A NEW CLASS OF GRACEFULL TREETREES

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Abstract: The gracefulfulness of Tp-tree of size (5n, 5n-1) is obtained

Introduction:
Most graph labeling methods trace their origin to one introduced by Rosa [2] or one given Graham and Sloane [1]. Rosa defined a function f, a β-valuation of a graph with q edges if f is an injective map from the vertices of G to the set {0, 1, 2 ,…..q} such that when each edge xy is assigned the label \(|f(x)-f(y)|\), the resulting edge labels are distinct.


A. Solairaju and others [5,6,7,8,9] proved the results that (1) the Gracefulness of a spanning tree of the graph of Cartesian product of \(P_m\) and \(C_n\), was obtained (2) the Gracefulness of a spanning tree of the graph of cartesian product of \(S_m\) and \(S_n\), was obtained (3) edge-odd Gracefulness of a spanning tree of Cartesian product of \(P_2\) and \(C_n\) was obtained (4) Even-edge Gracefulness of the Graphs was obtained (5) ladder \(P_2 \times P_n\) is even-edge graceful, and (6) the even-edge gracefulness of \(P_n \circ nC_5\) is obtained.

Section I: Preliminaries

Definition 1.1: Let \(G = (V,E)\) be a simple graph with p vertices and q edges.

A map \(f : V(G) \to \{0,1,2,\ldots,q\}\) is called a graceful labeling if

(i) \(f\) is one – to – one
(ii) The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends.
A graph having a graceful labeling is called a graceful graph.

**Example 1.1:** The circuit $C_4$ is a graceful graph as follows:

**Definition 2.1:** $n \Delta P_5$ is a tree, becoming a path by moving edges between vertices of degree 3 defined in the following manner only. That is, it is a $T_p$-tree obtained from $n$ copies of $P_5$, and connected acyclic in the following manner.

**Section – II:** $T_p-(5n, 5n-1)$ tree
Main theorem 2.2: The connected graph $T_p$-tree with $p=5n$ and $q=5n-1$ is graceful.

Define $f: v(G) \rightarrow \{0,1,2,\ldots,q\}$

Proof: Due to definition (2.1), $T_p-(5n,5n-1)$ is a connected graph (see figures 1 and 2) according as $n$ is odd or even.

Case (1): $n$ is odd.

The labelings of vertices and edges for $T_p-(5n,5n-1)$ (Figure 1) are as follows:

Figure 1 (n is odd)

Figure 2 (n is even)
Define $f: E(G) \rightarrow \{1, 2, 3, \ldots, q\}$ by $f_e(u, v) = |f(u) - f(v)|$, $u, v \in E(G)$.

Hence, the bisection maps $f$ for vertices and $f_e$ for edges in $T_p - (5n, 5n-1)$ satisfies all condition of graceful labeling. Thus, $T_p - (5n, 5n-1)$ is a graceful if $n$ is odd.

**Case (2): $n$ is even.**

The labeling of vertices and edges for $T_p - (5n, 5n-1)$ (Figure 2) are as follows:

**Define $f: V(G) \rightarrow \{0, 1, 2, \ldots, q\}$**

By

\[
\begin{align*}
    f_{\{i\}} &= \frac{i - 1}{2}, & i &= 2, 4, 6, \ldots, 5n \\
    f_{\{i\}} &= q - \left(\frac{i - 1}{2}\right), & i &= 3, 5, 7, \ldots, 5n - 1
\end{align*}
\]
Define $f_+ : E(G) \rightarrow \{1,2,3..q\}$ by $f_+ (u,v) =$

$|f(u)-(v)|, \forall u,v \in E(G)$.

Hence, the bisection maps $f$ for vertices and $f_+$ for edges in $T_p - (5n,5n-1)$ satisfies all the conditions of graceful labeling. Thus, $T_p - (5n,5n-1)$ is a graceful if $n$ is even.

**Corollary 2.3:** $T_p - (5n,5n-1)$ is a graceful tree

**Example 2.1:**

The graph $5 \Delta P_5$ is a graceful graph.

**Example 2.2:**

The graph $6 \Delta P_5$ is a graceful graph.

**References:**


5. A. Solairaju and P. Sarangapani, even-edge gracefulness of $P_n \circ nC_5$, Preprint (Accepted for publication in Serials Publishers, New Delhi).


