APPLICATION OF POLYNOMIAL EIGEN VALUE DECOMPOSITION

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Abstract--The algorithm to compute the Eigen value decomposition of a Para Hermitian polynomial matrix is described. This lead to diagonalizing the polynomial matrix by the Para unitary “similarity” transformation. The algorithm uses Para unitary matrix and perform generalization of conventional Hermitian matrix diagonalization. A convergence proof is presented. The application to Broadband Signal Subspace Decomposition, signal processing, MIMO Precoding for Filter Bank Modulation Systems is highlighted in terms of strong decorrelation and multichannel data compaction. The performance parameters are presented to demonstrate the capability of the algorithm.

I. INTRODUCTION

Polynomial matrices have been used for many years in the area of control. They play an important role in the realization of multivariable transfer functions associated with MIMO systems. Few years back they have become more widely used in the context of (DSP) digital signal processing and communications [22]. Broadband subspace decomposition [12], characteristic areas of application include broadband adaptive sensor array processing [23], [24], MIMO communication channels [12] [26], and digital filter banks for sub band coding [25] or data compression [24].

A polynomial matrix is simply a matrix whose elements are polynomials. It may be viewed equivalently, as a polynomial with matrix coefficients. In this paper, we will use the term polynomial to include Laurent polynomials which can include negative powers of the indeterminate variable. We denote a polynomial matrix in the indeterminate variable.

Numerical procedures have previously been developed for a range of polynomial matrix factorization and reduction operations such as the Smith–McMillan decomposition [23]. To date, however, very little attention seems to have been devoted to polynomial matrix techniques equivalent to the eigenvalue decomposition (EVD) or singular value decomposition (SVD) for conventional matrices with scalar elements [11]. The development and implementation of such a technique is the subject of this paper. The Eigen value decomposition of conventional Hermitian matrices plays a major role in DSP. For example, it is at the heart of the Karhunen–Loeve transform for optimal data compaction. This paper comprises of the following sections as, PEVD technique in section II, application of PEVD in section III, and conclusion in section IV.

II. TECHNIQUE OF POLYNOMIAL EIGEN VALUE DECOMPOSITION.

The Eigen vectors are numbers and vectors associated to square matrices, and jointly they give the Eigen-decomposition of a matrix which analyzes the structure of this matrix. Although the Eigen decomposition does not be present for all square matrices, it has a mainly simple expression for a class of matrices frequently used in multivariate analysis such as correlation, covariance, or cross-product matrices. The Eigen-decomposition of this type of matrices is vital in data because it is used to find the maximum or minimum of functions linking these matrices. For example, principal component analysis is obtained from the Eigen-decomposition of a covariance matrix and gives the least square approximation of the original data matrix. Eigenvectors are also referred to as characteristic vectors and latent roots or characteristic equation. These Set of Eigen values of a matrix is also called its Spectrum.

A. NOTATIONS AND DEFINITION

There are a number of ways to describe Eigen vectors; the most frequent approach defines an eigenvector of the matrix A. As a vector u that satisfies the following equation

\[ Au = \lambda u \]  \hspace{1cm} (1)

When rewritten, the equation (1) becomes

\[ (A - \lambda I) u = 0 \]  \hspace{1cm} (2)

Where \( \lambda \) is a scalar called the eigenvalue associated to the eigenvector. In a similar manner, we can also state that a vector u is an Eigen vector of a matrix A if the length of the vector is distorted when it is multiplied by A. Traditionally we put together the set of eigenvectors of A in a matrix denoted U. Each column of U is an eigenvector of A. The Eigen values are stored in a diagonal matrix denoted as \( \Lambda \), where the diagonal
element gives the Eigen values and all other values are zeros and the first equation can be written as

\[ AU = UA \]  
\[ \text{Or} \]  
\[ A = UAU^{-1} \]

B. POSITIVE DEFINITE MATRICES

Positive semi definite matrices used extremely in statistics. The Eigen decomposition of these matrices always exists, and has a mostly appropriate form. A matrix is said to be positive semi-definite when it can be get as the product of a matrix by its reverse. These imply that a positive semi-definite matrix is always symmetric. So, formally, the matrix A is positive semi-definite if it can be obtain as

\[ A = X X^T \]

For a definite matrix X, Positive semi-definite matrices of particular relevance for multivariate analysis positive semi-definite matrices include correlation matrices. Covariance and, cross-product matrices. The important property of a positive semi-definite matrix that its eigenvalues are eternally positive or null, and that its eigenvectors are pair wise orthogonal when their eigenvalues are different. The eigenvectors are also poised of real values. Because eigenvectors corresponding to different eigenvalues are orthogonal, it is likely to accumulate all the eigenvectors in an orthogonal matrix. This implies the following equality

\[ U^{-1} = U^T \]

We can, therefore, express the positive semi-definite matrix A as

\[ A = UAU^T \]

Where \( U^T U = I \) are the normalized eigenvectors; if they are not normalize then \( U^T U \) is a diagonal matrix.

C. DIAGONALIZATION

When a matrix is positive semi-definite we can rewrite Equation 7 as

\[ A = UAU^T \]

This shows that we can change the matrix A into an equivalent diagonal matrix. As a consequence, the Eigen-decomposition of a positive semi-definite matrix is frequently referred to as its diagonalization.

D. EIGEN-DECOMPOSITION STATISTICAL PROPERTIES

The Eigen decomposition is important because it is disturbed in problems of optimization. For example, in main component examination, we want to analyze I×J matrix X where the rows are account and the columns are variables telling these observations. The goal of the study is to find row factor score, such that these factor scores give details as much of the variance of X as possible, and such that the sets of factor score are pair wise orthogonal. This sum to achieve matrix as

\[ F = XP \]

Under the constraint that

\[ F^T F = P^T X^T XP \]

Is a diagonal matrix /orthogonal matrix)

\[ P^T P = I \]

Where P is an orthonormal matrix. There are numerous ways of obtaining the solution of this problem. One possible advance is to use the technique of the Lagrangian multipliers where the limitation from Equation 11 is expressed as the multiplication with a diagonal matrix of Lagrangian multipliers denoted \( \Lambda \) as

\[ \Lambda (P^T P - I) \]

This amount to defining the following equation

\[ L = F^T F - \Lambda (P^T P - I) = P^T X^T XP - \Lambda (P^T P - I) \]

In order to find the values of P which give the maximum values of L, we first calculate the derivative of L relative to P

\[ \frac{\partial L}{\partial P} = 2X^T XP - 2\Lambda P \]

And then set this derivative to zero

\[ X^T XP - \Lambda P = 0 \iff X^T XP = \Lambda P \]

Because \( \Lambda \) is diagonal, this is obviously an Eigen-decomposition problem, and this indicates that \( \Lambda \) is the matrix of Eigen values of the positive semi-definite matrix \( X^T X \) controlled from the main to the smallest and that P is the matrix of eigenvectors of \( X^T X \) associated to \( \Lambda \). Lastly, we find that the factor matrix as

\[ F = PA^{1/2} \]

The variance of the factors scores is equal to the eigenvalues

\[ F^T F = A^{1/2} P^T PA^{1/2} = \Lambda \]

Consider that the sum of the eigenvalues is equal to the trace of \( X^T X \), this shows that the first factor scores pull out as much of the variances of the original data as possible, and that the second factor scores pull out as much of the variance gone uncertain by the first factor, and so on for the remaining factors. By the way, the diagonal elements of the matrix \( A^{1/2} \) are the singular values of matrix X.
III. APPLICATION OF PEVD

A. BROADBAND SIGNAL SUBSPACE DECOMPOSITION

In this the extraction of foetal ECG signals from cutaneous electrode recordings is considered. Most existing methods cannot account for the broadband nature of the ECG signals, producing (FECG) foetal ECG estimates that are not sufficiently accurate. A novel way of addressing this problem: the broadband equivalent of (PCA) principal component analysis is applied through the use of an algorithm that generalizes the EVD to polynomial matrices; FECG extraction is achieved by way of an orthonormal projection of the data into the broadband foetal subspace, whereby the maternal ECG and noise components are suppressed. The algorithm is contrasted with conventional PCA and the classical multi-reference adaptive noise cancelling method. The three methods are applied to real multi-channel ECG recordings obtained from a pregnant woman.

i). EXTRACTION OF FOETAL ELECTROCARDIOGRAM (ECG)

In prenatal diagnosis of foetal heart conditions, the electrocardiogram (ECG) signal from the foetal heart is of minute value [13]. The foetal ECG (FECG) is an electric signal that can be measured in a non-invasive manner by applying cutaneous electrodes to the abdomen of an expectant mother. This practice leads to a pollution of the recorded signals with interference mainly from the maternal heartbeat. Moreover, the signal-to-noise ratio of the foetal heartbeat is, in general, significantly lower than that of the maternal heartbeat [14]. Accurate measurement of the FECG is also hampered by the existence of other forms of interference and noise, such as uterine electromyographic signals, maternal respiration, thermal noise from the electronic equipment, etc. This creates the need for signal processing techniques that can recover the FECG components from the corrupted recordings.

ii). TECHNIQUES USED IN THE EXTRACTION OF FECG

Much research effort has been devoted to the detection and extraction of FECG, using techniques such as, neural networks [15], fuzzy logic [16], IIR adaptive filtering combined with genetic algorithms [17], Widrow’s multi-reference adaptive noise cancelling (MRANC) method [18],[19] and blind signal separation (BSS) [16]-[19]. BSS applies higher order statistical methods, such as independent component analysis (ICA), and uses multiple simultaneous recordings in order to exploit the spatial diversity between the different electrodes. These algorithms typically exploit second order statistics to perform principal component analysis (PCA), which generates uncorrelated sequences. Then higher order statistics are exploited in order to complete the separation process. However, if the total power of the FECG signal across all the channels is significantly different from the interferers, then the PCA carries out most of the separation. This is the philosophy behind the important class of foetal extraction techniques in [18], [19], which are based on the singular-value decomposition (SVD), or Eigen value decomposition (EVD), [18]. A drawback of the basic SVD-based technique is that signal separation performance is dependent on the position of the electrodes, which is still a matter of heuristic rules and trial-and-error [18].

iii). SECOND ORDER SEQUENTIAL BEST ROTATION (SBR2) ALGORITHM

A principal limitation imposed on the PCA (and ICA) technique is the assumption that the mixing process is linear and instantaneous, which is expressed as multiplication by a single scalar mixing matrix. This does not take into account the possible spatiotemporal dynamics of the underlying acoustical processes, e.g., propagation of the acoustic waves from the various sources, such as the maternal and foetal hearts; and the broadband muscle, observation and quantization noises [19]. These effects can be modeled by convolutive mixing. Following convolutive mixing, it is necessary to impose decorrelation, not just at the same time instant for all signals, but over a suitably chosen range of relative time delays. This is referred to as strong decorrelation [20], and a matrix of suitably chosen filters is required to achieve it. In [14], a technique that gives an extension of the EVD to polynomial matrices is proposed, which can perform strong decorrelation, to a good approximation. It is called the second order sequential best rotation (SBR2) algorithm. This uses the SBR2 algorithm in the context of broadband subspace decomposition.
In this we consider the design of a linearly precoded MIMO transceiver based on (FB) filter bank modulation for transmission on broadband frequency discriminatory fading channels. The modulation FB is capable of lowering the channel dispersion at subchannel level. Nevertheless, the sub-channels experience some level of inter-symbol interference. Therefore, the pre-coder and the equalizer are designed exploiting the polynomial singular value decomposition (PSVD). In particular, consider two types of FB system. The first method represents maximal frequency restricted pulses and it is referred to as filtered multi tone (FMT) modulation, while the second uses more time confined pulses with rectangular impulse response, i.e., it corresponds to the conventional (OFDM) orthogonal frequency division multiplexing system. The act of the measured systems in terms of capacity over typical WLAN channels, showing that PSVD precoding with FMT can outperform the performance of precoded OFDM in the 2 by 2 antenna case particularly for moderate to low SNRs.

i). PSVD ALGORITHM

A precoding method has been proposed in [21]. It is based on the (SVD) singular value decomposition of (PSVD) polynomial matrices which it is also referred to as (BSVD) broadband SVD. This method is characterized by high computational complexity that is $O(L^3)$ where $L$ is the order of the channel, or equivalently the channel length in number of taps. This allows dropping the difficulty of the PSVD algorithm since it operates at sub-channel level which has a length shorter than that of the broadband channel.

ii). APPLICATION OF PEVD

The approach has been presented on a MIMO transceiver based on FB modulation systems and on the PSVD decomposition. In exacting, we have considered two types of FB which deploys either time confined prototype pulses (OFDM) or frequency confined prototype pulses (FMT). We have shown the performance in terms of capacity of the considered systems in typical WLAN channels. MIMO-FMT can afford higher capacity compared to MIMO-OFDM only in certain conditions, in particular when the power level of the noise at the receiver is higher than the power level of the interferences. In presence of high interference power, MIMO-OFDM is the best option because due to the CP it can cope with limit imposed by the interference.
i). APPROXIMATE EVD (AEVD) ALGORITHM

More emphasis has thus been on obtaining an approximate EVD (AEVD) using realizable PU functions, such as (FIR) finite impulse response PU systems [21, 22]. An AEVD algorithm for PH systems via successive degree-1 FIR PU transformations. We show how to choose the parameters of such a FIR PU system to make the ZODE of the resultant PH system none decreasing. As more transformations are applied, the PH system approximately becomes more diagonal. As our aim is to maximize the ZODE, the entire impulse response of the PH system need not be known.

FIG.4 CONVERGENCE OF AEVD TECHNIQUE

The algorithm can thus run in the time or frequency domain, unlike SBR2 which must operate in the time domain. Also, at each stage of our algorithm, the FIR PU system degree increases by 1. This is in contrast to SBR2, which has a variable degree increase at each step.

D. ROBUST BROADBAND ADAPTIVE BEAMFORMING

A novel technique for robust broadband (ABF) adaptive beam forming is proposed. The technique, referred to as (DW-PEVD) domain-weighted polynomial matrix eigenvalue decomposition, is founded on a basic paradigm shift from one of broadband noise cancellation to one of signal separation. It uses the (SBR2) second-order sequential best rotation algorithm to perform second order convolutive blind signal separation after applying a simple transformation to the data. The transformation is designed to exploit prior knowledge in the form of an estimated steering vector. The method is quite distinct from existing algorithms for robust broadband ABF and can offer improved performance in many cases.

i). BROADBAND ABF

Adaptive beam forming (ABF) conjunction with sensor arrays for the purposes of interference suppression in diverse fields such as communications, radar, sonar and seismology. Much effort has been devoted to beam forming for narrowband signals where very good results can be obtained [2]-[5]. The theory of narrowband arrays has been well established because of its duality with spectral estimation. However, much work remains to be done on broadband adaptive arrays where both spatial and temporal sampling is exploited. The two most popular array structures that have been studied for broadband ABF are the linearly constrained minimum variance beam former introduced by Frost in [6] and its alternative the (GSC) generalized side lobe canceller, proposed by Griffiths and Jim in [7]. Their implementation is based on the premise of adaptive noise cancellation. These beam formers are very sensitive to array imperfections, such as calibration errors, which cause a mismatch between the assumed steering vector and the actual one required for the (SOI) signal-of-interest. In the presence of errors, the beam formers tend to suppress the SOI as if it were an interference signal, and so their performance degrades significantly. Over recent years, effort has been devoted to developing techniques for robust broadband ABF that alleviate the effects of array imperfections [8]-[11].

ii). DECOMPOSITION TECHNIQUES

The use of a novel time-domain algorithm introduced in [12], called the (SBR2) algorithm, which imposes strong decorrelation [13] on a set of signals. The strong decorrelation is achieved using a multichannel all-pass filter, which is guaranteed to preserve the combined power of the signals at every frequency. The SBR2 algorithm effectively extends the symmetric (EVD) eigenvalue decomposition [11] from narrowband to broadband applications in signal processing. The potential benefit of the SBR2 algorithm to underwater acoustic signal processing is illustrated in the context of robust ABF. It is shown how broadband ABF may be carried out in a very robust manner by extending the new robust ABF technique of [11] to broadband ABF. The resulting algorithm is called the DW-PEVD technique. It is based on a shift of paradigm away from that of adaptive noise cancellation towards one of convolutive semi-blind signal separation.

(BSS) Blind signal separation algorithms typically exploit second order statistics to perform a (PCA) principal components analysis, which generates uncorrelated sequences. Then (HOS) higher order statistics are exploited to determine the "hidden" rotation matrix and so complete the separation process. BSS can usually separate the SOI
from the interference signals with minimal underlying assumptions, but these assumptions do not emphasize the SOI over the unwanted signals, nor can the algorithms utilize prior information. If the total power of the SOI across all the channels is significantly different from the interferers, then the PCA carries out most of the separation. In the same way, convolutive BSS is possible using just a PEVD stage provided the spectra of the signals are distinct. The DW-PEVD exploits this fact by using information about the SOI and the sensor array to modify the total power of the desired signal so it becomes distinct from that of the interferers.

iii). POLYNOMIAL MATRIX EIGEN VALUE DECOMPOSITION (PEVD)

It uses the second-order sequential best rotation (SBR2) algorithm to compute polynomial matrix eigenvalue decomposition (PEVD). It allows available information about the SOI to be included in a soft manner. This avoids the problem of cancelling the SOI due to inaccurate prior knowledge without completely disregarding this information. The algorithm essentially implements a form of conclusive, semi-blind, second order signal separation where the approximate prior knowledge is used to pre-emphasize the SOI. Although the technique we have introduced uses a scalar parameter to enhance the SOI, it can easily have used an appropriate filter, such as one that is temporarily matched to the SOI. This filter can then be used to design an appropriate space-time blocking matrix that further enhances the correlation of the SOI components across all the channels being fed into the PEVD stage. Evaluation of this beam former has shown that it gives improved performances.

![FIG. 5 PERFORMANCE MEASURE OF PEVD ALGORITHM](image)

The performance measure of PEVD in different applications like broadband signal sub space decomposition, MIMO communication, FIR-PU FB (filter bank), Broadband ABF etc., and the decomposition technique uses various algorithms such as SBR2, Eigen value decomposition, singular value decomposition. These entire algorithms have been shown to convergence. With the usage of the algorithms the polynomial Eigen values obtained from various applications attains strong decorrelation. Hardware implementation of these techniques can be achieved by means of FPGA architecture can be used as future work.

REFERENCE


