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Algebraic models of transitions between mixed entangled states and specific eigenvalues of systems with two or three levels

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Abstract - In this study are examined the recent theoretical studies and applications of pure and mixed double and triple-entangled states. After getting acquainted with the basic concepts of the traditional methodologies for entanglement, are summarized the main phenomena and observations of the various approaches for multidimensional entanglement. More specifically, we explore the impact of the various parameters of these systems of the entanglement.in this research is proposed algorithmic model for transformation of mixed entangled states, a disappointing qubit can be removed by a GHZ state through the measurement of it along the spin axis perpendicular to the axis of entanglement and with the aid of the result of the measurement to be made a correction of the phase.

Key words: Quantum computing, multidimensional entanglemen, operators, gates

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1. INTRODUCTION

Any multipartite unitary transformation can be factorized as a product of unipartite gates and bipartite CNOT gates [1]. The controlled multipartite interaction between the qubits creates the so-called entangled states, which are interesting except for the fundamental study of the quantum mechanics, and also find application in the ultra-precision spectroscopy [7] and in the quantum information [1]. An entangled state is a multipartite state, the wave vector of which cannot be represented as a Tensor product of the individual single-part wave vectors. As an example of such entangled state is the two-qubit Bell state

$$|Bell\rangle = \frac{1}{\sqrt{2}}(|0_1\rangle|0_2\rangle + |1_1\rangle|1_2\rangle)$$

Where $|n_{1,2}\rangle\,(n = 1,2)$ is the state, respectively of the first and second qubit. According to the probabilistic interpretation of the quantum mechanics, if the first qubit is found in the state $|0_1\rangle$ or $|1_1\rangle$, then the second qubit will be in the state $|1_2\rangle$ or $|0_1\rangle$, even when there is no physical interaction between them.

In this study are examined the recent theoretical studies and applications of pure and mixed double and triple-entangled states. After getting acquainted with the basic concepts of the traditional methodologies for entanglement, are summarized the main phenomena and observations of the various approaches for multidimensional entanglement. More specifically, we explore the impact of the various parameters of these systems of the entanglement. The specific advantages of the use of the atomic Wehrl and Shannon entropy are highlighted. On the basis of this result, we propose a general model for the reduction of triple-entanglement to a system with two levels. We reveal new normal algebraic models for transitions between mixed entangled states and specific eigenvalues of systems with two and three levels, as well as some remarkable properties of the entanglements, which may reveal a new look on the quantum correlations, which are present in the models on several levels. In addition, we propose an intuitive idea for the behavior of mixed entangled state in the presence of the decoherence. In this study numerically is identified and demonstrated the region of the parameters, in which can be obtained a significant entanglement.

Entangled states are experimentally demonstrated in various physical systems, such as ions in ion trap, photons, atoms in a resonator, Bose-Einstein condensate in an optical grid, quantum points, etc. A step to the understanding of the role of the entangled states in the quantum information is the introduction of the model of the unidirectional quantum computer [8]. In this new model, the system of qubits is prepared in an entangled cluster state. The creation of various one-qubit and two-qubit gates is carried out by a measurement of a certain number of qubits, in this way the cluster state is destroyed, therefore the process is irreversible (unidirectional). An interesting problem in the quantum information is the use of systems with more than two states, called quNits. The reason for this is the fact that in a system of N states the information is encoded in 2 (N-1) real parameters. As a comparison, the qubit information is encoded in two parameters: one population and one phase. Therefore, the use of quNits instead of qubits would lead to a significant reduction of the number of parts, necessary for the carrying out of a given quantum algorithm. A major problem in the quantum information is the unwanted interaction between the qubits and their surrounding environment, leading to an irreversible loss of coherence. An example of such incoherent processes are the dephasing and the spontaneous emission.

2. THE APPROACH

General requirements when measuring entanglement:

• C = 0 for multiplication of states $\rho = \rho_A \otimes \rho_B$.

• C is a constant for local unitary transformations. The measurement is independent of the choice of basis.

The measure, which fulfills these requirements for the pure states is the entropy of entanglement. This is one of the simplest measures for quantum entanglement. It uses the von Neumann entropy of the operator for density

$$S(\rho) = -Tr\{\rho \, log_2(\rho)\} \qquad (1)$$

It disappears for a pure state, when all populations are 0 or 1 and reaches its maximum for a completely mixed state, when

$$S\left(\frac{1}{N}1\right) = -\frac{1}{N}Tr\left(\rho \, log_2\frac{1}{N}1\right) = log_2 N \qquad (2)$$

where N is a dimension of the Hilbert space. The Von Neumann entropy is related to the measure for information of Shannon, which is important in the context of information capacity and to the Gibbs entropy from the statistical mechanics. An useful interpretation of the von Neumann entropy is that it represents the minimum number of the bits needed for storing the result of a random variable: A pure state $\rho_1 = |\Psi\rangle\langle\Psi|$ can always be stored in its eigenbase as

$$\rho_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (3)$$

Its entropy disappears,

$$S(\rho_1) = 1\, log_2(1) + 0 log_2(0) = 0 \qquad (4)$$

An appropriate measurement of the observed σ_z which always gives the result +1 and information obtained from such a measurement disappears. For the maximum mixed state

$$\rho_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (5)$$

However, the entropy reaches its maximum value

$$S(\rho_2) = -Tr\left\{\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} log_2 \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\right\} =$$

$$Tr\left\{\begin{pmatrix} log_2 2 & 0 \\ 0 & log_2 2 \end{pmatrix}\right\} = 1 \qquad (6)$$

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Here each binary variable generates completely random values. Therefore each result must be represented in one bit, compression is not possible.

The entropy of entanglement for bipartite pure states is determined by the von Neumann entropy of one of the reduced states:

$$E(\rho) = S(\rho_A) = S(\rho_B) \qquad (7)$$

where $\rho_A = Tr_B(\rho)$ and vice versa. If ρ is a product state, as $|\uparrow\uparrow\rangle$, ρ_A and ρ_B are pure states and the entropy disappears. If the state is maximally entangled, e.g.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \qquad (8)$$

the subsystems become completely mixed, $\rho_A = \rho_B = \frac{1}{2}1$. The corresponding entropy of entanglement, the entropy of the maximally entangled 2-qubit states is $E(\rho) = S(\rho_A) = S(\rho_B) = 1$

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Concurrence of pure 2-qubit states

$$|\Psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle + \delta|\downarrow\downarrow\rangle \qquad (9)$$

is

$$C := 2|\alpha\delta - \beta\gamma| \geq 0 \qquad (10)$$

C (Ψ_1) = 0, i.e. the state is not entangled. In the same way, for

$$\Psi_2 = \frac{1}{2}(|\uparrow\rangle + |\downarrow\rangle)\otimes(|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{2}(1,1,1,1) \qquad (11)$$

again we find C (Ψ_2) = 0.

The effect of an "entangling gate", is similar to the one of a CNOT gate, if ϕ = π,

$$CN = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \cos\frac{\varphi}{2} & -\sin\frac{\varphi}{2} \\ & & \sin\frac{\varphi}{2} & \cos\frac{\varphi}{2} \end{pmatrix} \qquad (12)$$

But

$$\Psi_3 = CN\,.\,\Psi_2 = \frac{1}{2}\left(1, 1, \cos\frac{\varphi}{2} - \sin\frac{\varphi}{2}, \cos\frac{\varphi}{2} + \sin\frac{\varphi}{2}\right) \qquad (13)$$

This corresponds to "pre-measurement" in the theory of the quantum measurement that entangles the system with the apparatus. For this state, the concurrence is $\Psi_3 = \sin\frac{\varphi}{2}$. Therefore, the state entangles for each end angle φ. The entanglement reaches its maximum of $1/2$ for ϕ = π, where CN ≈ CNOT, with the exception of the sign - and returns to 0 for ϕ = 2π.

Also the entropy of the entanglement for this state can be calculated. The operator of full density has the form

$$\rho_3 = \frac{1}{4}\begin{pmatrix} 1 & 1 & C_- & C_+ \\ 1 & 1 & C_- & C_+ \\ C_- & C_- & 1-\sin\varphi & \cos\frac{\varphi}{4} \\ C_+ & C_+ & \cos\frac{\varphi}{4} & 1+\sin\varphi \end{pmatrix} \qquad (14)$$

Where

$$C_{\mp} = \cos\frac{\varphi}{2} \mp \sin\frac{\varphi}{2} \qquad (15)$$

In this article is shown how to "erase" a qubit of GHZ state.

For the subsystems is obtained

$$\rho_A = Tr_B(\rho) = \frac{1}{2}\begin{pmatrix} 1 & \cos\frac{\varphi}{2} \\ \cos\frac{\varphi}{2} & 1 \end{pmatrix} \qquad (16)$$

$$\rho_B = Tr_A(\rho) = \frac{1}{2}\begin{pmatrix} 1-\frac{1}{2}\sin\varphi & \cos^2\frac{\varphi}{2} \\ \cos^2\frac{\varphi}{2} & 1+\frac{1}{2}\sin\varphi \end{pmatrix} \qquad (17)$$

Where we use trigonometric identity $1 + \cos\varphi/4 = \cos^2\varphi/2$. The difference between ρ_A and ρ_B reflects the asymmetric role between the control and the target bit in the CNOT operator.

The dependence is different from the one of concurrence $C(\Psi_3)$ for the same state, which starts

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linearly with φ and reaches a maximum value of 0.5. However, both entanglements reach the maximum for one and the same state and disappear when the state is separable.

For density matrices, the concurrence is defined as

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad (18)$$

Where are λ_i eigenvalues of a Hermitian operator in increasing order.

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}} \qquad \tilde{\rho} = (\sigma_y \otimes \sigma_y)\, \rho^* (\sigma_y \otimes \sigma_y) \quad (19)$$

The concurrence and the entropy determine quantitative the entanglement between 2 qubits. In a 3-qubit system ABC, the qubits can be more entangled by pairs, i.e. A can become entangled with B or C, but there are also three-way entangled states, which are not entangled by pairs.

The three-way entanglements can be quantitatively determined by several measures for entanglement, which are called "tangle".

$$\tau_2 = \frac{C_{12}^2 + C_{23}^2 + C_{13}^2}{3} \quad (20)$$

Where C_{ik} measures the entanglement by pairs between qubits i and k. Each of them is determined by tracing through the third qubit and then using equation (18) for the resulting 2-qubit subspace, which can be pure or entangled.
The entanglement between the one qubit and two others can be measured by bipartite concurrence

$$C_{i(jk)} = \sqrt{2 - 2Tr(\rho_i^2)} \qquad (21)$$

Where ρ_i is the subsystem of qubit i, obtained by tracing over the two other qubits. If the pure 3-qubit state is a product state, ρ_i is a pure state and therefore $\rho_i = \rho_i^2$ and $\mathrm{Tr}(\rho_i^2) = 1$ and $C_{i(jk)} = 0$. For an entangled state $\mathrm{Tr}(\rho_i^2) < 1$ and $C_{i(jk)} > 0$. For a maximally entangled state $\rho_i = \frac{1}{2}1$ and $C_{i(jk)} = 1$.

This bipartite concurrence indicates whether a given qubit i is entangled with only one of the two other qubits or with both. This can be determined quantitatively by triple-entanglement τ_3, which subtracts the entangled pairs of qubits i with j and k from the bipartite concurrence in order to obtain the essential three-way entanglement of a pure three qubit state:

$$\tau_3 = C_{i(jk)}^2 - (C_{ij}^2 + C_{ik}^2) \quad (22)$$

The difference between pure 2-way and 3-way entanglement can be viewed by considering the GHZ and W states:

$$|W\rangle_{001} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$
$$|GHZ_{\mp}\rangle = \frac{1}{\sqrt{2}}(|000\rangle \mp |111\rangle)$$

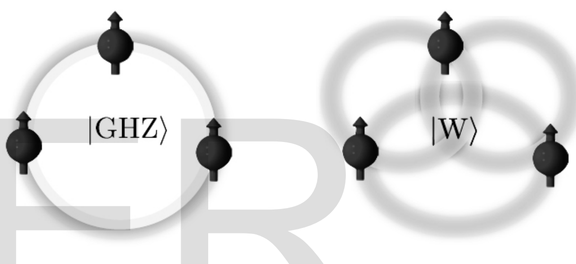

Figure 1 GHZ and W entanglements

The essential difference between these states becomes evident, if a measurement on one of the three qubits is carried out. In the case of the GHZ state, if for example is measured qubit 3 and is obtained the result 0, the system collapses in the state $|000\rangle$. Clearly this is not anymore an entangled state, and the measurement of each one of the qubits completely destroys the entanglement. This is due to the nature of the three-way entanglement. If the third qubit of the W state is measured and is obtained the result 0, the states $|010\rangle$ и $|100\rangle$ are preserved, at which the first two qubits are still maximally entangled. For that reason, this type of entanglement is called bipartite entanglement.

The various types of entanglements are complementary to each other: If the system is three-way entangled, its bipartite entanglements cannot

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be large. This can be expressed quantitatively for a system with three qubits

$$\tau_3 + \tau_2{}^{(k)} + S_k^2 = 1 \qquad (23)$$

Here, S_k characterizes quantitatively the single state of qubit κ. $\tau_2{}^{(k)}$ is the two-way entanglement of the qubit κ with the other qubits and τ_3 expresses the nature of the three-way entanglement.

GHZ Triplets and Bell Pairs

A Bell pair is a set of two qubits in superposition of all OFF and ON, i.e. in the state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$. A GHZ state is similar to a Bell pair, but with more involved qubits. For instance, a GHZ triplet is a set of three qubits in the state $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$.

It can be expected that, the qubits in a GHZ state are "more entangled" compared to the qubits in a Bell pair, because the superposition is greater, but in fact the opposite is true. Due to the monogamy of the entanglement, the qubits in a Bell pair are more entangled with one another than the qubits in a GHZ state. The third qubit in a GHZ triplet has a tendency to be rather unnecessary than useful.

Because the Bell pairs can be used for certain tasks, which cannot be carried out by GHZ states (e.g. superdense coding), it is good to be reduced a GHZ state into a Bell pair by removing one of the qubits. Previously it was accepted, that the only way for this is to find the unwanted qubit with a controlled NOT, controlled by one of the other participating qubits. This clears the unwanted qubit by reversing its value in the part all-ON of the superposition while leaving only in the part all-OFF of the superposition.

The approach with the controlled NOT works well, but requires the unwanted qubit to be in the same place as one of the other qubits (due to the quantum controlled operation). The satisfaction of this condition, usually, requires moving the qubits (e.g. if quantum channels with available bandwidth are necessary).

It appears that it is possible the payment of this price of the quantum bandwidth to be avoided. By finding the qubit with a Hadamard gate to cover its value, measuring it, and using the result of the measurement to fix the problem with parity of the phase, it is only necessary to be used a classical bandwidth. This is called the "erasing" of the qubit.

Manipulation of the circuit

The easiest way for understanding the approach with the "erasing" is by starting with the circuit for the approach with controlled NOT and applying several simple, apparently correct transformations.

For a start is given a circuit that creates a GHZ triplet, then uses a controlled NOT in order to eliminate the third qubit of the GHZ state:

Creates a GHZ state

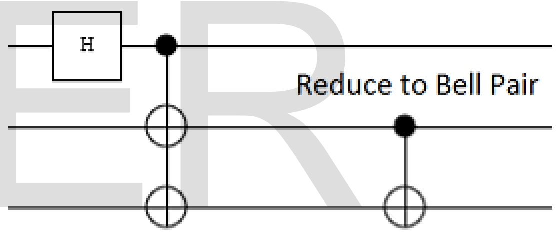

Figure 2 GHZ state 1

After the third qubit has been cleared, it can be found with any operation (since it is not used for anything else). With the aid of the power of the informed foresight let it be found with a Hadamard gate and then measured:

Creates a GHZ state

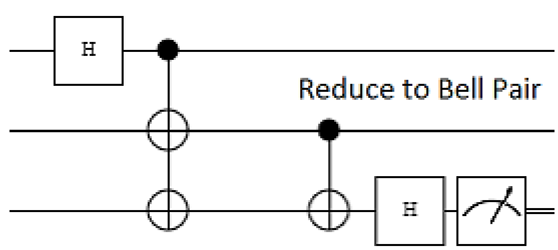

Figure 3 GHZ state 2

Now is the time to jump the Hadamard gate over the NOT gate. This is permitted, but transforms the

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value-shifting NOT gate into a phase-shifting Z gate (because $H \cdot X = Z \cdot H$):

Creates a GHZ state

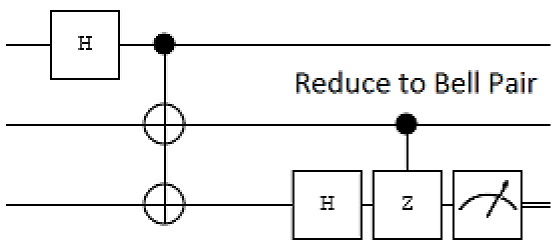

Figure 4 GHZ state 3

The Z gates are similar to controlled operations, in that they have no effect on qubits that are OFF. In the end, the exchange of a Z gate with one of its controls does not change its effect. Let's check this:

Creates a GHZ state

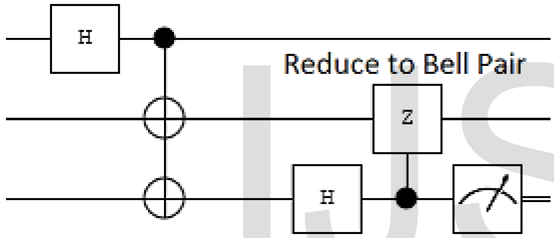

Figure 5 GHZ state 4

The presence of the control of the third wire is useful, because the controls are moving with measurements (i.e. the classical conditions are equivalent to the quantum conditions). This allows the performance of the phase correction after the measurement instead of before it:

Creates a GHZ state

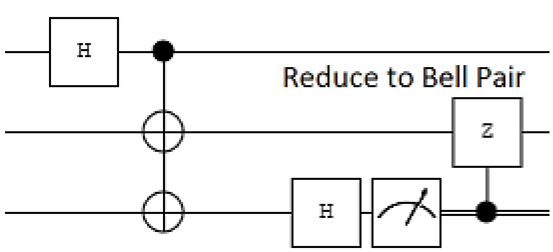

Figure 6 GHZ State

This is the final circuit:

1. It begins in a state $|000\rangle$.
2. It creates a GHZ triplet in state $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$.
3. If finds the third qubit with a Hadamard, passing to the state $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|001\rangle + \frac{1}{\sqrt{2}}|110\rangle + \frac{1}{\sqrt{2}}|111\rangle$.
4. It measures the third qubit, shrinking the system into either the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$, or in the state $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)|1\rangle$.
5. It fixes the minus sign in the result "the third qubit was ON" with a Z gate, controlled by the result of the measurement.
6. It ends with the first two qubits unconditionally in a Bell pair in the state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$.

Still is necessary the sending of information about the third qubit to the second qubit, but the transmitted information is classical (i.e. a result of measurement) instead of quantum (i.e. the original qubit). The method also works for larger GHZ states involving more qubits: qubits can be removed from the state one by one through the application of Hadamard+measurement+conditional Z qubit still in the state.

Updated solution of "Algorithm for switching 4 - bit packages in full quantum network with multiple network nodes"

Because the solution from the previous article for a puzzle for quantum network flow [20] involves removal of a qubit of a GHZ state, with the aid of the "reduction" is allowed several parts of the network to be downgraded from quantum to classical.

Here is a diagram of the improved decision:

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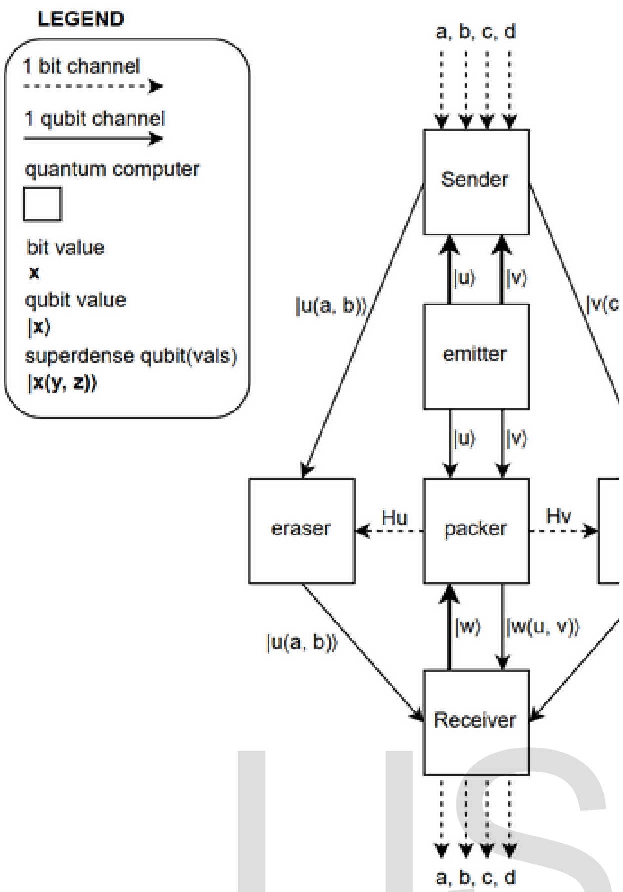

Figure 7 Algorithm for switching 4 - bit packages in full quantum network with multiple network nodes

3. CONCLUSION

Through the proposed algorithmic model for transformation of mixed entangled states, a disappointing qubit can be removed by a GHZ state through the measurement of it along the spin axis perpendicular to the axis of entanglement and with the aid of the result of the measurement to be made a correction of the phase.

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