listed in table II.

Results indicate that ratchetting is not observed at inner surface of inner wall at mid channel (location “A” shown in “Fig.14,” which is the most critical location being prone to cracking) when analysis is carried out using KINH model. "Fig. 17," shows the hoop stress-strain variation using this model. Smooth stress strain response is not seen when MISO model is used. "Fig. 16," shows the hoop stress-strain variation for this case. MISO+CHAB model showed the best stress strain response compared to other models. "Fig. 18," shows the hoop stress-strain variation using this model.
VI. CONCLUSION

In this paper, a detailed study of constitutive modeling of a special copper alloy using Chaboche nonlinear kinematic hardening plasticity model has been performed based on LCF tests conducted in a high temperature UTM. Chaboche parameters of the alloy are calibrated using a curve fitting technique through nonlinear regression technique available in ANSYS 14.5. A large number of trials were required to get a good fit of computed cyclic stress strain data to experimental results. Cyclic loading on a simple block was carried out to show the effectiveness of this model when combined with MISO model. It is possible to successfully capture cyclic hardening and smooth nonlinear stress strain behaviour with this combination for displacement loading.

These parameters are subsequently used for cyclic stress analysis of a rocket engine thrust chamber. Analyses with the conventional MISO and KINH models showed their failure in accurately capturing the cyclic response of the material. It was found that by using a combination of Chaboche and MISO models (combined isotropic and kinematic hardening model) better stress strain response is obtained compared to other models. It has the additional capability to capture cyclic hardening of the material and ratchetting effects. In this model combination, the Chaboche model represents the stabilized response while the MISO model simulates the cyclic hardening response of the material. This model is therefore, well suited to perform cyclic life prediction of the chamber considering low cycle fatigue, thermal ratchetting and creep.

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References