Availability and Reliability Analysis for System with Bivariate Weibull Lifetime Distribution

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Abstract: In this paper, analysis of a system consists of two dependent components with load sharing is introduced. The life and repair times of the units are assumed to follow bivariate Weibull distribution. Markov models are used to construct the mathematical model of the system. Analysis of the availability function, reliability function, steady state availability, and mean time to system failure are discussed. A numerical example is given for illustration.

Keywords: Bivariate weibull distribution, Bivariate exponential distribution, Availability, Reliability, Steady state availability, Markov models, Load-sharing systems, Dependent units, Mean time to system failure.

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1. Introduction

Availability is a measure of system performance which denotes the probability that the system is available for use (in operable condition) at any arbitrary instant t. Availability is therefore the probability that the system will be operational at the given time t. It combines aspects of reliability, maintainability and maintenance support and implies that the system is either in active operation or is able to operate if required. Availability pertains only to systems which undergo repair and are restored after failure. A high availability can be obtained either by increasing the average operational time until the next failure, or by improving the maintainability of the system. Gnedenko and Uskakov (1995) define different coefficients of availability for one-unit systems.

Reliability is a quantitative measure to ensure operational efficiency. The reliability of a product is the measure of its ability to perform its function when required for a specific time in a particular environment.

The scope of reliability engineering is extremely wide. It helps to obtain reliable transportation and telecommunication systems, provide a steady supply of power, and ensure successful operations of robotics, and so on. The growth of knowledge in several areas of reliability engineering and its applications has become increasingly important (Chung (1990)). The common-cause failures have gained considerable attention in the field of reliability (Dhillon (1979), Chung (1990), Shooman (1971), Dhillon (1981) etc.)

Reliability of a system is fairly simple when units fail independently of each other. In the presence of common-cause failures, the reliability calculation requires a set of simultaneous linear differential equations. Some of the reasons for systems with common-cause failures are:

(i) Equipment design deficiencies

(ii) Unforeseen external abnormal environments - dust, humidity, temperature

(iii) Operations and maintenance errors

(iv) External catastrophe

(v) Functional deficiencies

(vi) Common power source

A failure is a result of a joint action of many unpredictable, random processes going on inside the operating system as well as in the environment in which the system is operating (Gertsbakh (1989)). Functioning is therefore seriously impeded or completely stopped at a certain moment in time and all failures have a stochastic nature. In some cases the time of failure is easily observed, but if units deteriorate continuously determination of the moment of failure is not an easy task. Failure of a system is called a disappointment or a death and failure results in the system being in the down state. This can also be referred to as a breakdown (Finkelstein (1999a)).

Failure rate is the conditional probability that a device will fail per unit of time. The conditional probability is the probability that a device will fail during a certain interval given that it survived at the start of the interval (Lawless (1982)).

In most cases, independence is assumed across the components within the system which means that the failure of any component of the system does not affect the failure of another one in the system. However, if your system consists of multiple components sharing a load then the assumption of independence no longer holds true. If one component fails then the component(s) that are still operating will have to assume the failed unit's load. Therefore, the reliabilities of the surviving unit(s) will change. Calculating the system reliability is no longer an easy proposition. Huang and Xu (2010) presented a general closed-form expression for the lifetime reliability of load-sharing k-out-of-n: G hybrid redundant systems.

Parallel redundancy is a common method to increase system reliability and mean time to failure. Studies of reliability of systems assume independence among component lifetimes. In practice, components in a reliability structure are dependent as they may share the same load or may be failed with common-cause failures. Bivariate and multivariate lifetime distributions play important roles in modeling these dependencies. Many bivariate and multivariate exponential distributions have been proposed by Balakrishnan and Lai (2009). The bivariate exponential distribution of Marshall and Olkin (1967) is suited for modeling common-cause failures. Freund’s model (1961) can be applied to the situation that the failed component increases the stress on the surviving component and consequently increases the other component's tendency of failure.

Marshall and Olkin (1967) introduced a bivariate exponential distribution by considering a reliability model in which two components fail separately or simultaneously upon receiving a shock that is governed by a homogeneous Poisson process. They derived the bivariate exponential distribution in several ways: the bivariate lack of memory property, shock models, a random sum model, and a minima model.

Freund (1961) proposed a bivariate extension of the exponential distribution by allowing the failure rate of the surviving component to be affected after the failure of another component. Freund's bivariate distribution is absolutely continuous and possesses the bivariate lack of memory property (Bailey (1964)). Freund's model is one of the first to study bivariate distributions from reliability considerations, and it can be used to model load-sharing systems.

Weibull probability model plays an important role in reliability theory and life testing experiments. Mino et al. (2003) discussed the applications of a Marshall–Olkin bivariate Weibull distribution in lifespan. Rachev et al. (1995) considered a Marshall–Olkin bivariate Weibull distribution as a bivariate limiting distribution of the tumor latency time. Marshall and Olkin (1997) introduced a method of adding a parameter into a family of distributions, which result in the flexibility of the new distribution. Marshall and Olkin (1967) introduced two new distributions: an exponential distribution with two parameters and a Weibull distribution with three parameters.

Markov models are commonly used to perform reliability analysis of engineering systems and fault-tolerant systems. They are also used to handle reliability and availability analysis of repairable systems. First, we gave notations and several properties of stochastic processes. Next, we explore Markov chains focusing on criteria of recurrent/transient state, and long-run probabilities. We then discuss basic properties of the homogeneous Poison process, which is one of the most important stochastic processes. The discussion is then going to the continuous-time Markov chain, including the birth, the death, and the birth-death processes. It is not an easy task to solve the state equations. A number of solution techniques exist, such as analytical solution (see Rausand and Høyland (2004), Laplace-Stieltjes transforms (Pukite (1998)), numerical integration, and computer-assisted evaluation (Block and Basu (1974)).

In this paper, we present analysis for a system consists of two dependent non-identical units connected in parallel subject to load sharing. We consider that the failure and repair rates of the units follow bivariate Weibull distribution. Markov models are used to construct a block diagram and a mathematical model for a system. Availability analysis and steady state availability probability for a system are discussed. Reliability and mean time to system failure are introduced. A numerical example is introduced in order to show the results.

2. Marshall-Olkin Bivariate Weibull Distribution

Suppose $U_0, U_1,$ and U_2 are independent Weibull random variables with the same shape parameter α, and scale parameters $\lambda_0, \lambda_1,$ and λ_2, respectively. Now define $X_1 = min\{U_0, U_1\}$ and $X_2 = min\{U_0, U_2\}$, then (X_1, X_2) is said to have bivariate Weibull distribution with PDF given by

$$f(x_1, x_2) = \begin{cases} f_{WE}(x_1; \alpha, \lambda_1) f_{WE}(x_2; \alpha, \lambda_0 + \lambda_2) & if \quad 0 < x_1 < x_2 \\ f_{WE}(x_1; \alpha, \lambda_0 + \lambda_1) f_{WE}(x_2; \alpha, \lambda_2) & if \quad 0 < x_2 < x_1 \\ \dfrac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2} f_{WE}(x; \alpha, \lambda_0 + \lambda_1 + \lambda_2) & if \quad 0 < x_1 = x_2 = x \end{cases},$$

$$f_{WE}(x; \alpha, \lambda) = \alpha \lambda x^{\alpha - 1} e^{-\lambda x^{\alpha}}$$

Marshall and Olkin bivariate Weibull distribution has the following properties:

- It is also a singular distribution.
- The marginals are Weibull and hence very flexible.
- It can have decreasing and unimodal PDFs.
- It can have increasing or decreasing hazard functions also.
- It reduces to exponential distribution when the shape parameter α is one.

3. System Analysis

The system is considered to be consisted of two components connected in parallel and work dependently where the failure of any component affects the failure of the other one. In addition, it is assumed that there is a common failure between the two components. Life times of the units are assumed to follow bivariate Weibull distribution. Failures are assumed to be repairable and the repair times also follow bivariate Weibull distribution.

3.1 Notations

All notations for the system are given as follows.

$P_t[0, 0]$ is the probability that the system is up at time t.

$P_t[1, 0]$ is the probability that the first unit failed and the second unit is up at time t.

$P_t[0, 1]$ is the probability that the first unit is up and the second unit failed at time t.

$P_t[1, 1]$ is the probability that the system is down at time t.

$\lambda_1(t)$ is the dependent time failure rate from up state to failed state for the first unit.

$\lambda_2(t)$ is the dependent time failure rate from up state to failed state for the second unit.

$\lambda_{12}(t)$ is the dependent time common cause failure rate from up state to failed state for both units.

$\lambda_1'(t)$ is the dependent time failure rate from up state to failed state for the first unit after failure of the second unit.

$\lambda_2'(t)$ is the dependent time failure rate from up state to failed state for the second unit after failure of the first unit.

$\mu_1(t)$ is the dependent time repair rate of the first unit after failure.

$\mu_2(t)$ is the dependent time repair rate of the second unit after failure.

$\mu_{12}(t)$ is the dependent time repair rate of the both units after common failure.

$\mu_1'(t)$ is the dependent time repair rate for the first unit after it has failed and the second unit has failed.

$\mu_2'(t)$ is the dependent time repair rate for the second unit after it has failed and the first unit has failed.

3.2 System Availability

It is obvious that the system will have 4 states divided as follows: (0, 0), (1, 0), (0, 1) are the working states and (1, 1) is the failed state. All possible states and transition rates between them are shown in Figure 1.Continuous-time Markov model is used to construct mathematical model for the system as follows.

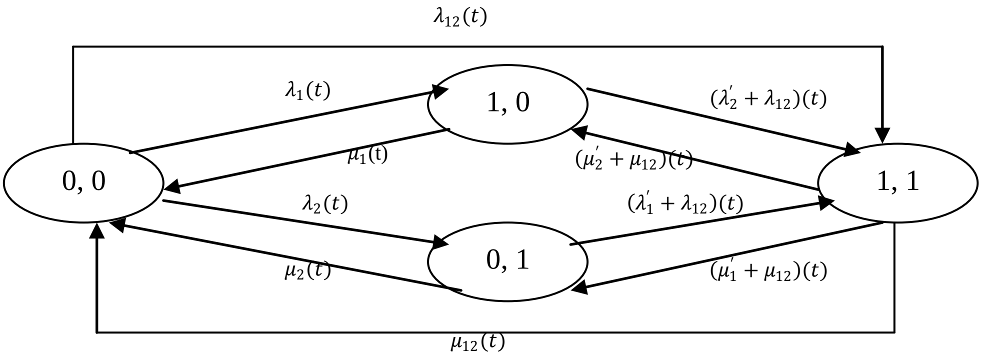

Fig.1: Block diagram for system of two dependent parallel units

$$\frac{dP_t[0,0]}{dt} = -[\lambda_1(t) + \lambda_2(t) + \lambda_{12}(t)]P_t[0,0] + \mu_1(t)P_t[1,0] +$$

$$+\mu_2(t)P_t[0,1] + \mu_{12}(t)P_t[1,1] \qquad (1)$$

$$\frac{dP_t[1,0]}{dt} = -[(\lambda_2^{'} + \lambda_{12})(t) + \mu_1(t)]P_t[1,0] + \lambda_1(t)P_t[0,0] + \\ +(\mu_2^{'} + \mu_{12})(t)P_t[1,1] \quad (2)$$

$$\frac{dP_t[0,1]}{dt} = -[(\lambda_1^{'} + \lambda_{12})(t) + \mu_2(t)]P_t[0,1] + \lambda_2(t)P_t[0,0] + \\ +(\mu_1^{'} + \mu_{12})(t)P_t[1,1] \quad (3)$$

$$\frac{dP_t[1,1]}{dt} = -[(\mu_1^{'} + \mu_{12})(t) + (\mu_2^{'} + \mu_{12})(t) + \mu_{12}(t)]P_t[1,1] + \lambda_{12}(t)P_t[0,0] + \\ +(\lambda_2^{'} + \lambda_{12})(t)P_t[1,0] + (\lambda_1^{'} + \lambda_{12})(t)P_t[0,1] \quad (4)$$

The initial conditions for the system are given by

$$P_0[0,0] = 1, P_0[1,0] = 0, P_0[0,1] = 0, P_0[1,1] = 0$$

According to Marshall and Olkin bivariate Weibull distribution the dependent time transition rates of the model are given by

$$\lambda_i(t) = \beta\lambda_i t^{\beta-1}, i = 1,2,12, \qquad (\lambda_i^{'} + \lambda_{12})(t) = \beta(\lambda_i^{'} + \lambda_{12})t^{\beta-1}, i = 1,2$$

$$\mu_i(t) = \beta\mu_i t^{\beta-1}, i = 1,2,12, \qquad (\mu_i^{'} + \mu_{12})(t) = \beta(\mu_i^{'} + \mu_{12})t^{\beta-1}, i = 1,2$$

The set of equations from (1) to (4) forms a system of first order differential equations which can be solved under the given initial conditions to obtain the state probabilities and the availability function can be calculated from the following sum of the probabilities of the working states.

$$A(t) = P_t[0,0] + P_t[1,0] + P_t[0,1] \quad (5)$$

3.3 Steady State Availability

The limiting or steady state availability A_∞ or simply A is the expected fraction of time that the system operates satisfactorily in the long run (Barlow & Proschan (1965)). It is defined as the probability that the system will be in an operational state at time t, when t is considered to be infinitely large.

$$A_\infty = \lim_{t\to\infty} A(t)$$

In Markov models, it is possible to go from one state to another one over a large long period of time. It can easily be shown that the limit $P[i,j] = \lim_{t\to\infty} P_t[i,j]$ always exists. One can get the steady state solutions by simply setting all the derivatives $\frac{dP_t[i,j]}{dt}$

equal zero, and hence the system of differential equations will be reduce to an equivalent system of algebraic equations as follows.

$$-[\lambda_1 + \lambda_2 + \lambda_{12}]P[0,0] + \mu_1 P[1,0] + \mu_2 P[0,1] + \mu_{12} P[1,1] = 0 \tag{6}$$

$$-[\lambda_2^{'} + \lambda_{12} + \mu_1]P[1,0] + \lambda_1 P[0,0] + (\mu_2^{'} + \mu_{12})P[1,1] = 0 \tag{7}$$

$$-[\lambda_1^{'} + \lambda_{12} + \mu_2]P_t[0,1] + \lambda_2 P_t[0,0] + (\mu_1^{'} + \mu_{12})P_t[1,1] = 0 \tag{8}$$

$$-[\mu_1^{'} + \mu_2^{'} + 3\mu_{12}]P[1,1] + \lambda_{12} P[0,0] + (\lambda_2^{'} + \lambda_{12})P[1,0] + (\lambda_1^{'} + \lambda_{12})P[0,1] = 0 \tag{9}$$

$$P[0,0] + P[1,0] + P[0,1] + P[1,1] = 1 \tag{10}$$

The previous set of equations is solved to obtain all possible probabilities and the results are given as follows

$$P[0,1] = \frac{P[0,0][\alpha\rho\lambda + \mu_1\alpha(\lambda_{12} + \lambda_2) + \lambda_2\mu_{12}(\rho + \mu_1) + \sigma\lambda_2\mu_1]}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho} \tag{11}$$

$$P[1,0] = \frac{P[0,0][\eta\sigma\lambda + \sigma\mu_2(\lambda_1 + \lambda_{12}) + \lambda_1\mu_{12}(\mu_2 + \eta) + \mu_2\alpha\lambda_1]}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho} \tag{12}$$

$$P[1,1] = \frac{P[0,0][\eta\rho\lambda + \rho\mu_2(\lambda_1 + \lambda_{12}) + \eta\mu_1(\lambda_2 + \lambda_{12}) + \mu_1\mu_2\lambda_{12}]}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho} \tag{13}$$

and

$$\begin{aligned} P[0,0] = \Bigg[1 &+ \frac{\alpha\rho\lambda + \mu_1\alpha(\lambda_{12} + \lambda_2) + \lambda_2\mu_{12}(\rho + \mu_1) + \sigma\lambda_2\mu_1}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho} \\ &+ \frac{\eta\sigma\lambda + \sigma\mu_2(\lambda_1 + \lambda_{12}) + \lambda_1\mu_{12}(\mu_2 + \eta) + \mu_2\alpha\lambda_1}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho} \\ &+ \frac{\eta\rho\lambda + \rho\mu_2(\lambda_1 + \lambda_{12}) + \eta\mu_1(\lambda_2 + \lambda_{12}) + \mu_1\mu_2\lambda_{12}}{\sigma\mu_1\mu_2 + \mu_1\mu_2\mu_{12} + \mu_2\mu_{12}\rho + \eta\sigma\mu_1 + \eta\mu_1\mu_{12} + \eta\rho\mu_{12} + \mu_1\mu_2\alpha + \mu_2\alpha\rho}\Bigg]^{-1} \end{aligned} \tag{14}$$

where

$$\lambda = \lambda_1 + \lambda_2 + \lambda_{12}, \eta = \lambda_1^{'} + \lambda_{12}, \rho = \lambda_2^{'} + \lambda_{12}, \alpha = \mu_1^{'} + \mu_{12}, \sigma = \mu_2^{'} + \mu_{12}$$

The steady state availability probability for the system can be obtained from the following sum.

$$A = P[0,0] + P[1,0] + P[0,1] \tag{15}$$

3.4 Reliability

In order to find the reliability function of the system, we assume that all failed states are absorbing states and hence set any transition from them equal zero. The system will reduce to the following model.

$$\frac{dP_t[0,0]}{dt} = -\beta(\lambda_1 + \lambda_2 + \lambda_{12})t^{\beta-1}P_t[0,0] \tag{16}$$

$$\frac{dP_t[1,0]}{dt} = -\beta(\lambda_2' + \lambda_{12})t^{\beta-1}P_t[1,0] + \beta\lambda_1 t^{\beta-1}P_t[0,0] \tag{17}$$

$$\frac{dP_t[0,1]}{dt} = -\beta(\lambda_1' + \lambda_{12})t^{\beta-1}P_t[0,1] + \beta\lambda_2 t^{\beta-1}P_t[0,0] \tag{18}$$

and the initial conditions are given by

$$P_0[0,0] = 1, P_0[1,0] = 0, P_0[0,1] = 0$$

The previous model forms a set of homogeneous first order differential equations which can be solved and the results are obtained as follows.

$$P_t[0,0] = e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta}, \qquad P_t[1,0] = \frac{\lambda_1\left[e^{-(\lambda_2'+\lambda_{12})t^\beta} - e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta}\right]}{\lambda_1 + \lambda_2 - \lambda_2'},$$

$$P_t[0,1] = \frac{\lambda_2\left[e^{-(\lambda_1'+\lambda_{12})t^\beta} - e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta}\right]}{\lambda_1 + \lambda_2 - \lambda_1'}$$

The reliability function of the model is the sum of all working states and is obtained as follows.

$$R(t) = P_t[0,0] + P_t[1,0] + P_t[0,1]$$

$$= e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta} + \frac{\lambda_1\left[e^{-\left(\lambda_2'+\lambda_{12}\right)t^\beta} - e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta}\right]}{\lambda_1 + \lambda_2 - \lambda_2'} + \frac{\lambda_2\left[e^{-\left(\lambda_1'+\lambda_{12}\right)t^\beta} - e^{-(\lambda_1+\lambda_2+\lambda_{12})t^\beta}\right]}{\lambda_1 + \lambda_2 - \lambda_1'} \tag{19}$$

3.5 Mean Time to System failure

Mean time to system failure (MTTF) is a measure of reliability for non-repairable systems. It is the mean time expected until the piece of equipment fails and needs to be replaced. MTTF is a statistical value and is calculated as the mean over a long period of time and a large number of units. Mean time to the system failure can be obtained from the following formula.

$$MTTF = \int_0^\infty R(t)\, dt \tag{20}$$

Substituting from equation (19) into equation (20) and computing the integral, an expression for mean time to system failure is obtained as follows.

$$MTTF = \Gamma\left(\frac{1}{\beta}+1\right)\left\{\frac{1}{(\lambda_1+\lambda_2+\lambda_{12})^{1/\beta}} + \frac{\lambda_1}{\lambda_1+\lambda_2-\lambda_2'}\left[\frac{1}{(\lambda_2'+\lambda_{12})^{1/\beta}} - \frac{1}{(\lambda_1+\lambda_2+\lambda_{12})^{1/\beta}}\right] + \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_1'}\left[\frac{1}{(\lambda_1'+\lambda_{12})^{1/\beta}} - \frac{1}{(\lambda_1+\lambda_2+\lambda_{12})^{1/\beta}}\right]\right\} \quad (21)$$

3.6 Special Case

As a special case of our system, let us suppose that $\beta = 1$ and hence the failure and repair rates of the system will be constant follow bivariate exponential distribution. The modified model with constant rates will be given by the following set of differential equations.

$$\frac{dP_t[0,0]}{dt} = -(\lambda_1+\lambda_2+\lambda_{12})P_t[0,0] + \mu_1 P_t[1,0] + \mu_2 P_t[0,1] + \mu_{12} P_t[1,1] \quad (22)$$

$$\frac{dP_t[1,0]}{dt} = -(\lambda_2'+\lambda_{12}+\mu_1)P_t[1,0] + \lambda_1 P_t[0,0] + (\mu_2'+\mu_{12})P_t[1,1] \quad (23)$$

$$\frac{dP_t[0,1]}{dt} = -(\lambda_1'+\lambda_{12}+\mu_2)P_t[0,1] + \lambda_2 P_t[0,0] + (\mu_1'+\mu_{12})P_t[1,1] \quad (24)$$

$$\frac{dP_t[1,1]}{dt} = -(\mu_1'+\mu_2'+3\mu_{12})P_t[1,1] + \lambda_{12} P_t[0,0] + +(\lambda_2'+\lambda_{12})P_t[1,0] + (\lambda_1'+\lambda_{12})P_t[0,1] \quad (25)$$

The initial conditions for the system are given by

$$P_0[0,0] = 1, P_0[1,0] = 0, P_0[0,1] = 0, P_0[1,1] = 0$$

In order to find solutions for the previous model, we take Laplace transformation of the set of equations from (22) to (25) and the result model is shown as follows.

$$(s+\lambda_1+\lambda_2+\lambda_{12})P_s[0,0] - \mu_1 P_s[1,0] - \mu_2 P_s[0,1] - \mu_{12} P_s[1,1] = 1 \quad (26)$$

$$(s+\lambda_2'+\lambda_{12}+\mu_1)P_s[1,0] - \lambda_1 P_s[0,0] - (\mu_2'+\mu_{12})P_s[1,1] = 0 \quad (27)$$

$$(s+\lambda_1'+\lambda_{12}+\mu_2)P_s[0,1] - \lambda_2 P_s[0,0] - (\mu_1'+\mu_{12})P_s[1,1] = 0 \quad (28)$$

$$(s+\mu_1'+\mu_2'+3\mu_{12})P_s[1,1] - \lambda_{12} P_s[0,0] - (\lambda_2'+\lambda_{12})P_s[1,0] - -(\lambda_1'+\lambda_{12})P_s[0,1] = 0 \quad (29)$$

The previous system is consisted of a set of linear equations which can be solved and the state probabilities of the system can be obtained by taking the inverse Laplace transformations of the results. The availability function can be calculated from the following sum of the probabilities of the working states.

$$A(t) = L^{-1}\{P_s[0,0] + P_s[1,0] + P_s[0,1]\}$$
$$= P_t[0,0] + P_t[1,0] + P_t[0,1] \tag{30}$$

4. Numerical Example

Suppose that the parameters of the system have the following numerical values.

$$\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_{12} = 0.03, \lambda_1' = 0.04, \lambda_2' = 0.05, \mu_1 = 0.06, \mu_2 = 0.07, \mu_{12} = 0.08,$$
$$\mu_1' = 0.09, \mu_2' = 0.095, \beta = 1.1$$

Substituting these values in the system of equations (1)-(4) and using Maple package to solve the system of differential equations. Numerical solutions for the availability function are obtained and the results are shown in Figure 2.

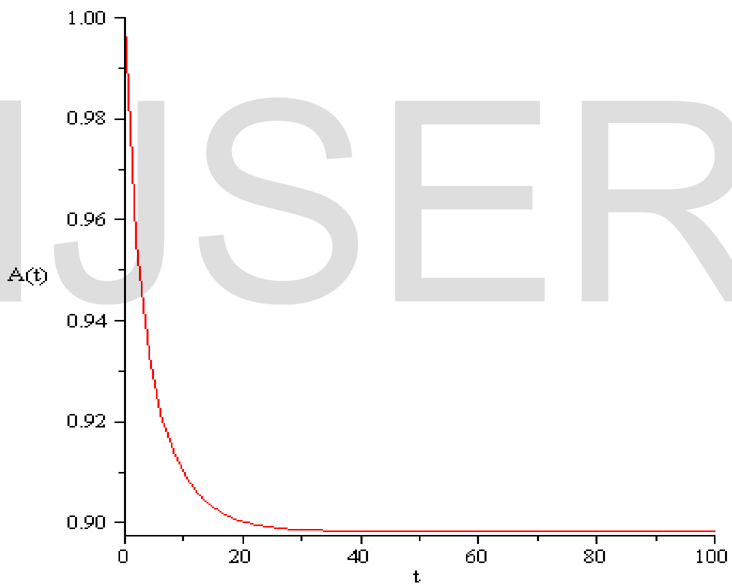

Fig.2: The availability function versus time

Steady state availability probability is computed from equation (15) and the result is

$$A = 0.898366606$$

In order to find the reliability function, we substitute the numerical values in equation (19) and hence the reliability function is obtained as follows and the result versus time is shown graphically in Figure 3.

$$R(t) = e^{-0.06\,t^{1.1}} + \left(2.000000000e^{0.01\,t^{1.1}} - 2.000000000\right) e^{-0.07\,t^{1.1}} + \left(0.5000000000e^{0.02\,t^{1.1}} - 0.5000000000\right) e^{-0.08\,t^{1.1}}$$

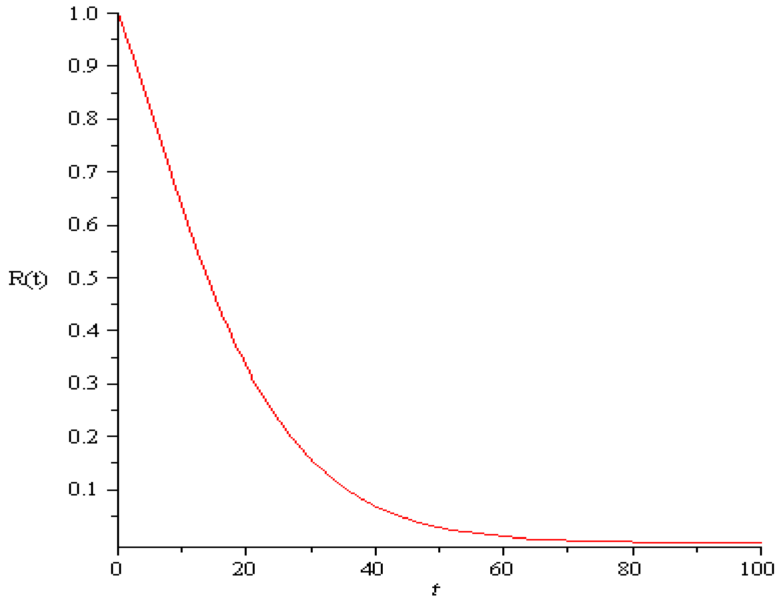

Fig.3: The reliability function versus time

The result for the mean time to system failure is obtained by using equation (21) and the results are illustrated graphically in Figures 4 and 5.

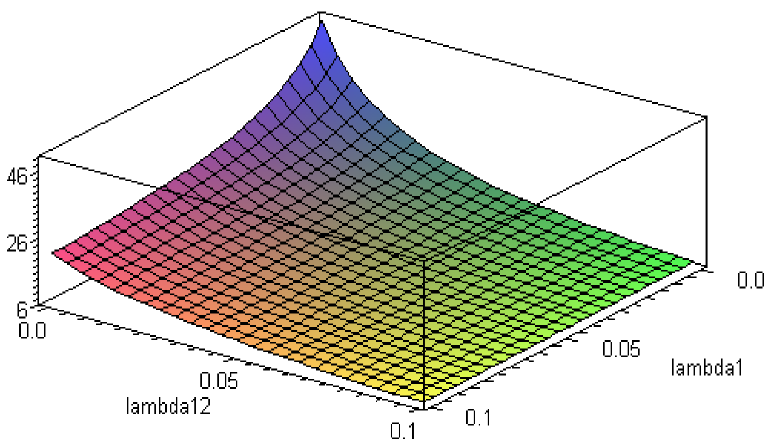

Fig.4: The mean time to system failure versus λ_1 and λ_{12}

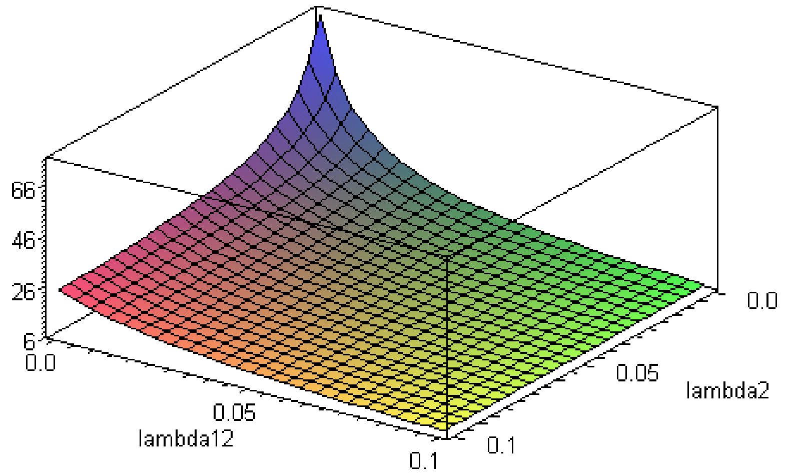

Fig.5: The mean time to system failure versus λ_2 and λ_{12}

As a special case we suppose that $\beta = 1$ and hence substituting the numerical values in the model (26)-(29) and using the equation (30), the availability function is obtained as follows and the results are shown graphically in Figure (6).

$$
\begin{aligned}
A(t) = {} & \big(0.1186239530 \\
& + 4.89760746510^{-10}\,\mathrm{I}\big)\, \mathrm{e}^{(-0.08440837706 - 8.\ 10^{-11}\,\mathrm{I})\,t} \\
& + \big(0.05977220731 \\
& + 1.99417958210^{-11}\,\mathrm{I}\big)\, \mathrm{e}^{(-0.3021007550 - 4.64101615\ 10^{-12}\,\mathrm{I})\,t} \\
& + \big(0.01864817306 \\
& - 4.02339306510^{-10}\,\mathrm{I}\big)\, \mathrm{e}^{(-0.1034908680 + 6.464101615\ 10^{-11}\,\mathrm{I})\,t} \\
& + 0.8029556650
\end{aligned}
$$

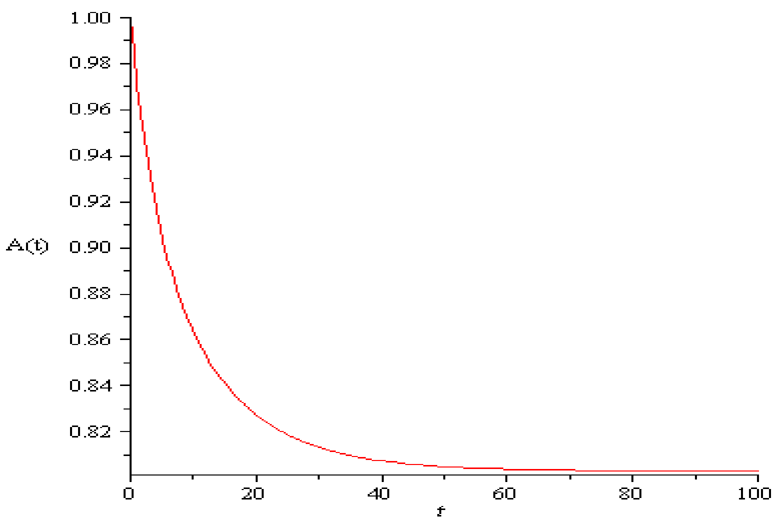

Fig.6: The availability function versus time

5. Conclusion

When components of a system fail, they do not necessarily fail independently of each other. The failures may be synchronised, and these cases have a common cause. Bivariate Weibull distribution is suitable and more flexible to model the life times of the dependent units. Bivariate Weibull model is a generalization of the bivariate exponential model. Markov model is a tool to analyze the availability and reliability of a system. It is not easy to solve system of differential equations to find the state probabilities of the model and in this case, numerical solutions can be obtained instead of analytical solutions.

References

Gnedenko, B.V. & Ushakov, I. (1995) Probabilistic Reliability Engineering, New York: Wiley & Sons.

Lawless, J.F. (1982) Statistical Models and Methods for Lifetime Data, John Wiley & Sons.

Chung, W.K. (1990) Common-cause failures and critical human errors in repairable and non-repairable systems, Microelectronics & Reliability, Vol. 30, No. 2, pp. 243-247.

Dhillon, B.S. (1979) On common-cause failures- a bibliography, Microelectronics & Reliability, Vol. 18, p. 533.

Dhillon, B.S. & Singh, C. (1981) Engineering reliability: new techniques and applications, John Wiley & Sons, New York, N.Y.

Shooman, M.L. (1968) Probabilistic reliability: an engineering approach. McGraw-Hill Co., New York, U.S.A.

Finkelstein, M.S. (1999a) Multiple availability on stochastic demand, IEEE Trans. Ref., Vol. 48, No. 1, pp. 19-24.

Gertsbakh, I.B. (1989) Statistical Reliability Theory, New York & Basel: Marcel Dekker.

Balakrishnan, N. & Lai, C.D. (2009) Continuous Bivariate Distributions, Springer, New York.

Marshall, A.W. & Olkin, I. (1967) A Multivariate Exponential Distribution, Journal of American Statistical Association, 62, 30–44.

Freund, J.E. (1961) A Bivariate Extension of the Exponential Distribution, Journal of American Statistical Association, 56, 971–977.

Bailey, N.T.J. (1964) The Elements of Stochastic Processes, Wiley, New York.

Rausand, M. & Høyland, A. (2004) System Reliability Theory: Models, Statistical Methods, and Applications, Wiley, Hoboken.

Pukite, P. & Pukite, J. (1998) Modeling for Reliability Analysis, IEEE, New York.

Block, H.W. & Basu, A.P. (1974) A Continuous Bivariate Exponential Extension, Journal of American Statistical Association, 69, 1031–1037.

Huang, L. & Xu, Q. (2010) Lifetime Reliability for Load-Sharing Redundant Systems with Arbitrary Failure Distributions, IEEE Transactions on Reliability, 59, 2.
Barlow, R.E. & Proschan, F. (1965) Statistical theory of reliability and life testing: probability models, Holt, Rinehart & Winston, Inc.