BER Analysis of 3x3 MIMO Spatial Multiplexing under AWGN & Rayleigh Channels for Different Modulation Techniques

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Abstract — The idea of using multiple receive and multiple transmit antennas has emerged as one of the most significant technical breakthroughs in modern wireless communications. MIMO systems uses rich scattering environment, which is capable of huge theoretical capacities. The multiple input multiple output (MIMO) antenna system provides very promising gain in capacity without increasing the use of spectrum, throughput, and power consumption. This is also less sensitivity to fading, hence leading to a breakthrough in the data rate of wireless communication systems.

This paper proposes the analysis and performance of Spatial Multiplexing technique of MIMO system. Here different fading channels like AWGN and Rayleigh are used for analysis purpose. Moreover we analyzed the technique using high level modulations (i.e. M-PSK for different values of M). Detection algorithms used are Zero-Forcing and Minimum mean square estimator.

Index Terms — MIMO, Spatial Multiplexing, Rayleigh channel, ZF, MMSE, AWGN and BER.

1 INTRODUCTION

The main trend in communications is more users and higher data rates per user. Hence, all new developments are aimed at increasing both the total system capacity as well as the capacity for individual users. For wireless communications with a limited amount of bandwidth, these goals require more spectrum efficient communication systems that don’t require extra bandwidth and satisfy all the users in the provided bandwidth. In today’s communication MIMO system has emerged as one of the most promising techniques in wireless communications due to its great potential to improve system reliability and increase channel capacity [2]. Two typical approaches in the MIMO systems are to provide diversity gain as in space-time coding (STC) or to allow spatial multiplexing (SM). While STC systems are capable of improving system reliability through coding across space and/or time, SM systems are capable of increasing data transmission rate through spatial multiplexing. In this paper we focus on SM technique. Spatial multiplexing is a transmission technique in MIMO wireless communication to transmit independent and separately encoded data signals, so-called streams, from each of the multiple transmit antennas. Therefore, the space dimension is reused, or multiplexed, more than one time. If the transmitter is equipped with Nt antennas and the receiver

\[ N_s = \min (N_t, N_r) \]  

The general concept of spatial multiplexing can be understood using MIMO antenna configuration. In spatial multiplexing, a high data rate signal is split into multiple lower data rate streams and each stream is transmitted from a different transmitting antenna in the same frequency channel. If these signals arrive at the receiver antenna array with different spatial signatures, the receiver can separate these streams into parallel channels thus improving the capacity. Thus spatial multiplexing is a very powerful technique for increasing channel capacity at higher SNR values. The maximum number of spatial streams is limited by the lesser number of antennas at the transmitter or receiver side. Spatial multiplexing can be used with or without transmit channel knowledge.

MIMO spatial multiplexing achieves this by utilizing the multiple paths and effectively using them as additional channels to carry data such that receiver receives multiple data at the same time. The tenet in spatial multiplexing is to transmit different symbols from each antenna and the receiver discriminates these symbols by taking advantage of the fact that, due to spatial selectivity, each transmit antenna has a different spatial signature at the receiver. This allows an increased number of information symbols per MIMO symbol. In any case for MIMO spatial multiplexing, the number of receiving antennas must be equal to or greater than the number of transmit antennas such that data can be transmitted over different antennas. Therefore the space dimension is reused or multiplexed more than one time. The data streams can be separated by equalizers if the fading processes of the spatial channels are nearly independent. Spatial multiplexing requires no bandwidth expansion and provides additional data bandwidth in multipath radio scenarios [2].
2 MIMO

Multiple-input multiple-output (MIMO) systems are a natural extension of developments in antenna array communication. MIMO systems may be implemented in a number of different ways to obtain either a diversity gain to combat signal fading or to obtain a capacity gain. A MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment. Generally, there are three categories of MIMO techniques. The first aims to improve the power efficiency by maximizing spatial diversity. Such techniques include delay diversity, STBC and STTC. The second class uses a layered approach to increase capacity. One popular example of such a system is V-BLAST suggested by Foschini et al. [2] where full spatial diversity is usually not achieved. Finally, the third type exploits the knowledge of channel at the transmitter. It decomposes the channel coefficient matrix using SVD and uses these decomposed unitary matrices as pre- and post-filters at the transmitter and the receiver to achieve near capacity [3].

2.1 Modulation

Modulation is the process of mapping the digital information to analog form so it can be transmitted over the channel. Consequently every digital communication system has a modulator that performs this task. Closely related to modulation is the inverse process, called demodulation, done by the receiver to recover the transmitted digital information [4]. Modulation of a signal changes binary bits into an analog waveform. Modulation can be done by changing the amplitude, phase, and frequency of a sinusoidal carrier. There are several digital modulation techniques used for data transmission.

i Phase Shift Keying (PSK)

Phase Shift Keying is a digital modulation scheme that conveys data by changing or modulating, the phase of a reference signal (the carrier wave). In M-ary PSK modulation, the amplitude of the transmitted signals was constrained to remain constant, thereby yielding a circular constellation. Modulation equation of M-PSK signal is:

\[ s_i(t) = \sqrt{\frac{2E_s}{T}} \cos \left( 2\pi f_c t + \frac{2\pi i t}{M} \right) \quad i=0, 1, ..., M \]  

2.2 Channels

Channel is a transmission medium between the transmitter and receiver side. Channel can be air or space but it induces fading as well as distortions in the transmitted signal in such a way that the received signal is not same as that of transmitted signal but is a combination of reflected, diffracted and scattered copies of the transmitted signal. These copies are called multipath signal components. AWGN and Rayleigh fading channels have been taken into consideration for the analysis purpose.

i AWGN Channel

Additive white Gaussian noise (AWGN) channel is universal channel model for analyzing modulation schemes. In this model, the channel does nothing but add a white Gaussian noise to the signal passing through it. This implies that the channel’s amplitude frequency response is flat (thus with unlimited or infinite bandwidth) and phase frequency response is linear for all frequencies so that modulated signals pass through it without any amplitude loss and phase distortion of frequency components. Fading does not exist. The only distortion is introduced by the AWGN. AWGN channel is a theoretical channel used for analysis purpose only. The received signal is simplified to:

\[ r(t) = s(t) + n(t) \]  

where \( n(t) \) is the additive white Gaussian noise.

ii Rayleigh Channel

Constructive and destructive nature of multipath components in flat fading channels can be approximated by Rayleigh distribution if there is no line of sight which means when there is no direct path between transmitter and receiver. The received signal can be simplified to:

\[ r(t) = s(t) * h(t) + n(t) \]  

where \( h(t) \) is the random channel matrix having Rayleigh distribution and \( n(t) \) is the additive white Gaussian noise. The Rayleigh distribution is basically the magnitude of the sum of two equal independent orthogonal Gaussian random variables and the probability density function (pdf) given by:

\[ p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad 0 \leq r < \infty \]
where $\sigma^2$ is the time-average power of the received signal.

### 2.3 Signal Detection Techniques

There are numerous detection techniques available with combination of linear and non-linear detectors. The most common detection techniques are ZF, MMSE and ML detection technique.

**i Zero Forcing (ZF) Detection**

The ZF is a linear estimation technique, which inverse the frequency response of received signal, the inverse is taken for the restoration of signal after the channel. The estimation of strongest transmitted signal is obtained by nulling out the weaker transmit signal. The strongest signal has been subtracted from received signal and proceeds to decode strong signal from the remaining transmitted signal. ZF equalizer ignores the additive noise and may significantly amplify noise for channel.

The basic Zero force equalizer of 2x2 MIMO channel can be modeled by taking received signal $y_1$ during first slot at receiver antenna as:

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}][x_1 \ x_2]^T + n_1$$  \hspace{1cm} (6)

The received signal $y_2$ at the second slot receiver antenna is:

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}][x_1 \ x_2]^T + n_2$$  \hspace{1cm} (7)

Where $i=1, 2$ in $x_i$ is the transmitted symbol and $i=1, 2$ in $h_{i,j}$ is correlated matrix of fading channel, with $j$ represented transmitted antenna and $i$ represented receiver antenna. $n$ is the noise.

The ZF equalizer is given by:

$$W_{ZF} = (H^H)^{-1}H^H$$  \hspace{1cm} (8)

Where $W_{ZF}$ is equalization matrix and $H$ is a channel matrix. Assuming $M_R \geq M_T$ and $H$ has full rank, the result of ZF equalization before quantization is written as:

$$y_{ZF} = (H^H)^{-1}H^Hy$$  \hspace{1cm} (9)

**ii Minimum Mean Square Estimator**

Minimum mean square error equalizer minimizes the mean-square error between the output of the equalizer and the transmitted symbol. Instead of removing ISI completely; an MMSE equalizer allows some residual ISI to minimize the overall distortion. Compared with a ZF equalizer, an MMSE equalizer is much more robust in presence of deepest channel nulls and noise. The MMSE equalization is:

$$W_{MMSE} = \arg\min_E E_{x,n}[\|x - x^\ast\|^2]$$  \hspace{1cm} (10)

Where is $W_{MMSE}$ equalization matrix, $H$ channel correlated matrix and $n$ is channel noise.

### 3. RESULTS AND DISCUSSIONS

The system discussed above has been designed and results are shown in the form of SNR vs. BER plot for different modulations and different channels. Here antenna configuration 3x3 is analysed using ZF and MMSE detection techniques. Analyses have been done for two channels AWGN and Rayleigh channel.

**3.1 Using ZF detection algorithm**
Fig. 4(c) BER vs. SNR plots over AWGN & Rayleigh channel for SM technique using 3x3 MIMO System using ZF Equalization

Table 1
Comparison of different Modulation Techniques for Rayleigh & AWGN Channel for 3x3 MIMO Spatial Multiplexing using ZF Equalization

<table>
<thead>
<tr>
<th>Modulations</th>
<th>Rayleigh Channel</th>
<th>AWGN Channel</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>32-PSK</td>
<td>63 dB</td>
<td>57 dB</td>
<td>6 dB</td>
</tr>
<tr>
<td>64-PSK</td>
<td>71 dB</td>
<td>63 dB</td>
<td>8 dB</td>
</tr>
<tr>
<td>128-PSK</td>
<td>77 dB</td>
<td>69 dB</td>
<td>8 dB</td>
</tr>
<tr>
<td>256-PSK</td>
<td>82 dB</td>
<td>75 dB</td>
<td>7 dB</td>
</tr>
<tr>
<td>512-PSK</td>
<td>90 dB</td>
<td>82 dB</td>
<td>8 dB</td>
</tr>
<tr>
<td>1024-PSK</td>
<td>96 dB</td>
<td>87 dB</td>
<td>9 dB</td>
</tr>
</tbody>
</table>

From Table 1 we can say that at 64-PSK, 128-PSK, 512-PSK there is an improvement of 8dB, at 1024-PSK there is an improvement of 9dB and at 64-PSK improvement is 8dB at BER of 10^{-4}. Table 1 shows the improvement in terms of decibels shown by proposed system employing SM technique for 3x3 MIMO system for different modulation schemes over different channels.
3.2 Using MMSE detection

Fig. 5(a) 32-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5(b) 64-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5(c) 128-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5(d) 256-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5(e) 512-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5(f) 1024-PSK modulation with 3x3 MIMO for AWGN and Rayleigh with MMSE

Fig. 5 BER vs. SNR plots over AWGN & Rayleigh channel for SM technique using 3x3 MIMO System using MMSE Equalization
a)32 PSK  b) 64 PSK  c) 128 PSK  d) 256 PSK  e) 512 PSK  f) 1024 PSK
problem of BER (bit error increase) which increases as the order of modulations decreases at higher values of SNR so, that the effect of the distortions introduced by the channel will also goes on decreasing, as a result of this, the BER will also decreases at higher values of the SNR for high order modulations.

Several different diversity modes are used to make radio communications more robust, even with varying channels. These include time diversity (different timeslots and channel coding), frequency diversity (different channels, spread spectrum, and OFDM), and also spatial diversity. Spatial diversity requires the use of multiple antennas at the transmitter or the receiver end. Multiple antenna systems are typically known as Multiple Input, Multiple Output systems (MIMO). Multiple antenna technology can also be used to increase the data rate (spatial multiplexing) instead of improving robustness. In future, we can make a single integrated circuit that uses both methods combination.

### REFERENCES


