Comparison of Modified PTS and SLM Techniques for PAPR Reduction in MIMO OFDM Systems

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Abstract— Communication is one of the important aspects of life. With the advancement in age and its growing demands, there has been rapid growth in the field of communications. Signals, which were initially sent in the analog domain, are being sent more and more in the digital domain these days. For better transmission, even single – carrier waves are being replaced by multi – carriers. Multi – carrier systems like CDMA and OFDM are now – a – days being implemented commonly. In the OFDM system, orthogonally placed sub – carriers are used to carry the data from the transmitter end to the receiver end. Presence of guard band in this system deals with the problem of ISI and noise is minimized by larger number of sub – carriers. But the large Peak – to – Average Power Ratio of these signal have some undesirable effects on the system.

In this thesis I have focused on learning the basics of an OFDM System and have undertaken various methods to reduce the PAPR in the system so that this system can be used more commonly and effectively.

Index Terms— Interference, ISI, MIMO, OFDM, PAPR, PTS, SLM.

1 INTRODUCTION

FDM was first introduced in the year 1966. Here in this method large numbers of parallel narrow band subcarriers are used instead of a single wide band carrier in order to transfer information. It is an easy and efficient way to deal with multipath communications. It has also a very accurate narrow band interference. OFDM basically is a combination of multiplexing and modulation. Multiplexing means independent signals are produced by different sources so there should be some method in order to share the entire spectrum with these users. In OFDM, multiplexing technique is applied to independent signals and each of these signals are the subset of a single main signal. In OFDM the main signal itself is first split into independent channels, modulated by data and then re-multiplexed to create an OFDM carrier. OFDM is a special case of Frequency Division Multiplexing where a whole bunch of data in a single stream is divided into number of small sub streams as shown in Fig(1). The carrier signals are orthogonal to each other. Condition for orthogonality is given by two factor-cross correlation and auto correlation. When the cross correlation of two carriers is zero and auto correlation is maximum then the signal is said to be orthogonal. In OFDM, sub-carriers frequencies are selected such that each sub-carrier is perpendicular to the other one, so by maintaining this condition the crosstalk between the sub channels are eliminated. This simplifies the design of both transmitter and receiver unlike FDM where a separate filter is required in each sub-channel.

K is a positive integer with a numerical value 1. Therefore for M subcarriers the total pass band bandwidth is

\[ B = N \cdot K \Delta f \text{ Hz} \]  

2.1 FFT Algorithm Implementation

DSP components are generally low cost and they efficiently calculate FFT so OFDM can be used for wideband communications. OFDM employs low symbol rate modulation so it suffers very less intersymbol interference(ISI) caused by multipath propagation. It is beneficial to transmit a number of low rate streams in parallel instead of single high rate stream, since the duration of each symbol is long it is feasible to insert a guard interval between OFDM symbols thus eliminating ISI.

2 CONDITION FOR ORTHOGONALITY AND SYSTEM MODEL

If N subcarriers are used and each subcarrier is modulated using M alternative symbols then the OFDM symbol alphabet consist of M^N combined symbols. Condition of orthogonality is given as

\[ \Delta f = k/T_u \text{ Hz} \]  

\[ T_u = \text{receiver side window size} \]  

Subcarriers spacing

2.2 Channel coding and interleaving

Interleaving means inserting frequencies or blocks in between. OFDM uses channel coding, frequency and time interleaving. Frequency interleaving improves immunity against fading of signal. Simultaneously time interleaving ensures bit stream rather than being concentrated. The fundamental idea to go for interleaving is to spread the error out in bit
stream that is presented to the error correction decoder because such decoders will be unable to correct the errors in all the bits. Another criteria maintained in OFDM system is space diversity that is to transmit the same signal simultaneously over the same channel frequency which is called single frequency network.

2.3 System Model

In OFDM carrier signal is the sum of a number of orthogonal sub-carriers, with baseband data on each sub-carrier being independently modulated commonly using some type of Quadrature Amplitude Modulation (QAM) or Phase Amplitude Modulation (PSK). This composite baseband signal is typically used to modulate a main radio frequency (RF) carrier. If \( s(n) \) is a serial stream of binary digits then by inverse multiplication these are first demultiplexed into \( N \) parallel streams and each one is mapped to a symbol stream using some modulation constellation type like Quadrature Amplitude Modulation (QAM) or Phase Amplitude Modulation (PSK). The constellation may vary so some stream may have higher bit rate than others. An inverse FFT is applied on each set of symbols, giving a set of complex time domain samples. These samples are quadrature mixed to passband in the standard form. The real and the imaginary components are connected to the digital to analog converter (DAC). Analog signals are then send to modulate cosine and sine wave at center frequency \( f_c \). The signals are then summed up to give transmission signal \( s(t) \) as shown in Fig (2).

At the receiver side, receiver picks up the signal \( r(t) \) which is quadrature mixed down to baseband using cosine and sine waves at \( f_c \). This also creates signals centered at \( 2f_c \) so a low pass filter is used to eliminate higher frequencies. The baseband signal is then passed to analog to digital converter (ADC) and FFT to convert back to frequency domain signal as shown in Fig (2).

\[
\nu(t) = \sum X(k) e^{j2\pi kt/T}, 0 \leq t < T. \tag{3}
\]

where \( X(k) \) = data symbols

\[ T=\text{OFDM symbol time} \]

Subcarrier spacing of \( 1/T \) make the sub carriers perpendicular to each other over each symbol period. This property is expressed as

\[
1/T \int (e^{j2\pi k_1 t/T}) \ast (e^{j2\pi k_2 t/T}) dt \tag{4}
\]

\[
= 1/T \int (e^{j2\pi (k_1 - k_2) t/T}) dt \tag{5}
\]

\[
= \delta k_1 k_2. \tag{6}
\]

\( * \) is a complex conjugate operator and \( \delta \) is kronecker delta.

To avoid ISI a guard length of \( T_g \) is inserted in OFDM block. Cyclic prefix (CP) is transmitted such that the signal in the interval \( -T_g \leq t \leq 0 \) equals the signal in the interval \( (T - T_g) \leq t \leq T \). OFDM with CP is

\[
\nu(t) = \sum X(k) e^{j2\pi kt/T}, -T_g \leq t < T \tag{7}
\]

Transmitted signal is

\[
s(t) = R(\nu(t) e^{j2\pi f_c t}) \tag{8}
\]

\[
= \sum |X(k)| \cos \left( 2\pi \left( fc + \frac{k}{T} \right) t + \arg\{X(k)\} \right). \tag{9}
\]

3 MULTIPLE INPUT MULTIPLE OUTPUT (MIMO)

OFDM technique provides security of the data transmission while MIMO provides guarantee that the data is transmitted and received at the receiving end.

MIMO transmits independent data \((x_1, x_2, \ldots, x_N)\) on different transmit antennas simultaneously and in the same frequency band. At the receiver end MIMO decoder uses \( M \geq N \) antennas as shown in Fig (3). Multiple data streams are transmitted in parallel from different antennas so there is a linear increase in the throughput with every pair of antennas added to the system. They do not increase bandwidth to increase throughput. They simply do it by increasing the number of paths. If \( N \) is the number of receiving antennas then representing signal received by each antenna as \( r_j \) we have

\[
r_1 = h_{11} x_1 + h_{12} x_2 + \ldots + h_{1N} x_N \tag{10}
\]

\[
r_2 = h_{21} x_1 + h_{22} x_2 + \ldots + h_{2N} x_N \tag{11}
\]

\[
r_N = h_{N1} x_1 + h_{N2} x_2 + \ldots + h_{NN} x_N. \tag{12}
\]

In making their path from transmitter to receiver the independent signals are all combined. Traditionally this combination has been treated as interference however by treating the channel as matrix, independent transmitted streams \( x_i \) can be recovered by assigning every individual channel a weight such as \( h_{ij} \).

2.4 Mathematical Desription

Let the low pass equivalent OFDM signal is
4 PEAK TO AVERAGE POWER RATIO (PAPR)

The major problem in OFDM signal is an increased PAPR, requiring linear transmission circuit which suffers from poor power efficiency. PAPR also increases system complexity. The transmitted signal is a combination of a number of sinusoidal waves which can add coherently and results in a high peak magnitude but the average value of the signal is still maintained at a low level due to destructive interference and therefore PAPR ratio is high. As a result it results in high power consumption.

4.1 Peak Reduction Carriers (PRC)

A reduction in PAPR is achieved by adding extra carriers called PRC. The phase and amplitude are varied to minimize the overall PAPR. The original information remains unaffected and can be obtained back by normal decoding process. At the receiving end either these extra bits can be discarded or can be used for error detection method.

The frequency and relative position of PRC can be varied with respect to the information carrier depending upon the application. An optimal setting of PRC means an appropriate combination of phase and amplitude which can be used to achieve the lowest PAPR of the entire OFDM system. This method is used only for carriers ranging up to 16 numbers.

4.2 Signal Scrambling Techniques

Instantaneous output of an OFDM system often has large fluctuations compared to traditional single carrier system. If power amplifiers, ADC, DAC are not operating in their linear dynamic range then peak signal goes into non-linear region of the devices at the transmission side and hence intermodulation distortion occurs.

PAPR = peak output power/average output power

\[ PAPR = \log_{10}\frac{\text{max}[|x|^2]}{E[|x|^2]} \]  

E[.] = expected value

\[ x(n) = \text{transmitted signal obtained by IFFT} = 1/\sqrt{N}\sum_{k=0}^{N-1} x_k \cdot W_{n}^{nk} \]  

For an OFDM system with N subcarriers the peak power of the received signal is N times the average power when phase values are kept same. PAPR is dependent on modulation schemes like BFSK, QPSK, QAM and number of sub carriers. One of the solution to PAPR problem is provided by signal scrambling technique. In this method OFDM signal is scrambled with different scrambling sequences and then it selects the one which is having the smallest PAPR value for transmission. In this paper two scrambling techniques are used.

1) Selective Mapping Method

Selective Mapping or SLM scrambling applies rotation to all the sub carriers independently. It has no restriction to the number of carriers and type of modulation. In SLM first M statistical independent sequences which represent the same information are generated and next these M statistically independent data blocks \( S_m = [\text{seq}(0), \text{seq}(1), \ldots, \text{seq}(N-1)]^T, m = 1, 2, \ldots, M \) are then forwarded to IFFT operation simultaneously. At the receiving end OFDM symbols \( x_m = [x_1, x_2, \ldots, x_N]^T \) in discrete time domain are obtained and then the PAPR of these M vectors are calculated separately. Then \( x_d \) with the smallest PAPR will be selected for the final serial transmission. Data blocks \( S_m = [\text{seq}(0), \text{seq}(1), \ldots, \text{seq}(N-1)]^T, m = 1, 2, \ldots, M \) are statistically independent to each other as shown in Fig(4).

2) Partial Transmit Method

PTS divides the original OFDM sequence into several sub-sequences and each sub sequence is multiplied by different weights until the optimum value is reached. Data information in frequency domain \( X \) is separated into \( V \) non-overlapping sub blocks and each sub block vector has the same size \( N \). Hence for every sub block it contains \( N/V \) non-zero elements and the rest part is appended with zero. In the time domain signal obtained by IFFT one suitable factor combination is selected which makes the result optimum. Here \( b \) is selected which is assigned as phase value as shown in Fig(5).

3) Modified PTS Scheme

As discussed above, the traditional PTS method is a traversal algorithm, which requires all possible phase values to be evaluated, that actually constrain the real application. For this reason, a suboptimal algorithm is going to be introduced to reduce the number of phase patterns. It can reduce the complexity effectively and combine the advantage of
small degree of degradation compared to the optimum, although the solution retrieved from this algorithm is not the optimal solution.

The detailed steps are shown as follows:
1) Divide $N$ sub-carriers into $V$ non-overlapping sub-blocks.
2) Assume that $\mathbf{b}=[b_1, b_2, \ldots, b_V]=[1, 1, \ldots, 1]$, and then calculate the PAPR value of an intermediate sequence $x'$ in time domain, write it as $\text{PAR}_x'$. Initialize $\text{index}=1$, which represents the subscript of $\text{bind}$ex.
3) Assume that $\text{bind}$ex$=-1$, and calculate the PAPR of this new sequence, write it as $\text{PAR}_x$.
4) If $\text{PAR}_x > \text{PAR}_x'$, let $\text{bind}$ex$=-1$, otherwise, $\text{PAR}_x' = \text{PAR}_x$, and make $\text{index} = \text{index}+1$.
5) If $\text{index} < V+1$, go to step 3), otherwise jump to step 6). The iteration continues until $\text{index} = V$.
6) Obtain sub-optimal weighting factor $\mathbf{b}$ and corresponding PAPR is min $(\text{PAR}_x, \text{PAR}_x')$.

Adopting iterative algorithm for searching suboptimal weighting factor, only $V$-steps calculation are needed. In each step, the IFFT calculation only be performed for corresponding one sub-block rather than calculating for all $V$ sub-blocks, by this means, it can reduce complexity of the calculation significantly.

5 Simulation Results

Fig(6) shows the simulation result for Selective Mapping for different phase values. It can be observed from the graph that as the M values are increased there is a drastic change in the PAPR values for an OFDM signal. PAPR decreases as the number of phase increases. Here simulation is done using maximum value of $M = 32$. Without applying any technique the original PAPR is nearly 12.2 dB. But after applying SLM technique and by taking $M$ value = 32, the PAPR value is reduced to 6.8 dB.

Fig(7) shows the PAPR values of an OFDM signal using PTS technique for phase factors =32. The simulation result shows that as the V values are increased, PAPR values are decreased for the given original OFDM signal. As seen from the Fig(7) initial PAPR value without using any technique is nearly 11.3 dB and after using PTS technique for the same signal with phase value = 32 the PAPR is reduced to nearly 4.8 dB.

Fig(8) shows the comparison between SLM and PTS techniques. The graph shows that PTS is a better method in the reduction of PAPR values for a given phase value. Fig(9) shows that though suboptimal PTS reduces complexity of the design but it lags in effective PAPR reduction when compared to ideal PTS for the same value of $V$.

6 Conclusion

So in this paper we studied and analysed the matlab simulation results of two scrambling techniques that is SLM and PTS and found PTS technique is better in reducing PAPR when compared to SLM. We also found that with changing
and increasing phase values, PAPR value decreases.

Fig(8)

Fig(9)

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