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Algorithm for switching 4-bit packages in full quantum network with multiple network nodes

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Abstract - in this research are offered two versions of an algorithm for superdense encoding of a 4-dimensional qubit vector. The algorithm for switching of 4-bit packages in a full quantum network with multiple network nodes includes superdense encoding and is implemented as quantum logic circuit. The proposed two algorithmic solutions and the applied quantum circuit open new perspectives for more effective methods for establishment of a full quantum switching network.

Key words: Quantum computing, diffraction, simulator, operators, gates

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1. INTRODUCTION

In the quantum communication networks very different channels must share limited amount of resources. In the quantum communication through various properties of the quantum mechanics it is possible to share a new type of communication resources. In the quantum communication circuit, one communication line can combine the information from all users, and at the end points, the original quantum data are recovered through decoding.

The quantum information opens new perspectives in the field of the communication [1]. The more efficient quantum algorithms [2, 3] and the more secure quantum cryptography [4, 5] are only some of the applications of the quantum computing.

The interest in the quantum computer networks continuously increases. Structural solutions for small operational quantum distributed networks [6, 7, 8, 9] have been already proposed and also qubit networks for teleportation have been proposed, which are compatible with the existing infrastructure with optical fibers [10, 11, 12]. Formalizations for quantum network elements [13, 14] also have been proposed, as well as new applications for delayed switching [15], which use certain properties of the quantum mechanics for performance of tasks, which are impossible for realization with classic networks. In the multiuser networks, often arises a need to share a channel with multiple access. The possibilities of the quantum communication networks expand the classical capacity for transmission of information. The superdense encoding [18] can be applied to multiple scenarios for access, using systems with more dimensions [19, 20, 21]. There are theoretical studies, whose results determine the border capacity of the quantum channels for multiple access, which transmit classical information [22], including derivations for quantum optical channels [23].

The classical information capacity of the quantum communication network with multiple access can be increased by specific quantum encoding techniques. There are practical optical circuits that exploit the superadditivity of the quantum encoding, i.e. the ability to obtain a capacity increase more than the proportional size of the code, in order to improve the efficiency of transmission of information [24, 25].

With an appropriate design of the algorithms, protocols and decoding circuits for quantum communication, can be achieved further increase of the amount of the superdense encoded packages , which also can allow additional optimization of the capacity of the channel, and even to adapt the capacity of the channel according to the instantaneous needs of the commutators [26].

In the communications networks there is a limited amount of resources, which the users must share. Precisely in this area appear numerous techniques for access [29, 30]. The most communication systems use one or another form of multiplexing, i.e. the sharing of the channel from different users. In the multiplexing, the information of many users is transmitted over one channel. The information is transformed, in order to be reduced the use of the most critical resources of the communication system, as well as to be used at maximum the information capacity of the channel

2. THE NETWORK

Let's view the following network of quantum computers and channels:

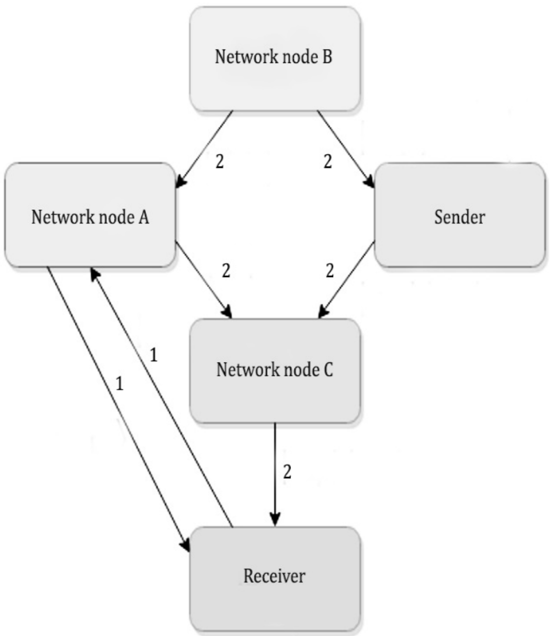

Figure 1 Quantum network

Each node in the diagram represents a quantum computer. The quantum computers can receive, process, introduce and send qubits.

Each end in the diagram represents a one-way quantum channel, and the number next to the end is the number of times the channel can be used to send a single qubit. For example, two qubits can be sent via the channel from the sender to network node C, but only one qubit can pass through the channel from the receiver to network node A .

There is no pre-existing entanglement between the nodes.

The goal is to find a way to pass **four** classical bits of information from the sender to the receiver, as the qubits are processed at each node, sending them along the given channels and observing the restrictions for capacity.

For example, the sending of two classical bits of information is easy. They simply have to be encoded in the obvious way, to be transmitted from the sender to network node C to the receiver and to be measured.

Of course, the actual solution is a little more complicated. A superdense encoding can be used for transmission of two classical bits through a quantum bit, by consuming a Bell pair. Another hint is that the classical bits aren't the only things amenable to superdense encoding.

Forced movements

Before the solution for the network to be explained, let's clarify several segments, which are defined by common constraints. The absolute maximum number of classical bits, which are sent per qubit via superdense encoding is two. The outer capacity of the sender is two, and four bits should be sent. That is why the entire outer capacity of the sender must be transmitted to superdense encoded bits. The superdense encoding requires a Bell pair. The sender must receive half of the Bell pair, in order to encode the message, and the receiver needs to end up with both halves of the pair, in order to decode the message. The only place at which the sender may receive Bell pair halves, where the other half eventually may end up at the receiver, is from network node B. So network node B must create two Bell pairs, called conditionally u and v, and must use the entire capacity of its channels to transmit half of u and v to the sender and network node A.

On the other side of the network must be observed, that the receiver has inner capacity 3 and outer 1. Without pre-existing entanglement the maximum number of classical bits, which can be received per qubit, is one. The outer quantum capacity must be changed as inner classical capacity. Namely, the receiver is forced to create a Bell pair, which is called conditionally w, and to send one of the halves of w through the outward link. Once these forced movements are noted, the following situation is observed:

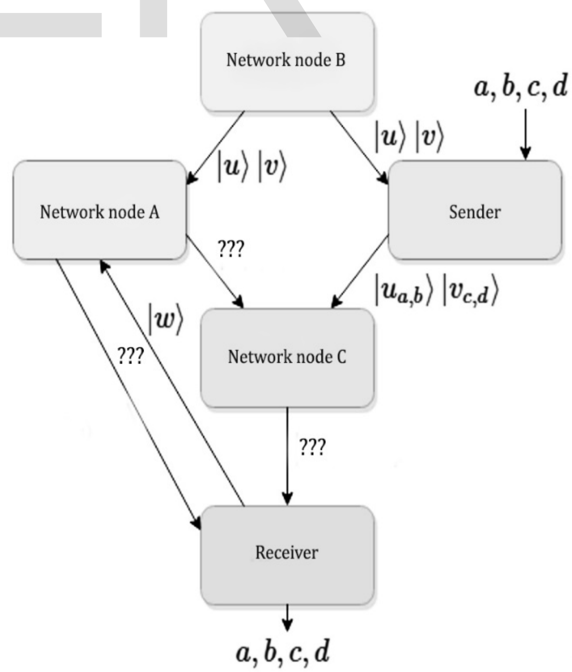

Figure 2 Quantum network

U, v, $u_{a,b}$ and $v_{c,d}$ must be sent to the receiver, in order for the superdense decoding to be performed. There are four qubits for sending, but the capacity for receiving is only three qubits. But it turns out that the superdense encoding is a little more flexible than it seems.

Superdense Bell Pairs

An entangled pair allows the placement of unitary matrix with 4 coefficients in the shared system (unlike the normal unit vector with 2 coefficients). The phase space of 2x2 unitary matrices can be parametrized as $U_{\phi,\theta,v} = e^{\phi i}(Icos\theta + \hat{v}\sigma_{xyz}sin\theta)$, where ϕ and θ are angles, and $\hat{v}$ is a unit vector in $\mathbb{R}^3$. Φ can be ignored because the global phase factors have no measurable effect. In addition θ could be unfold in $\hat{v}$, in order to be obtained a unit vector $\hat{v}_4$ in $\mathbb{R}^4$.

What that means is: it is likely to be encoded an arbitrary real unit 4-dimensional qubit vector into an entangled state. This in fact is possible and the vector can be decoded in amplitudes on the receiving side. In fact the existing process on superdense encoding is already sufficient. The ability to send a unit real 4-dimensional qubit vector is sufficiently significant achievement, as the ability to send two qubits. When two qubits are sent, in a normal way, this actually means sending a unit 4-dimensional qubit vector. It's just a complex unit 4-dimensional qubit vector instead of a real unit 4-dimensional vector. As a consequence, it turns out that the sending of qubits through superdense encoding is possible, as long as their phase information is limited to positive-vs-negative. Arbitrary qubits can not be sent, but "flat" qubits can be sent. But how often the qubits are flat? All intermediate states of the Grover's algorithm are flat. And the quantum compression preserves the flatness. The most useful example are the Bell pairs: qubits in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11)$ do not require phase information, because their phase is constantly zero.

Let's one shared Bell pair turn into two. The sender applies a Hadamard gate to two additional qubits, by putting them in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1)$, then superdense encodes the classical bits in the existing Bell pair half. After superdense decoding on the other side, the resulting qubits are entangled together with the sender's qubits. Two Bell pairs from one! But a qubit must be sent and a Bell pair must be used, in order to achieve this, so that in the end, the situation is "two steps forward and one step back". The sent qubit could simply be applied for sending a normal Bell pair half. There's probably cryptographic applications, using superdense encoding in a similar way, but it is not useful in respect of the capacity of the channel.

Next thing, which can be tried, is to send both halves of a Bell pair from a third side via superdense encoding. A problem appears immediately: The process of superdense encoding doesn't move the qubits into an entangled state, but copies them in such. This is a problem when a Bell pair half have to be sent, because it makes a third half! In other words, the Bell pair is no longer a Bell pair, but it's a GHZ state (i.e. three qubits in the state $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$). Why obtaining a GHZ state is a problem? For example, a superdense encoding can not be performed with GHZ state spread over three parts. And in order to cancel a qubit from a GHZ state, by turning back into a Bell pair, it is necessary some of the other qubits in the state to be in the same place, at the same time.

It may be expected that the sender, who has just made an additional copy, would have access to this copy and could use it to cancel out the original. However, since that copy is superdense encoded into a Bell pair, there is no way to extract it without both halves of the pair! If there was a way to do this, then the receiver would have it done immediately by his side, which means that superluminal communication would be possible. If the receiver have to do the canceling, this solution would work out, but then must be spent channel capacity to move the waste copy. This would make pointless the idea for using superdense encoding in the first place. So the only way is an intermediate node (network node *C*) to perform the cleanup!

Solution 1

First let network node *B* create two Bell pairs (u and v) and to transmit them to network node *A* and the sender. The sender receives one half from u and another half of v. The other halves go to network node *A*. At this stage the receiver creates a Bell pair w and sends one half to network node *A*. Then, network node *A* superdense encodes 4 bits of classical information (a, *b, c, and d*) in u and v. This creates $u_{a,b}$ and $v_{c,d}$, which the sender sends to network node *C*.

Here is the moment in which network node *A* superdense encodes u and v in w. This creates $w_{u,v}$, but turns the copies of u and v, still held by network node *A*, into waste. Network node *A* forwards this waste to network node C. Network node *A* also sends $w_{u,v}$ to the receiver. Network node C cleans up the waste by turning $u_{a,b}$ with controlled NOT in u, a $v_{c,d}$ and v. This cancels the waste u *and* v, leaving the qubits which have encoded b and d. After this network node C forwards $u_{a,b}$ and $v_{c,d}$ to the receiver. Finally, the receiver uses its half of w, to decode superdense $w_{u,v}$ in u and v. Then uses u and v, to decode superdense $u_{a,b}$ and $v_{c,d}$ in qubits, which return a, *b, c* and *d* upon measurement. Below is given the network with all ends, annotated with the information passing through them.

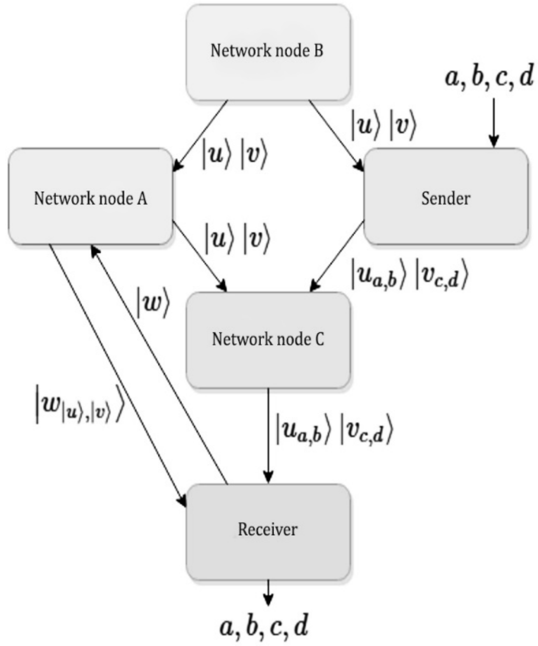

Figure 3 Quantum network

And here is given a circuit diagram, showing the exact operations that are occurring. Each area corresponds to a node in the network diagram:

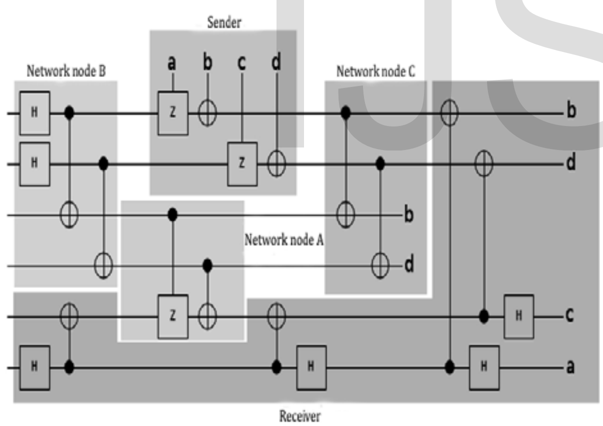

Figure 4 Quantum circuit

Solution 2
Splitting the node for cleaning of the unnecessary data into two parts

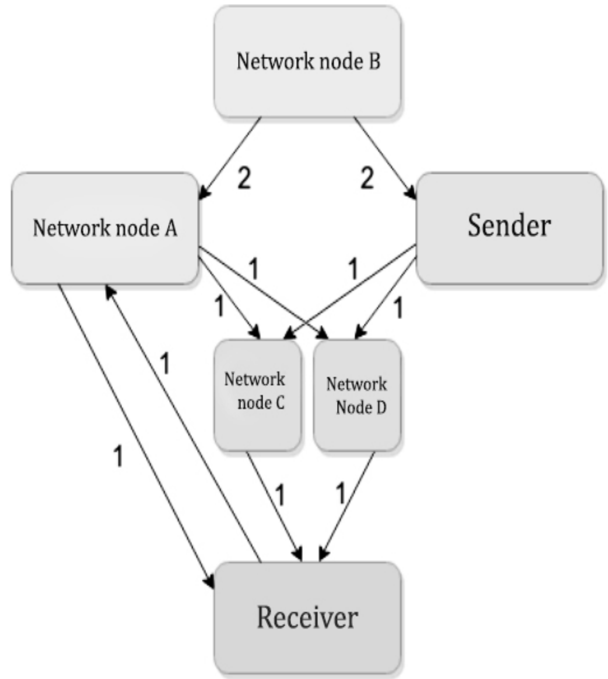

Figure 5 Splitting the cleaner network node

At the second proposed solution the problem connected with the clean up of the unnecessary information is divided into two subproblems, for each of which is corresponding by one network node, each of which sends by two bits.

The functionalities of the first two bits are already clarified. *B* creates an entangled pair of qubits and sends one half to A and the other half to the Sender. *A* forwards forward its entangled qubits directly to the receiver.

The sender superdense encodes two bits and sends the resulting qubit to the receiver through network node *C*.

The receiver decodes two bits in the usual way. This operation requires by one qubit for each of the links: B -> A, A -> Receiver, B -> Sender, Sender -> C, и C-> Receiver.

The operations on the other two bits are the following. *B* generates an entangled pair of qubits and sends one half to C through A, and the other half to the Sender. The sender superdense encodes both remaining bits and sends the encoded qubit to *C*. *C* decodes two bits.

The receiver generates entangled pair of qubits, retains the half, and sends the other half to C by A. *C* superdense encodes both, so this network node decodes the entangled qubit from the receiver and sends the encoded qubit to the receiver. The receiver decodes it.

This algorithm sends by one qubit for each link: B -> A, B -> Sender, Sender -> C, C -> Receiver and Receiver -> A, as well as two qubits on the link: A -> C.

If this logic is added in the circuit upwards, this would allow the entire possible capacity of all links to be used.

3. CONCLUSION

The superdense encoding of 4-dimensional qubit vector works, but the qubits must have "flat" phases, without amplitudes with imaginary components, also the qubits must be copied, and not to be moved.

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