ANALYSIS OF A THREE UNIT COMPLEX SYSTEM WITH CORRELATED LIFETIMES

Rakesh Gupta, Madhu Mahi, Arti Tyagi

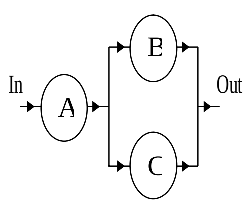
Abstract— The paper deals with profit function analysis of a three non-identical units namely, A B and C complex system model. The units are arranged in such a way that the system failure occurs if either unit-A or both the units B and C fail totally. A single repairman is always available with the system to repair the failed units in FCFS discipline. The failure time distribution of unit-A is taken exponential whereas time to failure of unit-B and unit-C are assumed to be correlated random variables having their joint distribution as Bi-variate exponential. The distributions of time to repair of unit B and C are taken exponential with different parameters whereas the distribution of time to repair of unit-A is taken arbitrary. Various performance measures of system effectiveness have been obtained by using regenerative point technique.

Index Terms— Transition probabilities, mean sojourn time, Bi-variate exponential distribution, MTSF, availability, expected busy period of repairman, expected numbers of repair and net expected profit.

1 Introduction

Due to the needs of Modern society, the systems are becoming complex day by day. Several researchers including [1, 2, 4] in the field of reliability have analyzed the complex and priority redundant system models under different sets of assumptions using supplementary variable and regenerative point techniques. In all these models it is assumed that the failure and repair times are uncorrelated random variables. Goel, Shrivastava and Gupta [3] introduced the concept of correlation between failure and repair times in two-unit cold standby system and thereafter some authors including [5, 6, 7] further extended the concept of correlation between failure and repair, lifetimes of units working in parallel and working and rest time of repairman by analyzing the two unit system models under different model formulations.

The aim of present chapter is to analyze a three unit complex system model using the concept of correlation between the life times. The three non-identical unit- A, B and C are arranged in such a way that the system failure occurs if either unit-A or both the units B and C fail totally. The configuration of the system model with units A, B and C is shown in figure.



The joint distribution of failure times of units B and C working in parallel configuration is taken to be bivariate exponential i.e. the life times of the units B and C are assumed to be correlated random variables. The failure time distribution of unit-A and repair time distributions of units B and C are taken exponentials with different parameters whereas repair time distribution of unit-A is taken arbitrary. Using regenerative point technique, the following economic measures of system effectiveness are obtained-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and Mean time to system failure.
iii. Point-wise and steady state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of the repairman in repair of the failed units during time interval (0, t).
v. Expected number of repairs of failed units during time interval (0, t).
vi. Net expected profit in the interval (0, t) and in steady-state under two different policies.

2. Model Description and Assumption

i) The system comprises of three non-identical units A, B and C. For successful operation of the system the unit A and at least one of the units B and C should function.
ii) System fails when either unit A or both the units B and C fail.
iii) A single repair facility is available to repair the failed units on first come first served basic i.e. the later failed unit waits for repair till the repair of the unit already in hand is completed.
iv) Failure time of unit-A is taken exponential whereas that of unit-B and unit-C are assumed to be correlated random variables having their joint distribution as bivariate exponential (B. V. E.) with density function as follows-

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right);$$
$$0 \le r < 1, \quad \alpha_1, \alpha_2, x_1, x_2 > 0$$

where,

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$$I_0 = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$$

v) The distributions of time to repair of unit B and C are taken exponential with different parameters whereas the distribution of time to repair of unit-A is taken arbitrary.

vi) Each repaired unit acts as good as new.

3. Notations and States of the System

a) Notations :

E : Set of regenerative states i.e. S_0 to S_7.

$X_i(i=1,2)$: Random variable representing the failure time of unit-B and unit-C respectively for i=1, 2.

$f(x_1, x_2)$: Joint pdf of (x_1, x_2)

$$= \alpha_1 \alpha_2 (1-r) e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0\left(2\sqrt{\alpha_1 \alpha_2 r x_1 x_2}\right)$$

$$0 \le r < 1, \qquad \alpha_1, \alpha_2, x_1, x_2 > 0$$

$$I_0\left(2\sqrt{\alpha_1 \alpha_2 r x_1 x_2}\right) = \sum_{j=0}^{\infty} \frac{(\alpha_1 \alpha_2 r x_1 x_2)^j}{(j!)^2}$$

$f_i(x)$: Marginal p.d.f. of X_i

$$= \alpha_i (1-r) e^{-\alpha_i (1-r) x}$$

$k_1(u \mid X_2 = x)$: Conditional p.d.f. of X_1 given $X_2 = x$

$$= \alpha_1 e^{-\alpha_1 u - \alpha_2 r x} I_0\left(2\sqrt{\alpha_1 \alpha_2 r x u}\right)$$

$k_2(u \mid X_1 = x)$: Conditional p.d.f. of X_2 given $X_1 = x$

$$= \alpha_2 e^{-\alpha_2 u - \alpha_1 r x} I_0\left(2\sqrt{\alpha_1 \alpha_2 r x u}\right)$$

$K_1(u \mid X_2 = x)$: Conditional C.d.f. of X_1 given $X_2 = x$.

$K_2(u \mid X_1 = x)$: Conditional C.d.f. of X_2 given $X_1 = x$.

λ : Constant failure rate of unit-A.

$G(\cdot)$: C.d.f. of repair time of unit-A.

β_i : Constant repair rate of a failed unit-B and unit-C respectively for i=1,2.

b) Symbols for the states of the systems:

A_0, A_g : Unit-A is operative / good operable condition.

B_0, B_g : Unit-B is operative / good operable condition.

C_0, C_g : Unit-C is operative / good operable condition.

A_r, A_w : Unit-A is under repair / waits for repair.

B_r, B_w : Unit-B is under repair / waits for repair.

C_r, C_w : Unit-C is under repair / waits for repair.

Using the above symbols and keeping in view the assumptions stated in section-2, the possible states of the system are shown in transition diagram (Fig. 1). The epochs of transitions into all the states are regenerative.

4. Transition Probabilities

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \ge 0\}$ constitutes a continuous parametric Markov-Chain with state space E. First we obtain the direct conditional and unconditional transition probabilities in terms of

$$\alpha_1' = \frac{\alpha_1}{\lambda + \alpha_1 + \beta_2}, \qquad \alpha_2' = \frac{\alpha_2}{\lambda + \beta_1 + \alpha_2}$$

as follows-

$$p_{01} = \int \alpha_1 (1-r_1) e^{-\{\lambda + \alpha_1(1-r) + \alpha_2(1-r)\}u} du$$

$$= \frac{\alpha_1 (1-r_1)}{\lambda + \alpha_1 (1-r) + \alpha_2 (1-r)}$$

Similarly,

$$p_{02} = \frac{\alpha_2 (1-r_2)}{\lambda + \alpha_1 (1-r) + \alpha_2 (1-r)}$$

$$p_{03} = \frac{\lambda}{\lambda + \alpha_1 (1-r) + \alpha_2 (1-r)}$$

$$p_{30} = p_{42} = p_{51} = p_{63} = p_{73} = 1 \qquad (1)$$

$$p_{10|X} = \beta_1 \int e^{-(\lambda+\beta_1)u} \bar{K}_2(u \mid x) du$$

$$= \beta_1 \int e^{-(\lambda+\beta_1)u} \left(\int_u^{\infty} \alpha_2 e^{-(\alpha_2 z + \alpha_1 r x)} \sum_{j=0}^{\infty} \frac{(\alpha_1 \alpha_2 r x z)^j}{(j!)^2} dz \right) du$$

$$= \frac{\beta_1}{\lambda + \beta_1} \left\{ 1 - \alpha_2' e^{-\alpha_1 r x (1-\alpha_2')} \right\}$$

Similarly,

$$p_{14|X} = \alpha_2' e^{-\alpha_1 r x (1-\alpha_2')}$$

$$p_{16|X} = \frac{\lambda}{\lambda + \beta_1} \left\{ 1 - \alpha_2' e^{-\alpha_1 r x (1-\alpha_2')} \right\}$$

$$p_{20|X} = \frac{\beta_2}{\lambda + \beta_2} \left\{ 1 - \alpha_1' e^{-\alpha_2 r x (1-\alpha_1')} \right\}$$

$$p_{25|X} = \alpha_1' e^{-\alpha_2 r x (1-\alpha_1')}$$

$$p_{27|X} = \frac{\lambda}{\lambda + \beta_2} \left\{ 1 - \alpha_1' e^{-\alpha_2 r x (1-\alpha_1')} \right\}$$

We observe that

$$p_{01} + p_{02} + p_{03} = 1$$

$$p_{10|X} + p_{14|X} + p_{16|X} = 1$$

$$p_{20|X} + p_{25|X} + p_{27X} = 1 \qquad (2\text{-}4)$$

ISSN 2229-5518

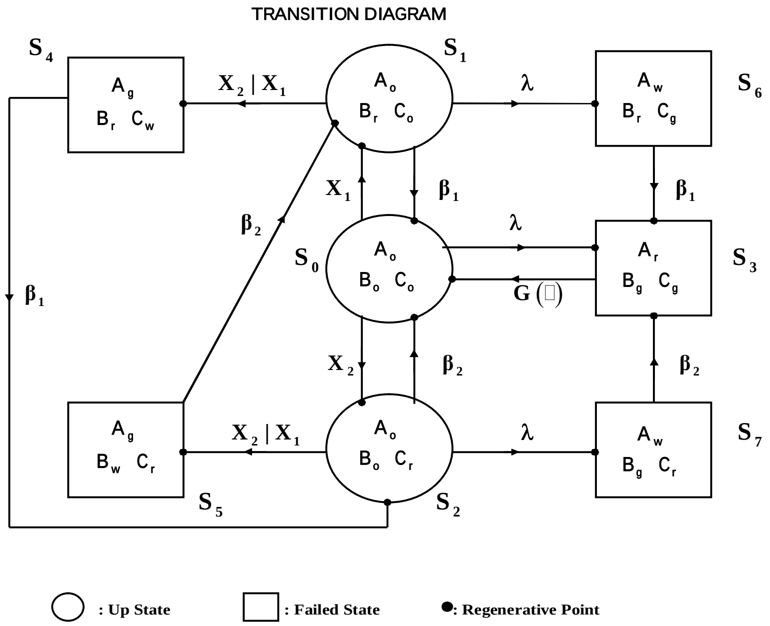


Fig.1

The unconditional transition probabilities with correlation coefficient can be obtained as follows:

$$p_{10} = \int p_{10|X} f_1(x)\,dx$$
$$= \frac{\beta_1}{\lambda+\beta_1}\int\left\{1-\alpha_2' e^{-\alpha_1 r x(1-\alpha_2')}\right\}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx$$
$$= \frac{\beta_1}{\lambda+\beta_1}\left[1-\frac{\alpha_2'(1-r)}{1-r\alpha_2'}\right]$$

Similarly,

$$p_{14} = \frac{\alpha_2'(1-r)}{1-r\alpha_2'}$$
$$p_{16} = \frac{\beta_1}{\lambda+\beta_1}\left[1-\frac{\alpha_2'(1-r)}{1-r\alpha_2'}\right]$$
$$p_{20} = \frac{\beta_2}{\lambda+\beta_2}\left[1-\frac{\alpha_1'(1-r)}{1-r\alpha_1'}\right]$$
$$p_{25} = \frac{\alpha_1'(1-r)}{1-r\alpha_1'}$$
$$p_{27} = \frac{\lambda}{\lambda+\beta_2}\left[1-\frac{\alpha_1'(1-r)}{1-r\alpha_1'}\right]$$

It can be easily verified that,

$$p_{10}+p_{14}+p_{16} = 1$$
$$p_{20}+p_{25}+p_{27} = 1 \quad (5\text{-}6)$$

5. Mean Sojourn Times

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P[U_i > t]\,dt$$

Therefore, its values for various regenerative states are as follows.

$$\psi_0 = \int e^{-(\alpha_1+\alpha_2)(1-r)t}dt = \frac{1}{(\alpha_1+\alpha_2)(1-r)} \quad (7)$$

Similarly,

$$\psi_{1|X} = \int e^{-(\lambda+\beta_1)t}\left(\int_t^\infty \alpha_2 e^{-(\alpha_2 u+\alpha_1 r x)}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 r x u)^j}{(j!)^2}du\right)dt$$
$$= \frac{1}{\lambda+\beta_1}\left\{1-\alpha_2' e^{-\alpha_1 r x(1-\alpha_2')}\right\}$$

so that,

$$\psi_1 = \int \psi_{1|x} f_1(x)\,dx$$
$$= \frac{1}{\lambda+\beta_1}\int\left\{1-\alpha_2' e^{-\alpha_1 r x(1-\alpha_2')}\right\}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx$$
$$= \frac{1}{\lambda+\beta_1+(1-r)\alpha_2} \quad (8)$$

$$\psi_{2|X} = \frac{1}{\lambda+\beta_2}\left\{1-\alpha_1' e^{-\alpha_2 r x(1-\alpha_1')}\right\}$$

so that

$$\psi_2 = \frac{1}{\lambda+\beta_2+(1-r)\alpha_1} \quad (9)$$

$$\psi_3 = \frac{1}{\mu} = m \text{ (say)}, \qquad \psi_4 = \psi_6 = \frac{1}{\beta_1}$$

$$\psi_5 = \psi_7 = \frac{1}{\beta_2} \quad (10\text{-}12)$$

6 Analysis of Characteristics

a) Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_3 to S_7 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*} \quad (13)$$

Where, $Z_i^*(i = 0,1,2)$ are the L. T. of

$$Z_0(t) = e^{-\{\lambda + (\alpha_1+\alpha_2)(1-r)\}t}$$
$$Z_1(t) = e^{-(\lambda+\alpha_1)t} \int K_2(t \mid x) g_1(x) dx$$
$$Z_2(t) = e^{-(\lambda+\alpha_2)t} \int K_1(t \mid x) g_2(x) dx$$

Taking the Inverse Laplace Transform of (13), one can get the reliability of the system when it starts from state S_0.

The MTSF is given by

$$E(T_0) = \int R_0(t) dt = \lim_{s\to 0} R_0^*(s)$$
$$= \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \quad (14)$$

b) Availability Analysis

Let $A_i(t)$ be the probability that the system is operative at epoch t when it initially starts operation from regenerative state $S_i \in E$. Using regenerative point technique and the tools of L. T., one can obtain the value of $A_0(t)$ in terms of its L.T. as follows-

$$A_0^*(s) = N_1(s)/D_1(s) \quad (15)$$

Where,

$$N_1(s) = \left(1 - q_{14}^* q_{42}^* q_{25}^* q_{51}^*\right) Z_0^* + \left(q_{01}^* + q_{02}^* q_{25}^* q_{51}^*\right) Z_1^* + \left(q_{02}^* + q_{01}^* q_{14}^* q_{42}^*\right) Z_2^*$$

$$D_1(s) = \left(1 - q_{14}^* q_{42}^* q_{25}^* q_{51}^*\right)\left(1 - q_{03}^* q_{30}^*\right) - \left(q_{01}^* + q_{02}^* q_{25}^* q_{51}^*\right)\left(q_{10}^* + q_{16}^* q_{63}^* q_{30}^*\right) - \left(q_{02}^* + q_{01}^* q_{14}^* q_{42}^*\right)\left(q_{20}^* + q_{27}^* q_{73}^* q_{30}^*\right)$$

The steady state availability of the system is given by-

$$A_0 = \lim_{s\to 0} s A_0^*(s) = N_1/D_1' \quad (16)$$

Where,

$$N_1 = (1 - p_{14}p_{25})\psi_0 + (p_{01} + p_{02}p_{25})\psi_1 + (p_{02} + p_{01}p_{14})\psi_2$$

$$D_1' = (1 - p_{14}p_{25})(\psi_0 + p_{03}\psi_3) + (p_{01} + p_{02}p_{25})(\psi_1 + p_{16}\psi_3 + p_{14}\psi_4 + p_{16}\psi_6) + (p_{02} + p_{01}p_{14})(\psi_2 + p_{27}\psi_3 + p_{25}\psi_5 + p_{27}\psi_7)$$

The expected up time of the system during time interval (0, t) is given by-

$$\mu_{up}(t) = \int_0^t A_0(u) du$$

So that

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s} \quad (13) \quad (17)$$

c) Busy Period Analysis :-

Let $B_i(t)$ be the probability that the repairman is busy in the repair of the failed units at epoch t, when initially the system starts operation from state $S_i \in E$. Using regenerative point technique and the tools of L. T., one can obtain the value of $B_0(t)$ in terms of its L.T. as follows-

$$B_0^*(s) = N_2(s)/D_1(s) \quad (18)$$

Where,

$$N_2(s) = \left(1 - q_{14}^* q_{42}^* q_{25}^* q_{51}^*\right) q_{03}^* Z_3^* + \left(q_{01}^* + q_{02}^* q_{25}^* q_{51}^*\right)\left(q_{14}^* Z_4^* + q_{16}^* q_{63}^* Z_3^* + q_{16}^* Z_6^*\right) + \left(q_{02}^* + q_{01}^* q_{14}^* q_{42}^*\right)\left(q_{25}^* Z_5^* + q_{27}^* q_{73}^* Z_3^* + q_{27}^* Z_7^*\right)$$

and $D_1(s)$ is same as define in the expression (16) of section 6(b).

Where, $Z_3^*, Z_4^*, Z_5^*, Z_6^*$ and Z_7^* are the L. T. of

$$Z_3(t) = \bar{G}(t), \qquad Z_4(t) = Z_6(t) = e^{-\alpha_1 t}$$
$$Z_5(t) = Z_7(t) = e^{-\alpha_2 t}$$

The steady state result for the above probability is given by-

$$B_0 = \lim_{s\to 0} s B_0^*(s) = N_2/D_1' \quad (19)$$

Where,

$$N_2 = (1 - p_{14}p_{25})p_{03}\psi_0 + (p_{01} + p_{02}p_{25})(p_{16}\psi_3 + p_{14}\psi_4 + p_{16}\psi_6) + (p_{02} + p_{01}p_{14})(p_{27}\psi_3 + p_{25}\psi_5 + p_{27}\psi_7)$$

and D_1' is same as expressed in the expression (14) of section 6(b).

The expected busy period of repairman in repair of failed units during time interval (0, t) is given by-

$$\mu_b(t) = \int_0^t B_0(u) du$$

So that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s} \tag{20}$$

d) Expected number of Repairs

Let $V(t)$ be the expected number of repairs during time interval (0, t) when system initially starts from regenerative state S_i. By using the simple probabilistic arguments in the theory of regenerative point technique and applying the tools of L. S. T., one can obtain the values of $V_0(t)$ in terms of its L. S. T. i.e. $\tilde{V}_0(s)$ as follows-

$$\tilde{V}_0(s) = \frac{N_3(s)}{D_2(s)} \tag{21}$$

Where

$$N_3(s) = \left(1-\tilde{Q}_{14}\tilde{Q}_{42}\tilde{Q}_{25}\tilde{Q}_{51}\right)\tilde{Q}_{30}\tilde{Q}_{03} + \left(\tilde{Q}_{01}+\tilde{Q}_{02}\tilde{Q}_{25}\tilde{Q}_{51}\right)\left(\tilde{Q}_{14}\tilde{Q}_{42}+\tilde{Q}_{16}\tilde{Q}_{63}\tilde{Q}_{30}+\tilde{Q}_{16}\tilde{Q}_{63}\right) + \left(\tilde{Q}_{02}+\tilde{Q}_{01}\tilde{Q}_{14}\tilde{Q}_{42}\right)\left(\tilde{Q}_{25}\tilde{Q}_{51}+\tilde{Q}_{27}\tilde{Q}_{73}\tilde{Q}_{30}+\tilde{Q}_{27}\tilde{Q}_{73}\right)$$

and $D_2(s)$ can be obtain from $D_1(s)$ on replacing q_{ij}^* by $\tilde{Q}_{ij}$

Now the expected number of repairs per unit time in steady state is given by-

$$V_0 = \lim_{t\to\infty} V_0(t)/t = \lim_{s\to 0} s\tilde{V}_0(s) = N_3/D_1' \tag{22}$$

Where,

$$N_3 = (1-p_{14}p_{25})p_{03} + (p_{01}+p_{02}p_{25})(p_{02}+p_{01}p_{14})$$

and D_1' is same as expressed in the expression (16) of section 6(b).

7. Cost Benefit Analysis

We are now in the position to obtain the two profit functions during (0, t) in two different policies by considering the characteristics obtained in earlier sections-

Let us suppose

K_0 =revenue per unit time by the system when it is operative.

K_1 =cost per-unit time when repairman is busy in repairing a failed unit.

K_2 = per-unit time repair cost.

Now, the net expected profit incurred in time interval (0, t) in two policies are as follows

$$P_1(t) = K_0\mu_{up}(t) - K_1\mu_b(t)$$
$$P_2(t) = K_0\mu_{up}(t) - K_2V_0(t) \tag{23-24}$$

The above expected profits per unit time in steady state are respectively given by

$$P_1 = \lim_{t\to\infty} P_1(t)/t = \lim_{s\to 0} s^2P_1^*(s) = K_0A_0 - K_1B_0$$
$$P_2 = \lim_{t\to\infty} P_2(t)/t = \lim_{s\to 0} s^2P_2^*(s) = K_0A_0 - K_2V_0$$

(25-26)

8 Graphical Represntation

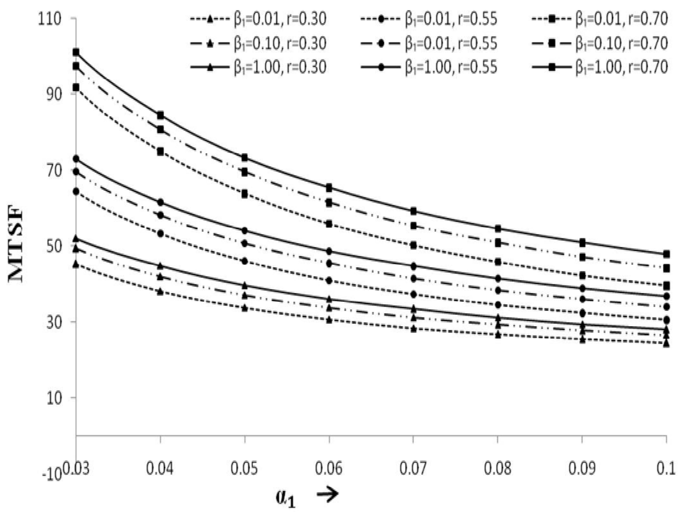
The curves for MTSF and profit functions are drawn for different values of parameters. Fig. 2 depicts the variations in MTSF with respect to failure parameter (α_1) of unit-B when unit-C is good for three different values (0.30, 0.55 and 0.70) of correlation coefficient (r) and three different values (0.010, 0.015 and 0.020) of repair parameter (β_1) of unit-B when other parameters are kept fixed as $\alpha_2 = 0.01$, $\beta_2 = 0.5$, $\lambda = 0.05$ and $\mu = 1.5$. We may clearly observe from Fig. 2 that MTSF decrease uniformly as the values of α_1 increase. It also reveals that MTSF increases with the increase in r and β_1.

Similarly, Fig. 3 reveals the variations in profit (P_1) with respect to α_1 for three different values of r (0.30, 0.55 and 0.70) and three different values of β_1 (0.010, 0.015 and 0.020) when the values of other parameters are kept fix as $\alpha_2 = 0.10$, $\beta_2 = 0.5$, $\lambda = 0.05$, $\mu = 1.5$, $K_0 = 50$ and $K_1 = 60$. In this figure the curves represented by dot reveal that the system is profitable only if α_1 is less than 0.0113, 0.0280 and 0.0555 respectively for $r = 0.30$, 0.55 and 0.70 when β_1 is taken as 0.010. From the curves denoted by dash we conclude that the system is profitable only if α_1 is less than 0.0215, 0.0472 and 0.0880 respectively for $r = 0.30$, 0.55 and 0.70 for fixed value of $\beta_1 = 0.015$ and from smooth curves we conclude that the system is profitable only if α_1 is less than 0.0113, 0.0280 and 0.0555 respectively for $r = 0.30$, 0.55 and 0.70 for fixed value of $\beta_1 = 0.020$.

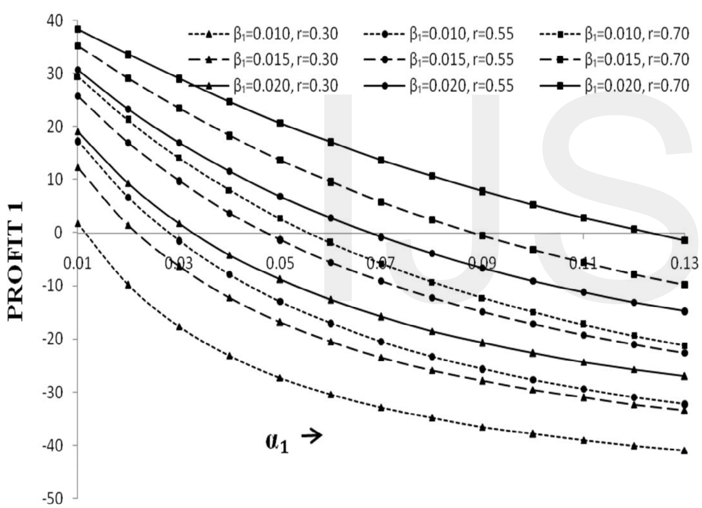
Similarly, fig. 4 shows the variations in profit (P_2) with respect to α_1 for three different values of r (0.30, 0.55 and 0.70) and three different values of β_1 (0.010, 0.015 and 0.020) when the values of other parameters are kept fix as $\alpha_2 = 0.10$, $\beta_2 = 0.5$, $\lambda = 0.05$, $\mu = 1.5$, $K_0 = 50$ and $K_2 = 300$. From fig it is clearly observed that profit-2 is decrease uniformly as the values of α_1 increase. It also reveals that MTSF increases with the increase in r and β_1.

ISSN 2229-5518

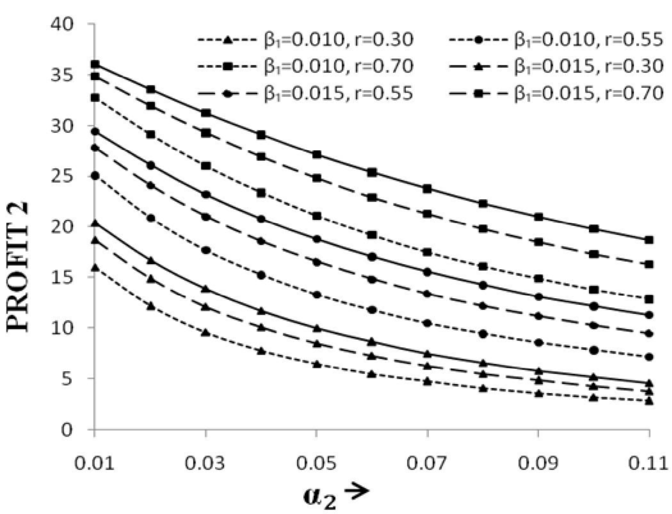
Behavior of MTSF with respect to α_1, β_1 and r



Behavior of Profit (P_1) with respect to α_1, β_1 and r



Behavior of Profit (P_2) with respect to α_1, β_1 and r



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