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A Systematic Study on Applications of Graph Theory in Image Processing With a Focus on Image Segmentation

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Abstract— Graph theory has an important role to play in computer science in particular in image processing. Image processing is the process of analysing the digital image by extracting its features and there by classifying it. The output of image processing is an image or set of features of it. The steps of image processing are pre-processing, image segmentation, feature extraction and finally classification. In this paper we have presented systematically how the graph theory is useful in image segmentation and its applications. Also we have used number of graph theoretical concepts which are used in segmenting the digital image.

Index Terms— Graph theory, image processing, cut-vertex, bi-partite graph, graph cut, normalized cut, MST.

1 Introduction

Image processing is a fundamental process in computer vision. The steps of image processing are shown in the following figure.

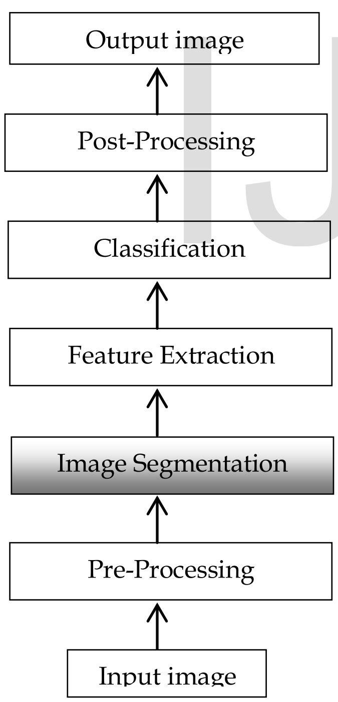

Figure 1. Steps in image processing.

Firstly the input digital image is pre-processed i.e. the image consisting of noise and other disturbances are removed by using filtering, noise removal algorithms. Then the resultant image is segmented using image segmentation algorithm which is the main focus of our study [1, 5]. This is most difficult step in image processing. Then features of that image are extracted. Finally using these features the image classification is done. The applications of image processing includes face recognition, Optical Character Recognition (OCR), biometric, finger print recognition, satellite image detection, medical image analysis and much more. However we have focussed on image segmentation in this paper using graph theoretical methods. Section 2 contains the terms and definitions of graph theory that are used in image processing. Implementation of graph theoretical concepts for image segmentation is explained in section 3. Section 4 contains some of the experimental results using image segmentation. And finally we have conclusions in section 5.

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2 Related Terms and Definitions

Graph: A graph is ordered pair of vertex set and edge set and is denoted by the symbol $G = (V, E)$. Where $V = \{v_1, v_2, v_3, \ldots v_n\}$ is set of vertices and $E = \{e_1, e_2, \ldots.. e_n\}$ is set of edges where edge is a line or curve connecting any two vertices of a graph G.

Cut-vertex: It is an edge of a graph in which removal of it increases the number of components of graph.

Bi-partite graph: It is a resultant of given graph such that the given vertex set V is divided into two subsets V_1 and V_2 so that if we consider any edge of resultant graph, one vertex is from the set V_1 and another is from the set V_2.

Minimal Spanning Tree (MST): The summation of graph vertices is equal to the minimal sum on the defined edge weights, and the graph-partition is obtained by removing edges to form different sub-graphs. Where edge weights are the weights assigned on the edges of given graph.

3 Implementation

Graph cut: In graph theoretic definition, the degree of dissimilarity among both components can be calculated in the form of

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a graph cut. A cut is related to a set of edges by which the graph G will be divided into two disjoint sets A and B. As a consequence, the segmentation of an image can be understood in form of graph cuts, and the cut value is usually defined as:

$$Cut = \sum_{u \in A, v \in B} w(u, v) \quad (1)$$

where u and v mention to the vertices in the two different components. In image segmentation, noise and other ambiguities bring doubts into the accepting of image content. The exact solution to image segmentation is hard to obtain. So, it is more suitable to solve this problem with optimization methods. The optimization-based approach expresses the delinquent as a minimization of some established criterion, whereas one can find an exact or approximate answer to the unique undefined pictorial problem. In this case, the optimal bipartitioning of a graph can be taken as the unique that reduces the cut value in Eq. (1). In a large amount of literature, image segmentation is also formulated as a labelling problem, where a set of labels L is assigned to a set of sites in S. In two-class segmentation, for example, the problem can be described as assigning a label f_i from the set $L = \{object, background\}$to site $i \in S$ where the elements in S are the image pixels or regions. Labeling can be performed separately from image partitioning, while they achieve the same effect on image segmentation. We will see in this survey that many methods perform both partitioning and labelling simultaneously. An example to illustrate the relationship between graph cuts and the corresponding vertex labelling is given in Fig. 1, where a graph is segmented by two cuts and thus has 3 labels in the final segmentation.

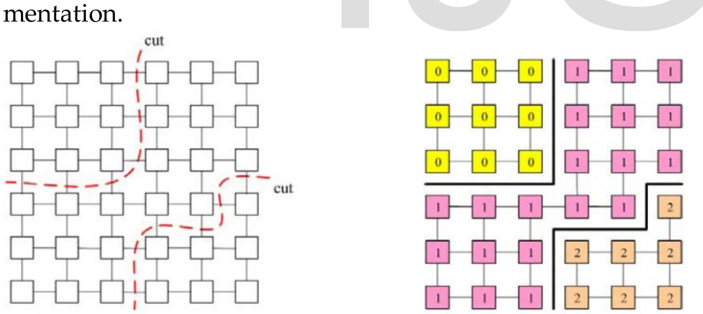

Fig 2. (a) Graph cuts (b) labeling

Minimal spanning tree (MST) based methods

The minimal spanning tree (MST) (also called shortest spanning tree) is an important concept in graph theory. A spanning tree T of a graph G is a tree such that $T = (V, E')$, where $E' \subseteq E$. A graph may have several different spanning trees. The MST is then a spanning tree with the smallest weights among all spanning trees [3]. The algorithms for computing the MST can be found in [34-36]. For instance, in Prim's algorithm [36], the MST is constructed by iteratively adding the frontier edge of the smallest edge-weight. The procedure is in a avaricious style and runs in polynomial time. MST based segmentation methods are essentially connected to the graph based gathering. The general study of graph clustering can be dated back to 1970s or earlier. Now graph-based gatherings, the data to be gathered are represented by an undirected adjacency graph. To represent the empathy, edges with sure weights are defined between two vertices if they are neighbours according to a given neighbourhood system. Clustering is then achieved by removing edges of the graph to form mutually exclusive sub-graphs. The clustering process usually emphasizes the importance of the gestalt principles of similarity or proximity in the graph vertices.

An advanced work of MST based algorithm proposed in [40] makes use of both the differences across the both sub-graphs and the variances inside a sub-graph. The segmentation is performed in conjunction with a region unification process and produces outcomes that satisfy some global properties. The key of this algorithm is adaptive thresholding. In contrast to solo linkage gathering which uses a constant K to set the threshold, the threshold here is a variable and is well-defined on the dimensions of collections. It allows two components to be merged if the linkage between them is smaller than the biggest edge in either of the modules MST plus this threshold. The formal definition of the merging criterion is given as below:

$$|e_t| < min\left(Int(C_1) + \frac{k}{|C_1|}, +Int(C_2)\frac{k}{|C_2|}\right) \quad (2)$$

where K is a constant, $|C_1|$ and $|C_2|$are the dimensions of components C_1 and C_2, respectively. $Int(C)$ is the largest edge weight in the MST of C. $|e_t|$ is the edge with the minimum weight which connects $|C_1|$ and $|C_2|$. From Eq. (2), we can see that the algorithm is sensitive to edges in suave parts and fewer sensitive to areas with high variability. In Fig. 2, we present the segmentation outcomes found with this algorithm. Both images contain regions with large variations or different levels of details, however segmentation outcomes preserve most of the perceptually important structures without any bias on the size of regions.

Normalized cut

The minimal cut criterion has a bias en route for finding small components. To overcome this problem Normalized cut concept is introduced [4].

Definition of Normalized cut

Another way of computing the degree of dissimilarity among the two components is by means of defining bi-partite graph. It is defined as the original vertex set V is divided into two subsets V_1 and V_2 , if you consider every edge of the bi-partite graph, it is joined by the two vertices such that $v_1 \in V_1$ and $v_2 \in V_2$ i.e. one vertex from the set V_1 and another from the set V_2 where both V_1 and V_2 are the subsets of the set V. This way of interpretation in the form of normalized cut and is defined as:

$$Ncut(A, B) = \frac{cut(A,B)}{vol(A)} + \frac{cut(A,B)}{vol(B)} \quad (3)$$

Where $vol(A) = \sum_{v_i \in A, v_j \in V} w(v_i, v_j)$ and

$vol(B) = \sum_{v_i \in B, v_j \in V} w(v_i, v_j)$

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4 Experimental Results

Fig. 3(a) and Fig 3(b): Segmentation of images using Minimal Spanning Tree.

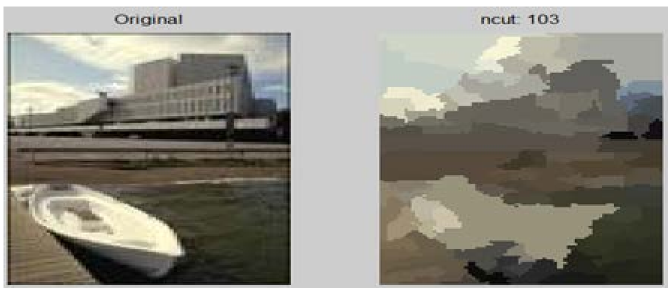

Fig 4(a)

Fig 4(b) Segmentation of images using normalized cut.

5 Conclusions

In this paper we have presented the systematic study of some of the graph theoretical concepts and their applications in image segmentation. Cut-vertex is used implement graph cut, bipartite graph is used to implement normalized cut. Minimal spanning tree (MST) is also used to segment the digital image. This study will definitely useful for those researchers who want to work on image segmentation. The results found using MST and normalized cut are very encouraging.

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