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Conflicts resolving by symmetry breaking

Nikolay Raychev

Abstract - In this report is examined an algorithm for conflict resolution by symmetry breaking, which requires the exchange of only one pair of qubits in order to be reached to a solution of the problem related to conflict resolution by symmetry breaking.

Index Terms— boolen function, circuit, composition, encoding, gate, quantum.

1 Introduction

The quantum logarithm for conflict resolution by symmetry breaking shows the ability to detect correct decisions, which can not be similarly achieved through the laws of the classical physics.

The algorithms for conflict resolution have been implemented as quantum logic circuits, which include paired qubits, which can be represented as unitary matrices. The solution can be found through the restrictions, on which these matrices must comply. The algorithm is implemented and simulated with the developed by the author of the article quantum simulator [11, 12, 13, 14, 15].

2 Quantum Algorithm for Conflict Resolution by Symmetry Breaking

One of the many primitives used by the distributed algorithms is the primitive for conflict resolution by symmetry breaking. This primitive is used when there are two nodes with the same values, its purpose is to set a value of 1 to one of the nodes, and a value of 0 to the other. The algorithms for conflict resolution by symmetry breaking usually rely on some form of chance. For example, it may be necessary to go round the circle in a cycle, in which each node selects a random value between 0 and 1, after which the result shall be determined. Each time, when the random selection of the nodes coincides, the cycle is repeated. Once the nodes have already chosen different random values in a given cycle, one of the nodes receives a value of 1, and the other receives a value of 0. Until a satisfactory solution is found to this problem, may be needed hundreds of inversions of the cycle in each message. The quantum algorithms are able to coordinate better similar problems in many esoteric cases such as this one.

Solution with a quantum algorithm

For the solution of this problem is needed a quantum circuit, which should comply with the following two conditions:

1. Symmetry. On both sides must be applied identical quantum gates. All the operations, which are performed on the lines A1 and A2 must be performed also on the lines B1 and B2.
2. Anti correlation. The exit should not contain amplitudes in the states, where A1 and B1 are the same.

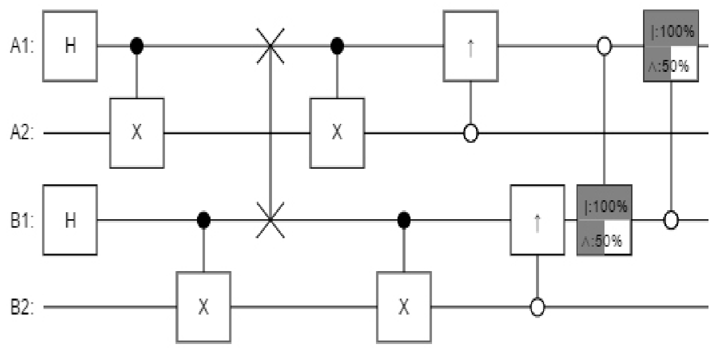

Figure 1: The composed with ISISKA - full quantum circuit

Implementation of the circuit from right to left.

Each side must include:

1. Entangled pair of qubits
2. Application of a Swap gate - half of the pair for the relevant half of the other side.
3. Application of controlled NOT gate - CNOT.
4. Application of controlled $\frac{1}{2}$ CNOT gate on the opposite side.
5. A1 and B1 have different values in all cases.

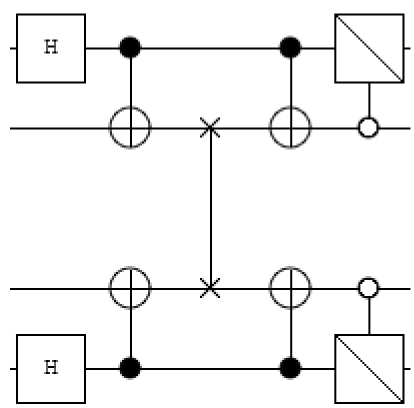

Figure 2: Component circuit diagram, showing the symmetry

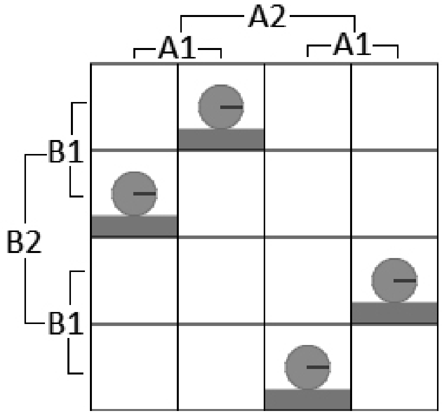

Figure 1.3: Diagram of the amplitudes, forming the final state, the cells in which A1 and B1 have the same values - they do not have any amplitude

It should be noted that the described solution would not work completely identically upon implementation of a quantum computer. ISISKA approximatively minimizes the interferences, which would have been optionally bigger upon implementation with a quantum computer. However, the thus established quantum algorithm (scheme) ensures at 100%, that the sought solution will be found only after one cycle, as is also the goal. In addition, the algorithm needs only one exchange of qubit.

Working solution

Detailed examination of the solution and monitoring of the development/change of states of the system during the implementation of the circuit. In order to simplify the verification will be ignored the factors from $\sqrt{2}$.

The initial state is: $|0000\rangle$

The first operation, which each side performs is: the creation of a twisted pair. In this way the first and the last two qubits are initialized in the same superposition on two- $|11\rangle$ or on: two-$|00\rangle$: $|0000\rangle$

$\rightarrow bell(|00\rangle+|11\rangle)\otimes(|00\rangle+|11\rangle)$

$=|0000\rangle+|0011\rangle+|1100\rangle+|1111\rangle$

At the second step, each of both sides sends its second qubit to the other side, in this way their second qubits are effectively transferred. This is the only communication step in the algorithm:

$|0000\rangle+|0011\rangle+|1100\rangle+|1111\rangle$

$\rightarrow swap|0000\rangle+|0110\rangle+|1001\rangle+|1111\rangle$

At the third step, both sides apply controlled NOT(CNOT) operation on their own qubit, and the one they have received, this enables switching of the received qubit, where the value of the own is: 1

$|0000\rangle+|0110\rangle+|1001\rangle+|1111\rangle$

$\rightarrow cnot|0000\rangle+|0111\rangle+|1101\rangle+|1010\rangle$

At the end, both sides shall apply square root of NOT gate, i.e. ↑ **Gate / Splitter,** which performs 90-degree rotation around the X basis (sends $|0\rangle$ to $(1-i)|0\rangle+(1+i)|1\rangle$ and $|1\rangle$ to $(1-i)|0\rangle+(1+i)|1\rangle$) to store the value of its qubit when the received qubit has the value of 0:

$|0000\rangle+|0111\rangle+|1101\rangle+|1010\rangle$

$\rightarrow split1((1-i)|0000\rangle+(1+i)|0010\rangle)+|0111\rangle+|1101\rangle+((1-i)|1010\rangle+(1+i)|1000\rangle)$

$=(1-i)|0000\rangle+(1+i)|0010\rangle+|0111\rangle+|1101\rangle+(1-i)|1010\rangle+(1+i)|1000\rangle$

$\rightarrow split2(-i|0000\rangle+|1000\rangle)+(|0010\rangle+i|1010\rangle)+|0111\rangle+|1101\rangle+(-i|1010\rangle+|0010\rangle)+(|1000\rangle+i|0000\rangle)$

$=(i-i)|0000\rangle+(1+1)|1000\rangle)+(1+1)|0010\rangle+(i-i)|1010\rangle)+|0111\rangle+|1101\rangle$

$=|1000\rangle+|0010\rangle+|0111\rangle+|1101\rangle$

$=(|1\_0\_\rangle+|0\_1\_\rangle)\otimes(|\_0\_0\rangle+|\_1\_1\rangle)$

As can be seen from the above described scheme, at the final state of the first and third qubit - both sides always differ

3 Conclusion

This report describes the quantum algorithm that require the exchange of only a single pair qubits to reach the solution to the problem associated with the resolution of conflicts by symmetry breaking.

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