An Inventory Model for Deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging

R.Amutha, Dr.E.Chandrasekaran

Abstract - The Paper presents an inventory model for deteriorating products with demand as linear function of time and time dependent holding cost. A two-parameter weibull distribution is used to represent the distribution of the time to deterioration. In which shortages are allowed and partially backlogged, backlogging rate is variable and is dependent on the length of the next replenishment.

Index Terms— Deteriorating Products, Partial backlogging, shortage, time varying holding cost, Weibull distribution.

1 INTRODUCTION

Inventory is defined as an idle resource which helps us to run the business successfully and effectively. The inventory products can be classified into three categories based on their shelf life. They are obsolescence (b) Deterioration (c) Without deterioration. Deterioration is damage caused due to Spoilage, Dryness, etc. The equation for Inventory model for Deteriorating products is \( \frac{dI(t)}{dt} + \theta I(t) = -f(t) \) where \( \theta \) is the constant decay rate, \( I(t) \) the inventory level at time \( t \), and \( f(t) \) the demand rate at time \( t \).

In the classical inventory model the demand rate was assumed to be constant. But it is not always possible. For example seasonal goods (like greetings cards, Umbrella etc) demand rate is not a constant throughout the year. The demand rate may be time dependent, price dependent and stock dependent. Ajantha Roy developed an inventory model where demand rate is a function of selling price. Vipin Kumar, Sr Singh, Sanjay Sharma [7] have developed a production inventory model with constant demand rate is with which implies an uniform change in the demand rate of the product per unit time. Time – proportional demand was developed by Dave and Patel S.K.Ghosh and K.S.Chaudhuri[4] had discussed Quadratic time demand in their inventory model. Similarly the constant deterioration rate was relaxed by Covert and Philip. A two – parameter weibull distribution to represent the distribution of time to deterioration were considered by Zhao Pei-xin [15], S.K.Ghosh and K.S Chaudhuri [4], Azizul Baten and Anton Abdulbasah kamil [3] etc.

In general holding cost is assumed to be known and constant. But in realistic holding cost may not always be constant. Many researchers like C.K.Tripathy and U.Mishra [1], V.K.Mishra and L.S.Singh[12], Ajantha Roy[2] etc have discussed with time dependent holding cost.

In real life some times all the demand cannot be satisfied with the existing inventory. When shortages occur, all customers may wait until the arrival of the next order, all customers may leave the system or some customers are willing to wait for the next replenishment. V.K.Mishra and L.S.Singh[12 ] have developed an inventory model with variable as backlogging rate. Inventory model with partial Backlogging rate was developed by Liang-Yuh OUYAHG, K.S.W.U, M.C. Chany [9], G.P Samanta, Ajanta Roy [ 11], etc.

In this paper, an inventory model for deteriorating products is developed with time dependent on demand and holding cost. Shortages are allowed and partially backlogged. Backlogging rate is variable and is dependent on the length of the next replenishment. We use the same notation and assumptions as C.K.Tripathy, L.M.Panth [14] except some.

2 MATERIALS AND METHODS

Assumptions and Notations
The following are the assumptions and notations applied in the proposed model.

- The inventory system deals with single item
- The lead time is zero
- Shortages are allowed and are partially backlogged. During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative
3 MATHEMATICAL MODEL

During the period \((0, \mu)\) the inventory level is decreasing and at time \(t_1\) the inventory reaches zero level, where the shortages starts, and in the period \((t_1, T)\) some demands are backlogged.

The rate of Change of inventory during positive stock period \((0, t)\) and Shortage period \((t, T)\) is governed by the differential equations

\[
\frac{dI(t)}{dt} = -\mu \quad 0 \leq t \leq \mu
\]

\[
\frac{dI(t)}{dt} = (a+bt) \quad \mu \leq t \leq t_1
\]

\[
\frac{dI(t)}{dt} = -(a+bt) \quad t \leq t \leq T
\]

With boundary condition \(I(0)=S, I(t_1)=0\).

Solving the equations (1),(2) and (3) and neglecting higher powers of \(t\)

\[
I(t) = \int_{t_1}^{T} \left( 1 - e^{-\lambda(T-t)} \right) \ (a+bt) \ dt
\]

\[
I_1 = \frac{arT^2}{2} + \frac{bT^2}{3} - a\beta Ti + \frac{arTi^2}{2} + \frac{bTi^2}{3}
\]

Total amount of lost sales \(I_1\) during the period \((0, T)\) is

Total amount of shortage units \(I_s\) during the period \((0, T)\) is

Total amount of deteriorated items \(I_d\), during the period \((0, T)\) is

\[
I_d = \int_{t_1}^{T} \left( e^{-\lambda(T-t)} + \frac{bTi^2}{3} \right) \ dt
\]
Therefore total unit cost per unit time is given by

\[
P = \frac{1}{T} \left[ \text{ordering cost} + \text{Carrying cost} + \text{backordering cost} + \text{lost sale cost} + \text{purchase cost} \right]
\]

\[
= \frac{1}{T} \left[ A + C_{\text{In}} + C_{\text{Do}} + C_{\text{ls}} + C_{\text{Ch}} \right]
\]

\[
= \frac{1}{T} \left[ A + \frac{C_{\text{bd}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+3}}{\beta+3} + \frac{C_{\text{act}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+2}}{2(\beta+2)} \right. \\
\left. + \frac{C_{\text{act}}t_1^{\beta+1}}{2(\beta+1)} \right]
\]

\[
= \frac{1}{T} \left[ \frac{C_{\text{bd}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+3}}{\beta+3} + \frac{C_{\text{act}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+2}}{2(\beta+2)} \right. \\
\left. + \frac{C_{\text{act}}t_1^{\beta+1}}{2(\beta+1)} \right]
\]

\[
= \frac{1}{T} \left[ \frac{C_{\text{bd}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+3}}{\beta+3} + \frac{C_{\text{act}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+2}}{2(\beta+2)} \right. \\
\left. + \frac{C_{\text{act}}t_1^{\beta+1}}{2(\beta+1)} \right]
\]

\[
= \frac{1}{T} \left[ \frac{C_{\text{bd}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+3}}{\beta+3} + \frac{C_{\text{act}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+2}}{2(\beta+2)} \right. \\
\left. + \frac{C_{\text{act}}t_1^{\beta+1}}{2(\beta+1)} \right]
\]

\[
= \frac{1}{T} \left[ \frac{C_{\text{bd}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+3}}{\beta+3} + \frac{C_{\text{act}}t_1^{\beta+4}}{2(\beta+4)} + \frac{C_{\text{bct}}t_1^{\beta+2}}{2(\beta+2)} \right. \\
\left. + \frac{C_{\text{act}}t_1^{\beta+1}}{2(\beta+1)} \right]
\]

From equation (15)

\[
\frac{\partial P}{\partial t_1} = 0
\]

Optimal Value of \( t_1 \) can be obtained by solving the equation

\[
\frac{\partial P}{\partial t_1} = 0
\]

4 CONCLUDING REMARKS

In this paper, we developed a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. This model is useful not only for time dependent demand also for time dependent holding cost.

REFERENCES


Inventory Model with Time Dependent Demand and Partial Backlogging, Applied Mathematical Sciences, Vol. 4 No.72, 3611-3619.


