# Bianchi Type-III String Cosmological Models in The Presence of Magnetic Field in General Relativity

Kandalkar S.P.[1], Samdurkar S.W.[2] and Gawande S.P.[3]

[1]Department of Mathematics, Govt. Vidarbha Institute of Science & Humanities, Amravati (India).

[1] spkandalkar2004@yahoo.com,[2] seema_samdu@yahho.co.in.[3] sskandalkar2010@yahoo.co.in

**Abstract**— In this paper we have examined Bianchi type-III string cosmological model in the presence of magnetic field. To get determinate solutions, the Einstein's field equations have been solved for two cases (i) Reddy string and (ii) Nambu string. The physical and geometrical behaviour of these models are discussed.

**Index Terms**— Bianchi type-III model, Reddy string, Nambu string, Magnetized
.

## 1 Introduction

At very early stage of evolution of universe, it is assumed that during the phase transition, the symmetry of universe is broken spontaneously. It can give rise to topological stable defects such as domain walls, strings and monopoles (Kibble1976).Of all these cosmological structures, cosmic string play a vital role in structure formation in cosmology (Zel'dovich1990).It is believed that the vacuum strings give rise to density fluctuations sufficient in formation of galaxies (Zel'dovich1990).The cosmic strings have their stress-energy coupled to the gravitational field. Therefore, the study of gravitational effects of such strings will be of interest.

The present day configuration of the universe is not contradicted by large scale network of strings in the early universe. The general relativistic formation of cosmic strings, are given by Letelier (1979 1983) and Stachel (1980).In string theory, the myriad of particle type is replaced by single fundamental building block, a string. These strings can be closed, like a loop or open, like a hair. As the string moves through time it tress out a tube or a sheet, according whether it is close or open. Since strings are not observed at present time of evolution of universe one can illuminate strings and end up with cloud of particles.

String cosmological models have been studied by many authors. A number of general relativistic exact solution were investigated for Bianchi type II,VI0 ,VIII & IX string cosmological models by Krori et al.(1990). A class of cosmological solutions of massive strings for Bianchi type VI0 space time has been obtained by Chakraborty (1991). Roy and Banerjee (1995) have investigated some LRS Bianchi type II string cosmological models which representgeometrical and massive strings. Wang (2004 2005 2006) has investigated and discussed some cosmological models and their physical implication in some Bianchi type space times.

The magnetic field has important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomenon has been studied in many papers. Melvin (1975) has pointed out that during the evolution of universe, the matter is in highly ionized state and is smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Therefore considering the presence of magnetic field in strings universe is not unrealistic and has been investigated by many authors (Banerjee et al. 1990;Shri Ram and Singh1995; Singh and Singh1999; Baliand Upadhaya2003).Banerjee et al.(1990) investigated some cosmological solutions of massive strings for Bianchi type I space time in the presences and absence of magnetic field. Tikekar and Patel (1992) obtained some exact solutions of massive string of Bianchi type III space time presence and absence of magnetic field. Recently Upadhaya and Dave (2008) have investigated Bianchi type III massive string cosmological model in the presence and absence of magnetic field, under the assumption that the expansion $(\theta)$ in the model is proportional to the shear $(\sigma)$ which lead $C = A^n$ and have obtained particular solution for $n = 1$.

In this paper we have investigated Bianchi type III cosmological model in the presence and absence of magnetic field. To obtained exact solution field equations, we have considered Reddy string and Nambu string. The physical behaviour of models are also discussed.

## 2. The Metric and Field Equations

We consider the Bianchi type III space time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2 \quad (1)$$

where $A, B$ and $C$ are functions of $t$ only and $\alpha$ is a constant.

The energy momentum tensor for a cloud of string dust with magnetic field along the z-direction of the string is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + \frac{1}{4\pi}\left( g^{lm} F_{il} F_{jm} - \frac{1}{4} g_{ij} F_{lm} F^{lm} \right) \quad (2)$$

where $u_i$ and $x_i$ satisfy the conditions

$$u_i u^j = -x_i x^j = -1 \quad u^i x_i = 0 \quad (3)$$

$\rho$ is the energy density for a cloud string with particles attached to them, $\lambda$ is the string tension density, $u^i$ is the four-velocity of the particles, and $x^i$ is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u^i = (0,0,0,1)\ ,\ \ x^i = (0,0,\frac{1}{C},0), \tag{4}$$

The particle density of the configuration is given by

$$\rho = \rho_p + \lambda \tag{5}$$

The electromagnetic field tensor $F_{ij}$ has only the non-zero component $F_{12}$ because the magnetic field is assumed to be along the $z$-direction. Subsequently Maxwell equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad \text{and} \quad \left[F^{ij}(-g)^{1/2}\right]_{;k} = 0$$

lead to

$$F_{12} = Ke^{-\alpha x} \tag{6}$$

where $K$ is a constant so the magnetic field depends upon the space coordinate x only. From (2), (4) and (6), it follows that $F_{14} = 0$.

The Einstein's field equation

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{7}$$

for the metric (1) lead to the following system of equations:

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} - \frac{K^2}{A^4} = 8\pi\rho \tag{8}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{K^2}{A^4} = 0 \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{K^2}{A^4} = 0 \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} - \frac{K^2}{A^4} = 8\pi\lambda \tag{11}$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{12}$$

where a dot (.) over a variable denotes ordinary differentiation with respect to time t. From equation (12), we have two cases $\alpha = 0$ and $A = B$. When $\alpha = 0$, the metric (1) degenerates into Bianchi type I. Since we have considered Bianchi III symmetry, we assume that $\alpha$ is non-zero and $A = B$. Thus the field equations (8)-(11) reduces as

$$2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} - \frac{K^2}{A^4} = 8\pi\rho \tag{13}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} - \frac{K^2}{A^4} = 8\pi\lambda \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{K^2}{A^4} = 0 \tag{15}$$

## 3 Solutions of the Field Equations

The field equations (13) -(15) are a system of three equations with four unknown parameters $\rho, \lambda, A$ and $C$. One additional constraint relating these parameter is required to obtain explicit solutions of the system. We assume that the expansion $(\theta)$ is proportional to the shear $(\sigma)$. This condition lead to

$$C = A^n \tag{16}$$

where $n$ is a constant.

To obtain exact solutions, we solve the field equations for the following two cases.

### 3.1 Case I: Reddy String

In this case

$$\rho + \lambda = 0 \tag{17}$$

From (13), (14) and (17) we obtain

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{A}}{A} - \frac{\alpha^2}{A^2} - \frac{K^2}{A^4} = 0 \tag{18}$$

Using (16), the above equation reduces to

$$\frac{\ddot{A}}{A} + (n+1)\frac{\dot{A}^2}{A^2} = \frac{\alpha^2}{A^2} + \frac{K^2}{A^4} \tag{19}$$

Let $\dot{A} = f(A)$ which impliesthat $\ddot{A} = ff'$, where $f' = \frac{df}{dA}$.

Hence (19) can be written as

$$\frac{d}{dA}\left(f^2\right) + \frac{2(n+1)}{A}f^2 = 2\frac{\alpha^2}{A} + 2\frac{K^2}{A^3} \tag{20}$$

By integrating (20) we find

$$f^2 = \frac{\alpha^2}{n+1} + \frac{K^2}{n}A^{-2} + MA^{-2(n+1)} \tag{21}$$

where $M$ is integration constant. Therefore, we have

$$\left(\frac{dA}{dt}\right)^2 = \frac{\alpha^2}{n+1} + \frac{K^2}{n}A^{-2} + MA^{-2(n+1)} \tag{22}$$

Thus the metric (1) reduces to the form

$$ds^2 = -\left(\frac{dt}{dA}\right)^2 dA^2 + A^2dx^2 + A^2e^{-2\alpha x}dy^2 + A^{2n}dz^2 \tag{23}$$

$$= -\frac{dT^2}{\frac{\alpha^2}{n+1} + \frac{K^2}{n}A^{-2} + MA^{-2(n+1)}} + T^2dx^2 + T^2e^{-2\alpha x}dy^2 + T^{2n}dz^2 \tag{24}$$

where $A = T, x = X, y = Y, z = Z$

In the absence of magnetic field i.e. when $K \to 0$ then the metric (24) reduces to the form

$$ds^2 = -\frac{dT^2}{\frac{\alpha^2}{n+1} + MA^{-2(n+1)}} + T^2dx^2 + T^2e^{-2\alpha x}dy^2 + T^{2n}dz^2 \quad (25)$$

### 3.1.1 Geometrical and physical significance of model

The energy density $(\rho)$, the string tension $(\lambda)$, the particle density $(\rho_p)$, the scalar of expansion $(\theta)$ and the shear $(\sigma)$ for the model (24) are given by

$$\rho = \frac{1}{8\pi}\left[\frac{n\alpha^2}{(n+1)T^2} + \frac{(n+1)K^2}{nT^4} + \frac{(2n+1)M}{T^{2(n+2)}}\right] \quad (26)$$

$$\lambda = -\frac{1}{8\pi}\left[\frac{n\alpha^2}{(n+1)T^2} + \frac{(n+1)K^2}{nT^4} + \frac{(2n+1)M}{T^{2(n+2)}}\right] \quad (27)$$

$$\rho_p = \frac{1}{8\pi}\left[\frac{2n\alpha^2}{(n+1)T^2} + \frac{2(n+1)K^2}{nT^4} + \frac{2(2n+1)M}{T^{2(n+2)}}\right] \quad (28)$$

$$\theta = (n+2)\left[\frac{\alpha^2}{(n+1)T^2} + \frac{K^2}{nT^4} + \frac{M}{T^{2(n+2)}}\right]^{1/2} \quad (29)$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{\alpha^2}{(n+1)T^2} + \frac{K^2}{nT^4} + \frac{M}{T^{2(n+2)}}\right] \quad (30)$$

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = const \quad (31)$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}$$

$$= \frac{1-n}{n+2} + \frac{3\left(\frac{K^2}{nT^4} + \frac{M(n+1)}{T^{2(n+2)}}\right)}{(n+2)\left(\frac{\alpha^2}{(n+1)T^2} + \frac{K^2}{nT^4} + \frac{M}{T^{2(n+2)}}\right)} \quad (32)$$

From the (32), we observe that

$$q > 0 \ \ if \ \ \frac{1 - \frac{n(n+1)M}{\alpha^2 T^{2(n+1)}}}{1 + \frac{(n+1)}{\alpha^2}\left(\frac{K^2}{nT^2} + \frac{M}{T^{2(n+1)}}\right)} + \frac{n}{3} > \frac{4}{3} \quad (33)$$

and

$$q < 0 \ \ if \ \ \frac{1 - \frac{n(n+1)M}{\alpha^2 T^{2(n+1)}}}{1 + \frac{(n+1)}{\alpha^2}\left(\frac{K^2}{nT^2} + \frac{M}{T^{2(n+1)}}\right)} + \frac{n}{3} < \frac{4}{3} \quad (34)$$

The energy condition $\rho \geq 0$ implies that

$$\frac{(n+1)K^2}{nT^2} + \frac{(2n+1)M}{T^{2(n+1)}} \geq \frac{-n\alpha^2}{n+1} \quad (35)$$

The particle density $(\rho_p)$ and tension density $(\lambda)$ of the cloud string vanish asymptotically in general if $(n+2) > 0$. The expansion in the model stops when $n = -2$. The model starts expanding with a big bang at $T = 0$ and the expansion in the model decreases as time increases if $(n+2) > 0$. The model (24) has singularity at $T = 0$. The physical parameters $\rho, \lambda, \rho_p$ are infinite at the singularity $T = 0$ and decreases as $T \to \infty$. The energy density and expansion in the model decreases rapidly in the presence of magnetic field. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$. Hence the model does not approach isotropy for large value of $T$. For the condition $\frac{1 - \frac{n(n+1)M}{\alpha^2 T^{2(n+1)}}}{1 + \frac{(n+1)}{\alpha^2}\left(\frac{K^2}{nT^2} + \frac{M}{T^{2(n+1)}}\right)} + \frac{n}{3} < \frac{4}{3}$, the solution gives accelerating model of the universe and for the condition $\frac{1 - \frac{n(n+1)M}{\alpha^2 T^{2(n+1)}}}{1 + \frac{(n+1)}{\alpha^2}\left(\frac{K^2}{nT^2} + \frac{M}{T^{2(n+1)}}\right)} + \frac{n}{3} > \frac{4}{3}$, our solution represents decelerating model of the universe and $q$ approaches at value -1 when $\frac{2K^2}{nT^2} + \frac{(n+2)M}{T^{2(n+1)}} = \frac{-\alpha^2}{n+1}$. Further under the constraint $\alpha = 1, K = 1$ and $M = 1$, the energy density $(\rho)$ and particle density $(\rho_p)$ is positive for $-2 < n < -1$ and $n > 0$ (Fig.1 and Fig.2).

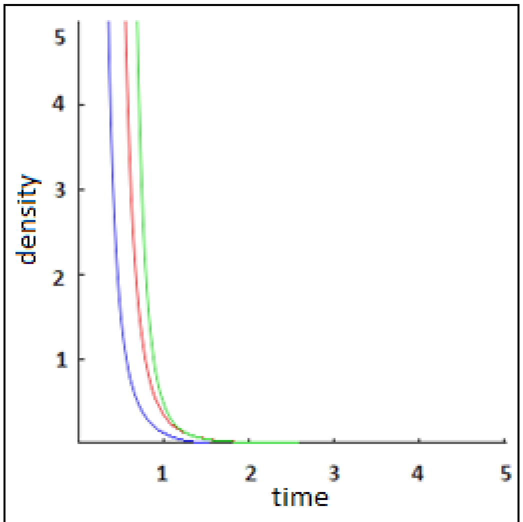

Fig. 1. n = -1.5, 0.5, 1.5

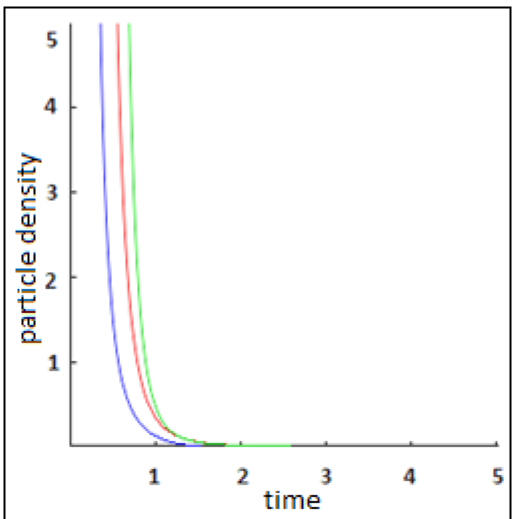

Fig. 2. n= -1.5, 0.5, 1.5

In the absence of magnetic field, the energy condition $\rho \geq 0$ lead to

$$\frac{(2n+1)M}{T^{2(n+1)}} \geq \frac{-n\alpha^2}{n+1} \tag{36}$$

and the physical parameters $\rho, \lambda, \rho_p, \theta$ and $\sigma$ are given

$$\rho = \frac{1}{8\pi}\left[\frac{n\alpha^2}{(n+1)T^2} + \frac{(2n+1)M}{T^{2(n+2)}}\right] \tag{37}$$

$$\lambda = -\frac{1}{8\pi}\left[\frac{n\alpha^2}{(n+1)T^2} + \frac{(2n+1)M}{T^{2(n+2)}}\right] \tag{38}$$

$$\rho_p = \frac{1}{8\pi}\left[\frac{2n\alpha^2}{(n+1)T^2} + \frac{2(2n+1)M}{T^{2(n+2)}}\right] \tag{39}$$

$$\theta = (n+2)\left[\frac{\alpha^2}{(n+1)T^2} + \frac{M}{T^{2(n+2)}}\right]^{1/2} \tag{40}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{\alpha^2}{(n+1)T^2} + \frac{M}{T^{2(n+2)}}\right] \tag{41}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = const \tag{42}$$

In the absence of magnetic field, the particle density $(\rho_p)$ and tension density $(\lambda)$ vanish when $T \to \infty$. The expansion in the model decreases as time increases if $n+2>0$ except $n=-1$. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$. Then the model does not approach isotropy for large values of $T$. Further under the constraint $\alpha = 1, K = 1$ and $M = 1$, the energy density $(\rho)$ and particle density $(\rho_p)$ is positive for $-2 < n < -1$ and $n > -\frac{1}{2}$.

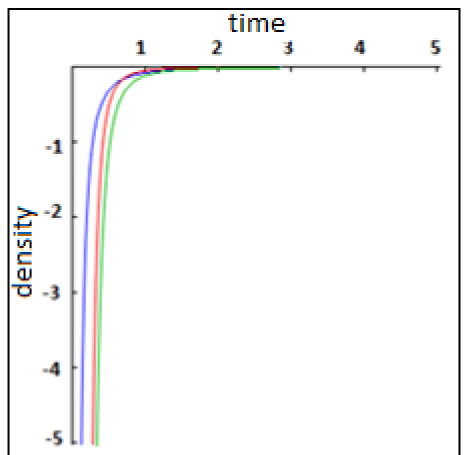

Fig. 3. n = -0.5, -0.33, -0.25

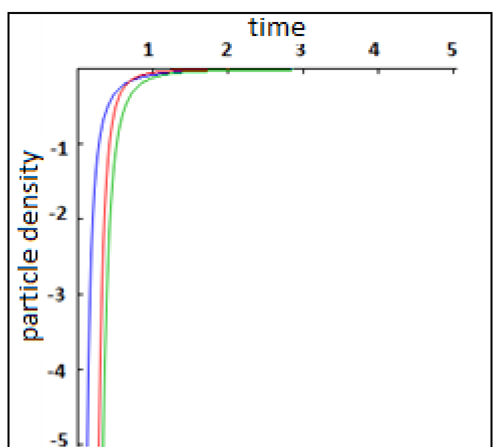

Fig. 4. n = -0.5, -0.66, -0.75

### 3.2 Case II: Nambu String

In this case

$$\rho = \lambda \quad (43)$$

From (13), (14) and (43) we obtain

$$\frac{\ddot{A}}{A} - \frac{\dot{A}\dot{C}}{AC} = 0 \quad (44)$$

Using (16), the above equation reduces to

$$\frac{\ddot{A}}{A} - n\left(\frac{\dot{A}}{A}\right)^2 = 0 \quad (45)$$

which on integration gives

$$A = (1-n)^{1/(1-n)}(at+b)^{1/(1-n)} \quad (46)$$

where $a$ and $b$ are constants of integration.
Hence we obtain

$$B = (1-n)^{1/(1-n)}(at+b)^{1/(1-n)} \quad (47)$$

$$C = (1-n)^{n/(1-n)}(at+b)^{n/(1-n)} \quad (48)$$

Thus the metric (1) reduces to the form

$$ds^2 = -dt^2 + (1-n)^{2/(1-n)}(at+b)^{2/(1-n)}dx^2 + (1-n)^{2/(1-n)}(at+b)^{2/(1-n)}e^{-2\alpha x}dy^2 + (1-n)^{2n/(1-n)}(at+b)^{2n/(1-n)}dz^2 \quad (49)$$

After a suitable transformation of coordinate, the metric (49) takes the form

$$ds^2 = -\frac{dT^2}{a^2} + (1-n)^{2/(1-n)}T^{2/(1-n)}dX^2 + (1-n)^{2/(1-n)}T^{2/(1-n)}e^{-2\alpha x}dY^2 + (1-n)^{2n/(1-n)}T^{2n/(1-n)}dZ^2 \quad (50)$$

where $T = ax+b,\ x = X,\ y = Y,\ z = Z$

### 3.1.2 Geometrical and physical significance of model

The energy density $(\rho)$, the string tension $(\lambda)$, the particle density $(\rho_p)$, the scalar of expansion $(\theta)$ and the shear $(\sigma)$ for the model (50) are given by

$$\rho = \lambda = \frac{1}{8\pi}\left[\frac{(2n+1)a^2}{(1-n)^2T^2} - \frac{\alpha^2}{(1-n)^{2/(1-n)}T^{2/(1-n)}} - \frac{K^2}{(1-n)^{4/(1-n)}T^{4/(1-n)}}\right] \quad (51)$$

$$\rho_p = 0 \quad (52)$$

$$\theta = \frac{a(n+2)}{(1-n)T} \quad (53)$$

$$\sigma^2 = \frac{(n+2)a^2}{3(1-n)^2T^2} \quad (54)$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} = const \quad (55)$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = \frac{1-4n}{n+2} \quad (56)$$

From (56), we observe that

$$q < 0 \text{ if } n > \frac{1}{4} \quad (57)$$

and

$$q > 0 \text{ if } n < \frac{1}{4} \quad (58)$$

The energy condition $\rho \geq 0$ implies that

$$\frac{(2n+1)a^2}{(1-n)^2} \geq \frac{\alpha^2}{(1-n)^{2/(1-n)}T^{2/(1-n)}} + \frac{K^2}{(1-n)^{4/(1-n)}T^{4/(1-n)}} \quad (59)$$

The energy density and string tension are infinite at singularity $T = 0$ and decreasing as $T \to \infty$. The expansion in the model stops when $n = -2$. The model starts expanding with a big bang at $T = 0$ and the expansion in the model decreases as time increases. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$. Hence the model does not approach isotropy for large value of $T$. The model (50) has singularity at $T = 0$. For the condition $n < \frac{1}{4}$ the solution gives accelerating model of the universe and for the condition $n > \frac{1}{4}$, ours solution represents decelerating model of the universe and $q$ approaches at value -1 when $n = 1$. Further under the constraint $\alpha = 1$ and $K = 1$, the energy density $\rho$ is positive for $n \geq 4$ (Fig.5).

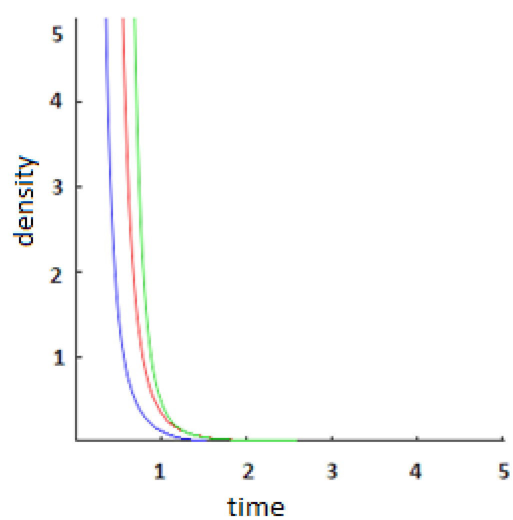

Fig. 5. n = 4, 6, 8

In absence of magnetic field, the energy condition $\rho \geq 0$ lead to

$$\frac{(2n+1)a^2}{(1-n)^2} \geq \frac{\alpha^2}{(1-n)^{2n/1-n} T^{2n/1-n}} \tag{60}$$

and the physical parameters $\rho, \lambda, \rho_p, \theta$ and $\sigma$ are given by

$$\rho = \lambda = \frac{1}{8\pi}\left[\frac{(2n+1)a^2}{(1-n)^2 T^2} - \frac{\alpha^2}{(1-n)^{2/1-n} T^{2/1-n}}\right] \tag{61}$$

$$\rho_p = 0 \tag{62}$$

$$\theta = \frac{a(n+2)}{(1-n)T} \tag{63}$$

$$\sigma^2 = \frac{(n+2)a^2}{3(1-n)^2 T^2} \tag{64}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} = const \tag{65}$$

In the absence of magnetic field, the particle density and tension density vanish when $T \to 0$. The expansion in the model decreases as time increases. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$. Hence the model does not approach isotropy for large value of $T$. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ .Hence the model does not approach isotropy for large value of $T$ .Further under the constraint $\alpha = 1$ and $K = 1$, the energy density $(\rho)$ is positive for $n \geq 3$ (Fig.6).

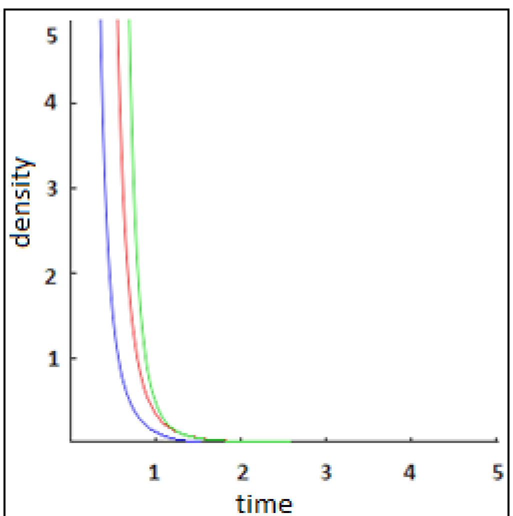

Fig. 6. n = 3, 5, 7

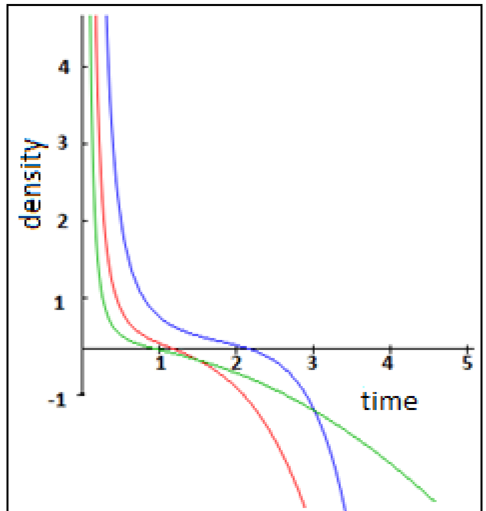

Fig. 7. n = 1.5, 2, 3

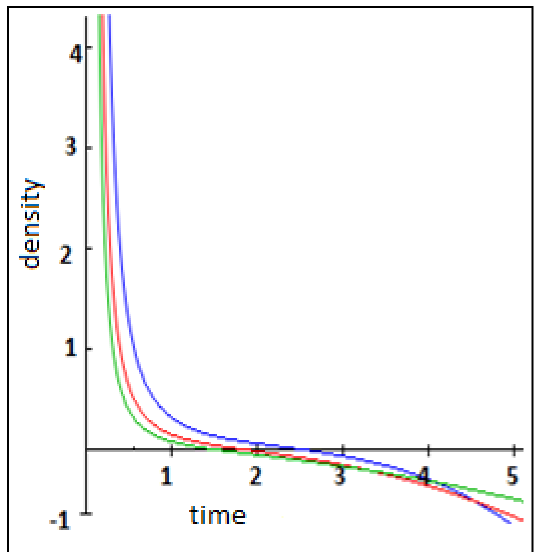

Fig. 8. n = 1.5, 1.7, 2

## 4 Conclusion

We have discussed Bianchi type III cosmological model for two cases (i) Reddy string and (ii) Nambu string. It is found that in both the cases, the model always represent accelerating and decelerating universe under the conditions (33), (34) and (57), (58). It has been shown that in the case of Reddy string and Nambu string, the models are not free from singularities. It is reasonable to say that cosmological model is required to explain acceleration in present universe. Therefore, our theoretical models are in agreement with recent observations (Perlmutter et al.1999; Garnavich et al.1998; Riesset al.1998; Schmidt et al.1998). Further under the constraint $\alpha = 1, K = 1, M = 1$ for Reddy string, in the presence of magnet, the particle density is negative in $-1 < n < 0$ and in the absence of magnet in $-1 < n \leq -\frac{1}{2}$ (Fig. 3 and Fig. 4). For Nambu string under the constraint $\alpha = 1, K = 1$, in the presence of magnet, the energy density is negative in $n \leq 3$ and in the absence of magnet in $n \leq 2$ (Fig. 7 and Fig. 8).

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